Phase Structure of the $SU(3)$ Gauge-Higgs System. II

--- Adjoint Higgs ---

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(Received February 21, 1985)

The phase structure of the $SU(3)$ adjoint Higgs model is studied by a Monte Carlo simulation. We report the results of the simulation carried out on a four-dimensional euclidean lattice of size $3^4$ and $6^3 \times 3$. We find clear signals of the transition from a symmetric phase to broken ones. At finite temperature, the broken Higgs phase consists of three parts, i.e., the $SU(2) \times U(1)$-confined phase, the $U(1)$-Coulomb and $SU(2)$-confined normal Higgs phase, and the finite temperature Higgs phase with $SU(2) \times U(1)$ Coulomb interactions. We conjecture that the last one will disappear at zero temperature. The symmetric phase is divided into two by a deconfining transition. The deconfined symmetric phase will disappear at zero temperature.

§1. Introduction

In our previous report,\(^1\) the system of $SU(3)$ gauge fields interacting with scalar matter fields (Higgs scalar) in the fundamental representation has been analysed by a Monte Carlo simulation. The observed phase structure of the system is very similar to that of $SU(2)$ model with Higgs fields.\(^2\) Study of $SU(N)$ gauge model with adjoint Higgs fields is of interest in unified theories and also in cosmology. In most of the existing unified models, such as the Georgi-Glashow model,\(^3\) the symmetry breaking by a Higgs scalar in the adjoint representation has been assumed. In such theories, the phase transition from a symmetric to a Higgs phase plays an important role in the inflationary universe scenario.\(^4\) Therefore, it is worthwhile to clarify the phase structure of various systems for the model building. In this paper, we discuss the phase structure of the $SU(3)$ adjoint Higgs theory on the four-dimensional euclidean lattice.

Several Monte Carlo studies of the $SU(2)$ Higgs model have been made,\(^5,6\) in which the norm of Higgs field is fixed. In this case, the symmetry in the broken phase is uniquely $U(1)$, so that the observed phase structure is rather simple. The $SU(3)$ adjoint Higgs theory has been investigated for the case of the unitary Higgs fields.\(^7\) The symmetry of the broken phase of this model is very simple and it is not satisfactory from the viewpoint of the model building. The model with the general $\lambda \phi^4$ Lagrangian without a constraint on the Higgs fields has also been investigated.\(^8\) The finite temperature phase structure, however, has not been studied. In this paper, we analyse the model with one adjoint Higgs scalar at finite temperature as well as at zero temperature. In our analysis the norm of Higgs fields is kept fixed as in the $SU(2)$ case. The only reason for the norm fixing is the economy of the computer time in a Monte Carlo simulation. Under this

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condition we can simulate a wider range of parameter space. The Higgs interaction causes the spontaneous symmetry breaking. Possible residual symmetries in the broken phase for the present model are $SU(2) \times U(1)$ and $U(1) \times U(1)$. From the naive arguments on the measure of the group space, it seems that the breaking to the direction of $SU(2) \times U(1)$ dominates. It will be verified by careful analysis. In fact, we will find clear signals of the deconfining transition of $SU(2)$ and of $U(1)$ in the broken Higgs phase by measuring new quantities.

It has been known that the $SU(3)$ pure gauge model has a first-order deconfining transition.¹⁹ Such a transition must occur also in our Higgs model. Study of its behaviour is important to understand not only the physics at zero temperature but also field theories at finite temperature. As suggested by several authors,¹⁶ the usual thermal-Wilson-loop is not a good parameter for looking for a deconfining transition in the Higgs phase, since it fluctuates badly as the Higgs coupling becomes larger. Therefore we will introduce some new gauge invariant quantities by combining a thermal-Wilson-loop and a loop with a Higgs field. On the basis of the results of our Monte Carlo simulation and the renormalization group equation, we will try to determine the phase structure of the adjoint Higgs model at zero temperature.

In § 2, we define the lattice action for the $SU(3)$ adjoint Higgs model and deduce various theoretical predictions in the limiting cases of parameters. In § 3, we present the results of Monte Carlo study and a phase diagram of the model at finite temperature is proposed at the end of the section. Section 4 is devoted to some conjectures and comments.

§ 2. Model and predictions

On the four-dimensional euclidean space-time, we formulate a lattice theory of the $SU(3)$ gauge fields interacting with a scalar field in the adjoint representation of the gauge group. There are a variety of models which contain Higgs fields. In this paper we use the usual Wilson action with a restriction of the constant norm for Higgs fields:

\[
S = S_U + S_\phi,
\]

\[
S_U = \beta \Sigma [1 - \frac{1}{3} \text{Re} \text{Tr}(U_{ij} U_{jk} U_{kl} U_{li})],
\]

\[
S_\phi = \gamma \Sigma [1 - \text{Tr}(\Phi_i U_{ij} \Phi_j U_{ij})],
\]

where $U_{ij}$ is a gauge field on the link $(i, j)$, $\Phi_i = \sum a \lambda^a \phi_i^a / \sqrt{2}$ is a Higgs field on the site $i$, and $\lambda^a (a = 1, 2, \ldots, 8)$ stands for the $SU(3)$ Gell-Mann matrix. Summations in (2.2) and (2.3) are taken over all plaquettes and all links of the whole lattice, respectively. The condition of the fixed norm for Higgs fields reads

\[
\text{Tr} \phi_i^2 = 1.
\]

The symmetry breaking parameter $\gamma$ is proportional to the vacuum expectation value of the norm squared of the Higgs fields in a naive continuum limit, and $\beta$ is the so-called inverse temperature related with the bare gauge coupling constant $g_0$ as $\beta = 6/g_0^2$.

In the limiting case $\gamma \to 0$, there appears the $SU(3)$ pure gauge system, for which many
investigations have been made.\(^{10}\) Their results show that the deconfining first-order phase transition appears at \(T_c \approx 200\text{MeV}\) in the finite temperature field theory, and the asymptotically free phase and the confining phase seem to be smoothly connected in the zero temperature limit. Deconfinement at finite temperature is characterized by a thermal-Wilson-loop defined by

\[
W = \frac{1}{3} \text{Tr} \left( \prod_{i=1}^{N_t} U_{ii+\hat{0}} \right),
\]

where \(N_t\) is a temporal size of the lattice and \(\hat{0}\) denotes the unit vector pointing the time direction. It measures the free energy of an isolated quark and vanishes in the confined phase. In the deconfined phase it has a finite value, showing the breakdown of the center of \(SU(3)\), i.e., \(Z(3)\). For small \(\gamma\) it will be a good order parameter also in the adjoint Higgs theory.

For \(\gamma \neq 0\), it is convenient to reduce the Higgs field degrees of freedom by choosing the unitary gauge:

\[
\Phi_i = (\lambda^3 \phi_i^3 + \lambda^8 \phi_i^8) / \sqrt{2}.
\]

As \(\gamma\) becomes large, the Higgs action dominates. In the limit \(\gamma \to \infty\), the most probable configuration must minimize \(S_\Phi\). The condition is given by

\[
\Phi_{i+\hat{\mu}} = U_{ii+\hat{\mu}} \Phi_i U_{ii+\hat{\mu}}^\dagger.
\]

In the unitary gauge, \(\Phi_i\) is diagonal, and there remains a finite symmetric group of permutations among eigenvalues. Therefore refixing the gauge such as \((\Phi_i)_{11} \cong (\Phi_i)_{22} \cong (\Phi_i)_{33}\), we have

\[
\Phi_i = \Phi, \quad [\Phi, U_{ij}] = 0.
\]

This reads that \(U_{ij} \in U(1) \times U(1)\) for non-degenerate \(\Phi\) and \(U_{ij} \in SU(2) \times U(1)\) for degenerate \(\Phi\). Thus the \(SU(3)\) gauge symmetry can break down into two different directions; the one is \(U(1) \times U(1)\) and the other is \(SU(2) \times U(1)\). The group space of the residual symmetry is larger for the latter than the former. Therefore it is a plausible guess that the \(SU(2) \times U(1)\) dominates as the parameter \(\gamma\) becomes large. Then the fields can be represented by (in an appropriate gauge)

\[
\Phi_i = \frac{1}{\sqrt{6}} \begin{pmatrix} 1 & 0 \\ 0 & -2 \end{pmatrix}, \quad U_{ij} = \begin{pmatrix} e^{-i\theta_{ij} \sigma_3} \bar{U}_{ij} & 0 \\ 0 & e^{i\theta_{ij}} \end{pmatrix}
\]

and the system reduces to the \(SU(2) \times U(1)\) pure gauge one with the action

\[
S = \beta \sum \left[ 1 - \frac{1}{3} \text{Re} (e^{-i\theta_{ij} \sigma_3} \text{Tr} \bar{U}_p + e^{i\theta_p}) \right],
\]

where \(\theta_{ij} (\bar{U}_{ij})\) is a gauge variable for \(U(1) (SU(2))\) and \(\theta_p (\bar{U}_p)\) denotes a sum (an ordered product) along a plaquette. Equation (2·10) shows that the \(SU(2)\) part cannot order as long as the \(U(1)\) part is in the disordered state. When we cool the system starting from the small \(\beta\), the most plausible behaviour of the system is as follows: At high temperature all the system is in the disordered phase. At some critical temperature
the $U(1)$ part will make a transition to the ordered state. While the $SU(2)$ part is disordered, the system is governed by the last term of (2.10). The transition point is estimated to be $\beta \approx 3$ by using a typical value of the critical temperature of the $U(1)$ gauge theory.\(^{13}\) Let us consider the behaviour of the $SU(2)$ part after the $U(1)$ transition. The effective action is a Wilson type with an effective inverse temperature $\langle \exp(-i\theta_p/2) \rangle \cdot \beta/3$. If we use the critical temperature of the $SU(2)$ deconfining transition on the $6^3 \times 3$ lattice,\(^{12}\) the critical point in our system is $\beta \gtrsim 3.3$. Thus we expect that the broken Higgs phase subdivides into three phases as far as considered at finite temperature.

Next we consider the $\beta=0$ case. The partition function is

$$Z = \int (dU)(d\Phi)e^{-S_p}.$$ \hspace{1cm} (2.11)

If the gauge fields $U_{i\alpha}$ are integrated out, we obtain an effective action for $\Phi_i$ as a function depending only on $D_i = \det \Phi_i$. We carry out the integration by using the small $\gamma$ expansion to order $\gamma^3$. After elementary calculations, we have

$$Z = \int \mathcal{D} \Pi \Pi dD_i M_i \exp(\gamma_D \sum_{i,\alpha} D_i D_{i+\bar{\alpha}}),$$ \hspace{1cm} (2.12)

where $\Lambda = 1/(3\sqrt{6})$, $\gamma_D = (9/40)\gamma^3$ and

$$M_i = \sum_{l=0}^2 (4X_i(l)-1)\sqrt{1-X_i(l)},$$ \hspace{1cm} (2.13)

$$X_i(l) = \text{sgn}(D_i)\cos\left(\frac{1}{3}\tan^{-1}\left[\frac{\sqrt{4/27-D_i^2}}{D_i}\right] + \frac{2\pi l}{3}\right).$$ \hspace{1cm} (2.14)

The probability distribution of $D_i$ (i.e., $M_i dD_i$) is a rather slowly varying function in the possible range of $D_i$, so that we may approximate the partition function as

$$Z = \Sigma \exp(\bar{\gamma} \sum_{i,\alpha} S_i S_{i+\bar{\alpha}}),$$ \hspace{1cm} (2.15)

where the summation is taken over all configurations of $S_i$ and

$$\bar{\gamma} = \langle D^2 \rangle \gamma_D,$$ \hspace{1cm} (2.16)

$$S_i = \text{sgn}(D_i),$$ \hspace{1cm} (2.17)

$$\langle D^2 \rangle = \int dD_i D_i^2 M_i / \int dD_i M_i = 1/(6\sqrt{6}).$$ \hspace{1cm} (2.18)

This is the very four-dimensional Ising model. The Migdal-Kadanoff recursion relations\(^{13}\) for the Ising system predict the critical point

$$\bar{\gamma}_c = \frac{1}{6} \left( = \frac{1}{2(\dim-1)} \right).$$ \hspace{1cm} (2.19)

Using this value and Eqs. (2.12) ~ (2.18), we have $\gamma_c = 5.4$. Though the small $\gamma$ expansion is used, it is strongly suggested that there exists some critical structure near $\gamma = 5$. The Higgs action $S_p$ is invariant under the global transformation $\Phi_i \rightarrow -\Phi_i$, but the gauge invariant quantity $D_i$ is not. Therefore the quantity $D_i$ can be a good order parameter to test the breaking of the $Z(2)$ symmetry for Higgs fields, and it may be used to distinguish the broken Higgs phase from the unbroken one, especially for small $\beta$. 
Finally we consider the case $\beta \rightarrow \infty$. The link variables $U_{ij}$ are frozen by the condition $\text{ReTr} (U_B)/3=1$ and gauged out by choosing a suitable gauge. The resultant action is that of the four-dimensional $O(8)$ Heisenberg model. It is well known to have a second-order phase transition. The value of the critical point can be estimated by the infra-red bound:\textsuperscript{14}

\[ \gamma_c \geq 4 \int_0^\infty ds [\exp(-s) \cdot I_0(s)]^4 = 1.238, \tag{2.20} \]

where $I_0(s)$ is the modified Bessel function.

As discussed above, the system is expected to have at least one critical point for each limiting value of parameters. In particular, we predict that the Ising-like transition will appear in the strong coupling region and that the $SU(2)$ deconfining transition will occur at a larger value of $\beta$ than the $U(1)$ transition in the limit $\gamma \rightarrow \infty$. In order to verify these predictions and clarify the phase structure of the inner region of the parameter space, we made a Monte Carlo simulation.

§ 3. Results and discussion

Let us mention some features of the Monte Carlo simulation before the presentation of the results. We studied the adjoint Higgs model in the unitary gauge defined by Eq. (2.6), using four-dimensional euclidean lattice of size $3^4$. At first sight, it seems rather small but we consider it is large enough for the investigation of the global phase structure. In order to study details of the obtained transitions and the phase structure of the finite temperature theory, we made some hysteresis runs on the $6^3 \times 3$ lattice as well as on the $3^4$ hypercubic one. Our measuring process consisted of 200 iterations of updating sweeps over a whole lattice. At every link (site), a gauge field $U_{ij}$ (a Higgs field $\Phi_i$) was updated 20 times under a Metropolis algorithm.\textsuperscript{15} In the hysteresis runs we made 60 (40) iterations at every parameter point in the case of $6^3 \times 3$ ($3^4$) lattice. Random numbers which are essential in Monte Carlo simulations were supplied from MIKY.\textsuperscript{16} They were obtained from the real physical processes (radiations), thus the randomness is better than usual quasi-random numbers.

We covered a square region $0 \leq \beta \leq 10$ and $0 \leq \gamma \leq 10$ in the parameter space. Quantities were measured at meshes of interval $\Delta \beta$, $\Delta \gamma = 1$ or 2. They are defined as follows:

1. Average link

\[ AL = 1 - \langle \text{Tr} (\Phi_i U_{ij} \Phi_i U_{ij}^\dagger) \rangle; \tag{3.1} \]

2. Average plaquette

\[ AP = 1 - \left\langle \frac{1}{3} \text{ReTr} (U_{ij} U_{jk} U_{kl} U_{li}) \right\rangle; \tag{3.2} \]

3. Determinant magnetization

\[ M_D = \langle \text{det} \Phi_i \rangle; \tag{3.3} \]

4. Squared thermal-Wilson-loop for $SU(3)$ gauge fields

\[ W_s^2 = \langle |\bar{W}|^2 \rangle; \tag{3.4} \]
(5) Squared thermal-adjoint-loop

\[ W_A^2 = \langle \overline{W}_A^2 \rangle, \tag{3.5} \]

\[ W_A = \frac{1}{8} (9|W|^2 - 1). \tag{3.6} \]

Here we denote the configuration average by a bar and the iteration average (over last 190 iterations) by a bracket. The first two quantities are well known in previous Monte Carlo simulations, but some comments may be needed for the others. The determinant magnetization is introduced to study the strong-coupling region where we expect an

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Fig. 1. Stereographical views of various quantities on \((\beta, \gamma)\)-plane: (a) average link; (b) determinant magnetization; (c) average plaquette; (d) squared thermal-Wilson-loop; (e) squared thermal-adjoint-loop; (f) squared thermal-Higgs-loop; (g) combination of (d) and (f). The phase diagram (h) is drawn for an eye-guide, where possible structures are given as line A to E.
Ising-like transition as discussed above. If it has a finite value, the system is in the spontaneously magnetized phase which corresponds, in our case, to the broken Higgs phase. The last two are for the study of the finite temperature structure. The squared thermal-Wilson-loop measures isolation energy of a static quark and the adjoint one does that of a gluon. They vanish in the confined phase.

Results of the measurements of the above quantities are shown in Figs. 1 (a)~(e) as stereographical views. We draw tentative lines of phase transitions with the name A to E in Fig. 1(h) for a guide. As for Figs. 1(f) and (g), see later discussions.

In Fig. 1(a) we plot the average link subtracted from unity, $\langle \text{Tr}(\Phi U \Phi U^\dagger) \rangle$. Since AL measures the energy density related to the Higgs system, it reflects the transition of $O(8)$ Heisenberg model at $(\beta, \gamma) \approx (\infty, 1.24)$. On the line $\beta = 10$ its behaviour is very similar to that of the pure Heisenberg model. Average link decreases like $1 - \gamma/8$ in the vicinity of $\gamma = 0$, has a break around $\gamma = 1.5$ and decreases again but like $7/(8\gamma)$ at larger $\gamma$. We made a hysteresis run across the break. Resultant AL is shown in Fig. 2(a). We show the data of $6^3 \times 3$ lattice only. Results of $3^4$ lattice have no difference except for a little larger errors. In the following, results of hysteresis runs mean those of $6^3 \times 3$ lattice. The configuration average for AL and AP in these runs was taken over space-like links and space-like plaquettes, respectively. A small loop is observed at $\gamma = 1.5$, but we confirmed that it shrank to a point by another 60 iterations starting from the last configurations obtained by the heating- and the cooling-process at this point. It is an artifact of a poor iteration in the hysteresis run. In Fig. 2(b) we also show the hysteresis result at $\beta = 6$. A similar loop behaviour is observable, but it is expected to disappear as well. We note that the break point moves to $\gamma = 2.1$ in this case. Thus the second-order nature of the transition of the Heisenberg model is still inherited in the region $\beta \geq 6$ and the critical line A turns upwards in the semi-strong-coupling region.

This line is seen more clearly in Fig. 1(b), since $M_B$ is an order parameter which vanishes in the phase where the Higgs fields are disordered. It shows that line A continues to line B in the cross-over region and then to C in the strong coupling region. On the line $\beta = 0$ it comes close to the point expected in the last section ($\gamma \sim 5$). Such a good agreement in spite of our naive estimation encourages us to interpret it as a transition of the gauge-invariant determinant-spin system. A sharply rising wall can be seen (Fig. 1(b)) along lines B and C, which imply their first-order nature. This point was confirmed by hysteresis runs, whose results are given in Fig. 3. At $\gamma = 3$ and $\gamma = 6$ both AL and AP show loop-like behaviour, which is a clear signal of the first-order transition across line B. As for line C, we adopt a different method, because $M_B$ needs rather long (more than 100) iterations for the thermalization and hysteresis-type analyses are not
Fig. 3. Hysteresis curves of average link (a) and average plaquette (b) at $\gamma = 3$ as a function of $\beta$. (c) and (d) are the same quantities at $\gamma = 6$.

Fig. 4. Long-run results of the configuration average $\bar{D}$, at $(\beta, \gamma) = (3.0, 6.4)$ (a) and $(2.0, 6.7)$ (b).

economical. Instead, we compare results of long-iteration measurements between an ordered-start- and a disordered-start-experiment near critical point. In Fig. 4 we show the configuration average of $D_i = \det \Phi_i$ at every iteration. At $(\beta, \gamma) = (3.0, 6.4)$ two series seem to lie apart. They, however, come together at about 130th iteration. It may be caused by an inertia of thermalization in the case of finite lattice, or it may imply that the parameters are set to the values slightly different from the critical line. At $(\beta, \gamma) = (2.0, 6.7)$ they exchange their locations after 200th iteration. These results suggest that line C is of the first order and it terminates as a second-order critical point on the $\beta = 0$ line.

As for Fig. 1(b) we note that $M_D$ approaches $1/(3\sqrt{6})$ as $\gamma$ increases. It suggests that the residual symmetry at $\gamma = \infty$ is not $U(1) \times U(1)$ but $SU(2) \times U(1)$. In order to make this point clear, we made a further analysis. Our method is to refix the gauge so that the diagonal elements of the Higgs fields should lie in order $((\Phi_i)_{11} \cong (\Phi_i)_{22} \cong (\Phi_i)_{33})$ and to average the absolute square of each element of matrix $U_{ij}$ over the whole lattice. If the residual symmetry is $SU(2) \times U(1)$, the averaged matrix is in the form of $2 \times 2$ and $1 \times 1$ block diagonal one. On the other hand, it is completely diagonalized, if the symmetry is $U(1) \times U(1)$. Obtained matrix is insensitive to $\beta$ in the broken phase, and for example,
at \((\beta, \gamma) = (4, 10)\) we have

\[
\frac{1}{|\langle U_{ij} \rangle_{ab}|^2} = \begin{bmatrix}
0.437 & 0.368 & 0.194 \\
0.372 & 0.435 & 0.193 \\
0.190 & 0.197 & 0.613
\end{bmatrix}.
\]

This matrix shows that the system does approach the \(SU(2) \times U(1)\) symmetric one. From the viewpoint of the Monte Carlo analysis, this fact can be understood by largeness of the measure of \(SU(2) \times U(1)\) (four degrees of freedom) compared to that of \(U(1) \times U(1)\) (three).

In Fig. 1(c) the spin transition line A is hard to see, since AP measures the energy density of transverse gauge fields and the alignment of spin affects the only longitudinal part of gauge fields. Line C does not appear, because the gauge fields are disordered in this region. We can see a wall along line B, which may imply that the first-order transition across this line is accompanied by an abrupt change of the gauge configuration from the disordered (confined) one to the ordered (Higgs) one.

In order to study the finite-temperature phase structure we measured the squared thermal-Wilson-loop \(W_s^2\) and results are shown in Fig. 1(d). New lines D and E appear in addition to line B. At \(\gamma = 0\) we adopted the pure gauge system for economy of computer time. Many people have discussed that the deconfining transition of the \(SU(3)\) pure gauge system is of the first order.\(^{10}\) In order to check this point, we made hysteresis experiments. As shown in Fig. 5, \(W_s^2\) has a clear loop at \(\beta = 5.6\) in the case \(N_x = 6\) (\(N_x\) is a spacial size of lattice). It confirms the existence of the first-order critical point at \(\beta = 5.5 \pm 0.1\). We did not elaborate to improve statistics to measure the latent heat. We also made a run in the case \(N_x = 3\), where the lattice is space-time symmetric. This time the critical point was located at \(\beta = 5.1 \pm 0.1\). If we assume that the lattice spacing of \(N_x = 3\) case is twice as large as that of \(N_x = 6\) one (in order to adjust the whole spacial volume), the renormalization group equations\(^{17}\) lead

\[
\exp \left[ -\frac{4\pi^2}{33}(\beta_c^{(3)} - \beta_c^{(6)}) - \frac{51}{121}\ln(\beta_c^{(6)}/\beta_c^{(3)3}) \right] = 2,
\]

where \(\beta_c^{(N)}\) represents a critical point of \(N_x = N\) case. From the above values, the

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Fig. 5. Hysteresis curve of squared thermal-Wilson-loop at \(\gamma = 0\) as a function of \(\beta\).

Fig. 6. Distribution of configuration averages of thermal-Wilson-loop on the Re-Im plane. Data points are taken from the cooling run at \(\gamma = 0\) from \(\beta = 4.6\) to 5.5. Dashed lines are of Arg\((W) = 0, 2\pi/3\) and \(4\pi/3\).
left-hand side of (3.7) becomes 1.3 – 2.0. Thus the movement is consistent with the renormalization group equation. As is well known, this transition is characterized by the Z(3)-symmetry, thus we try to visualize its breaking by plotting $\bar{W}$ at each iteration on the complex $\bar{W}$-plane. They are kinematically constrained in the equi-lateral triangle with vertices $1$, $\exp(2\pi i/3)$ and $\exp(4\pi i/3)$. Data points in Fig. 6 are taken from the cooling run in the interval $4.6 \leq \beta \leq 5.5$. They come together around the origin in the disordered phase as expected. After going through the critical point, they move towards one of the vertices, which implies that the Z(3)-symmetry is broken dynamically and then a massless Coulomb phase is realized.

For small values of $\gamma$ the above situation does not change, since the Higgs field belongs to the adjoint representation and the system still lives in $SU(3)/Z(3)$. On the other hand, the residual symmetry in the limit $\gamma \to \infty$ is $SU(2) \times U(1)$. Therefore above line A-B-C the usual thermal-Wilson-loop for $SU(3)$ gauge fields is not a good parameter to describe the phase. This can be seen from a large fluctuation in Fig. 1(d). The squared thermal-adjoint-loop (Fig. 1(e)) shows a similar behaviour, but the structure above line A-B-C seems to shift from $\beta = 3$ to $\beta = 4$. If we reexpress $W$ and $W_A$ in terms of $SU(2) \times U(1)$ defined in Eq. (2.9), then

$$W = \frac{\exp(-i\theta_r/2) \text{Tr}(\bar{U}_r) + \exp(i\theta_r)}{3}, \quad (3.8)$$

$$W_A = \frac{(\text{Tr}(\bar{U}_r)^2 + 2\cos(\theta_r/2) \text{Tr}(\bar{U}_r))}{8}, \quad (3.9)$$

where $\theta_r (\bar{U}_r)$ is a sum (an ordered product) along a thermal loop. Since $W_A$ is not affected by the $U(1)$-part in the disordered phase of the $SU(2)$-part, the shift of the structure suggests that the $SU(2)$-part is still disordered in the range $3 \leq \beta \leq 4$, that is, the transition of $U(1)$ and $SU(2)$ occurs at $\beta \sim 3$ and $\beta \sim 4$, respectively.

Let us define the thermal-Higgs-loop as

$$W_\Phi = \text{Tr}(\Phi \prod_{i=1}^{N} U_i e^{i\phi}). \quad (3.10)$$

In Fig. 1(f) we show the squared thermal-Higgs-loop

$$W_\Phi^2 = \langle |\bar{W}_\Phi|^2 \rangle, \quad (3.11)$$

which has a violent fluctuation in the broken phase. As in Eqs. (3.8) and (3.9), we express $W_\Phi$ in terms of the residual gauge fields,

$$W_\Phi = \frac{\exp(-i\theta_r/2) \text{Tr}(\bar{U}_r) - 2\exp(i\theta_r))}{3\sqrt{6}}. \quad (3.12)$$

Now let us define new interesting parameters which are meaningful in the $SU(3)$-broken phase:

$$W_1 = W - \sqrt{6} W_\Phi, \quad (3.13)$$

$$W_2 = W + \sqrt{6} W_\Phi/2. \quad (3.14)$$

The former extracts the $U(1)$ part of thermal-loop and the latter the $SU(2)$ part. We made a hysteresis run at $\gamma = 10$ with a step $\Delta \beta = 0.05$. In Fig. 7 we show $W_2^2$, $W_A^2$, $W_\Phi^2$ and $(W_1^2 + 2W_2^2)/3 = W_3^2 + 3W_\Phi^2$. The smooth behaviour is obtained for $W_3^2 + 3W_\Phi^2$, though $W_3^2$ and $W_\Phi^2$ fluctuate badly. The quantity $W_3^2 + 3W_\Phi^2$ has a hysteresis loop.
Fig. 7. Hysteresis curves at \( \gamma=10 \) as a function of \( \beta \): (a) squared thermal-Wilson-loop; (b) squared thermal-Higgs-loop; (c) combination of (a) and (b); (d) squared thermal-adjoint-loop.

Fig. 8. Hysteresis curves of thermal-Wilson-loops for \( U(1) \)-part (a) and \( SU(2) \)-part (b) at \( \gamma=10 \) as a function of \( \beta \).

around \( \beta=3.0 \) and it seems to have a break at \( \beta \approx 4.0 \). Above this point, the squared thermal-adjoint-loop has a finite value. Two-critical-point structure is more obvious in Fig. 8, where \( W_1^2 \) and \( W_2^2 \) are plotted separately at \( \gamma=10 \). Now we conclude that there exist two critical points corresponding to the \( U(1)- \) and the \( SU(2) \)-deconfining transition at \( \gamma=\infty \). Further the former seems to have a hysteresis loop but the latter being smooth. Two transition lines come close at \( \gamma=6 \) as shown in Fig. 9 and join to the first-order line B.

It is also interesting to plot \( \overline{W}_1 \) and \( \overline{W}_2 \) on the Re-Im plane. In Fig. 10 we plot the result of hysteresis run at \( \gamma=10 \). In order to obtain the usual thermal-loop of \( SU(2) \) pure gauge system, we rotated \( \overline{W}_2 \) by an angle of \( \text{Arg}(\overline{W}_1)/2 \). Then points accumulate near the real axis, as should be, if they are characters of \( SU(2) \). They fluctuate around the origin for \( 3.55 \leq \beta \leq 3.85 \). Then they start to deviate from zero as indicated by an arrow in the figure as \( \beta \) increases. Points of \( \overline{W}_1 \) are also plotted in the figure. They fall on a circle centering the origin, showing that \( U(1) \) is ordered in this parameter region (\( 3.55 \leq \beta \leq 4.10 \)).
Fig. 9. Hysteresis curves of thermal-Wilson-loops for $U(1)$-part (a) and $SU(2)$-part (b) at $\gamma=6$ as a function of $\beta$.

Fig. 10. Distributions of configuration average $\bar{W}$ and $\bar{\bar{W}}$ on the Re-Im plane, where $\bar{W}$ has been rotated by an angle of $\text{Arg}(\bar{W})/2$. Data points are taken from the cooling run at $\gamma=10$ from $\beta = 3.55$ to 4.10. Arrows show the movement of points as $\beta$ increases.

Fig. 11. The resultant phase diagram for the adjoint Higgs model at finite temperature. Phases (I)~(V) are described in the text. Shaded bands show possible phase boundaries.

In conclusion we have revealed five-phase structure of the $SU(3)$ adjoint Higgs system. It is summarized in the phase diagram shown in Fig. 11. Region (I) is the $SU(3)$-confined phase, (II) the $SU(3)$-deconfined symmetric phase of the finite temperature, (III) the $SU(2) \times U(1)$-confined phase, (IV) the $U(1)$-Coulomb and $SU(2)$-confined normal Higgs phase and (V) the finite-temperature Higgs phase with $SU(2) \times U(1)$ Coulomb interactions.

§ 4. Concluding remarks

We have clarified the phase structure of the $SU(3)$ gauge theory coupled with an adjoint Higgs scalar at finite temperature. In contrast to gauge-Higgs models investigated before, our system presents a rather complicated phase structure. It is due to the fact that there are many degrees of freedom because of the largeness of the gauge group. In the strong coupling limit gauge-invariant determinant-spins cause an Ising-like transition. After the breaking of $SU(3)$, a non-simple group, $SU(2) \times U(1)$, survives and each of subgroups has its own critical structure. We have verified all these features by the Monte Carlo method using order parameters suitable for each structure.

Here we conjecture the phase structure at zero temperature. The critical coupling-constants of deconfining transitions of $SU(3)$ and $SU(2)$ pure gauge systems vanish in the
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zero temperature limit according to the renormalization group equations. We have confirmed that it is the case for the SU(3) system in the $\gamma=0$ limit. For $\gamma \to \infty$, behaviour of the SU(2) part of the broken system will be the same as that of the pure SU(2) theory. Therefore we expect that the deconfining-transition point at $\beta \approx 4$ we have found will go to infinity as the temperature decreases to zero. Contrary to these behaviours of the SU(2)- and the SU(3)-system, the $U(1)$-transition point $\beta \approx 3$ would keep its location, because it is considered to be the fixed point of the renormalization group equation for the usual $U(1)$ gauge theory. Consequently, the five phases obtained at finite temperature will be reduced to three at zero temperature. The expectations are schematized in Fig. 12. Boundaries of three phases are all of the first order and they will terminate as a second-order critical point at each limiting point.

We have analysed the model with a constraint of fixed norm for the Higgs fields. It can be considered to be a limiting case of the $\lambda \phi^4$ model of Gupta and Heller\(^{31}\) as $\lambda, \mu^2 \to \infty$ with $\mu^2/\lambda$ fixed. It is difficult to compare our results with theirs, since the values of $\lambda$ and $\mu^2$ they took in their analysis are very small. Nevertheless, it seems that our resultant phase structure at zero temperature is very similar to theirs, if we extrapolate their phase boundaries to the region of large parameters.

Acknowledgements

The authors would like to thank Professor K. Yokoyama for encouragement. Numerical calculations were performed by using the HITAC M-200H at Hiroshima University Information Processing Center.

References