Characteristics of Modal Coupling in Nonclassically Damped Systems Under Harmonic Excitation

S. M. Shahruz. The authors study the nature of modal coupling in nonclassically damped systems under harmonic excitation. They consider the system

\[ \ddot{q}(t) + D \dot{q}(t) + Q(s) q(t) = f_1, f_2, \ldots, f_n u(t), \]

for all \( t \geq 0 \), where \( q(t) \in \mathbb{R}^n \), \( u(t) \) is a scalar-valued harmonic function of time of frequency \( \omega_f \) and amplitude one, \( D = D^T \in \mathbb{R}^{n \times n} \) is a positive definite matrix, and \( \Omega = \text{diag} \{ \omega_1, \ldots, \omega_n \} \).

For simplicity of the presentation, we let \( n = 3 \), and write (14) in Section 2 of the paper for \( i = 1 \):

\[ Q_i(s) = \frac{1}{s^2 + d_{11}s + \omega_1^2} \times \left[ f_1 - \frac{d_{12}f_2}{s^3 + (d_{22} + d_{21})Q_1(s)/Q_1(s) + d_{23}Q_1(s)/Q_2(s) + \omega_2^2} \right] U(s), \]

where \( Q_i(s) \) and \( U(s) \) are, respectively, the Laplace transforms of \( q_i(t) \) for \( i = 1, 2, 3 \) and \( u(t) \). The authors interpret the coefficient of \( U(s) \) in (D1) as the transfer function from the input \( U(s) \) to the response \( Q_i(s) \). We note that the coefficient of \( U(s) \) cannot be the transfer function from \( U(s) \) to \( Q_i(s) \), because in this coefficient the response \( Q_i(s) \) is present.

Referring to (D1), the authors define the coupling coefficients \( \gamma_{12}(s) = \frac{d_{12}f_2}{s^3 + (d_{22} + d_{21})Q_1(s)/Q_1(s) + d_{23}Q_1(s)/Q_2(s) + \omega_2^2} \) (D2) and \( \gamma_{13}(s) \), which is the third term in the square bracket in (D1). The authors state that the coupling coefficients \( \gamma_{12}(s) \) and \( \gamma_{13}(s) \) are, respectively, a measure of the effect of the coupling exerted by the second and third modes on the first mode \( Q_1(s) \). This statement is incorrect, because \( \gamma_{12}(s) \) and \( \gamma_{13}(s) \) are not functions of \( Q_1(s) \) and \( Q_1(s) \) only; they do incorporate \( Q_1(s) \). Therefore, \( \gamma_{12}(s) \) and \( \gamma_{13}(s) \) cannot determine the effect of \( Q_2(s) \) and \( Q_3(s) \) on \( Q_1(s) \) explicitly. The authors' argument could have been correct if \( Q_i(s) \) was not present in \( \gamma_{12}(s) \) and \( \gamma_{13}(s) \).

Next, the authors introduce the coupling index. The coupling index for the first mode is \( \Gamma_1(s) = \gamma_{12}(s) + \gamma_{13}(s) \). According to the authors' statement, a large magnitude of \( \Gamma_1(s) \) at \( s = j\omega_f \), \( f = \sqrt{-1} \), implies a significant effect of the modal coupling on \( Q_1(s) \). This statement is incorrect, because \( \Gamma_1(s) \) incorporates \( Q_1(s) \), and hence its direct effect on \( Q_1(s) \) is not clear. If the authors' statement was correct, then the computed value of \( |\Gamma_1(j\omega_f)| \) for each \( i \) could help decide on the approximate decoupling of the system (3) by neglecting the off-diagonal elements of the matrix \( D \). We, however, note that in order to compute \( |\Gamma_1(j\omega_f)| \) for any \( i \), we have to know the exact responses of the system, \( Q_i(s) \), for all \( i \). The question is: What do we achieve by computing \( |\Gamma_1(j\omega_f)| \) if we are supposed to solve the system for \( Q_i(s) \) for all \( i \)? If we solve the system, then there is no need to know the extent of the modal coupling that would possibly help us decide on the approximate decoupling of the system.

In Section 4, the authors analyze the effect of the separation of the natural and excitation frequencies on the decoupling of the modes. They first define \( \alpha_i = \text{Re}(\Delta_i(j\omega_f)) \) and \( \beta_i = \text{Im}(\Delta_i(j\omega_f)) \). Next, they derive expressions (22) and (23) for \( |\Gamma_i(j\omega_f)| \) and \( \text{arg}(\Gamma_i(j\omega_f)) \). We note that (22) and (23) express \( |\Gamma_i(j\omega_f)| \) and \( \text{arg}(\Gamma_i(j\omega_f)) \) implicitly, because they incorporate \( \alpha_i \) and \( \beta_i \), which in turn are functions of \( Q_i(s) \). Using (22) and (23), the authors draw conclusions (a), (b), (c), and (d). The validity of these conclusions is questionable, because (22) and (23) are not explicit expressions for \( |\Gamma_i(j\omega_f)| \) and \( \text{arg}(\Gamma_i(j\omega_f)) \); the response \( Q_i(j\omega_f) \) is embedded in the right-hand sides of (22) and (23).

Authors' Closure

The purpose of our paper is to demonstrate that, within the practical range of engineering applications, neither diagonal dominance of the modal damping matrix nor frequency separation of the natural modes would be sufficient for neglecting modal coupling. The discussion by Dr. S. M. Shahruz is much appreciated. However, the issues brought up in the discussion are rather minor and stylistic. These issues can be readily clarified in the following paragraphs.

It was stated in our paper that the coefficient of \( U(s) \) in Eq. (14) could be thought of as a transfer function of the modal response. While it is useful to intuitively look at Eq. (14) that way, it is also clear that the coefficient of \( U(s) \) cannot be a transfer function from a strict mathematical viewpoint. An obvious reason is that the Laplace transform of the input is not just \( U(s) \), the elements \( f_i \) (\( i = 1, 2, \ldots, n \)) of the amplitude vector must also be included. The coefficient of \( U(s) \) was never formally defined as a transfer function, nor was any property of a transfer function invoked on it. In our paper, it is suggested that the coupling term \( \gamma_{12} \), for example,