

f_1 and this results is a nonlinear decay with respect to $(x - x_0)$. For large distances downstream, however, the effect of f_1 becomes negligibly small and asymptotically,

$$u_0 \sim (x - x_0)^{-1} \quad (21)$$

exactly that for the nonswirling jet. A similar asymptotic condition applied to the swirling velocity shows that,

$$w_0 \sim (x - x_0)^{-2} \quad (22)$$

Thus far downstream, the swirl becomes of minor importance since it decays more quickly than the axial velocity. The approach to the self-preserving state will be rapid and we can consider that by approximately 10 diameters downstream the flow will have the essential characteristics of the free jet, with respect to the streamwise velocity component.

It is a little surprising that the definition of the lateral scale, $l_0 \sim (x - x_0)$ is so effective in stretching the mean profiles within the development region. One might expect a better fit by employing a function similar to f_1 or f_2 . It has been found, however, in most studies of free shear flows, that the length scales are not as sensitive to change as the velocity scales. In this present situation the exponent of $(x - x_0)$ must be different than unity in this pre-self-preserving region of the jet. Our mean flow results in Figs. 3 and 4 are just not definitive enough to pick out the differences.

We would expect a truly self-preserving flow to display scale variations for the turbulence which are identical to the mean field. Sufficiently accurate measurements were possible only for the normal components of turbulent intensities, however. Generally, the results show that the turbulence does attain this state, but at a somewhat later stage in the development of the flow. This is not unexpected since the fine-scale structure of the flow characteristically has longer development time.

Less success was obtained with the mean static pressure. Using arguments similar to those for the mean velocity field and ignoring the effect of the turbulence upon the magnitude of the mean pressure, it was found that

$$P_0 \sim (x - x_0)^{-4}$$

The measurements showed, however, that the turbulence is at least of the same order as mean terms. Since the analytical argument cannot handle the turbulence explicitly it is perhaps more reasonable to use the measured decay function,

$$P_0 \sim (x - x_0)^{-2}$$

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References

- Loitsyanskii, L. G., "The Propagation of a Twisted Jet in an Unbounded Space Filled With the Same Fluid," *Prikladnaya Matematika i Mekhanika*, Vol. 17, 1953, pp. 3-16.
- Gortler, H., "Decay of Swirl in an Axially Symmetrical Jet Far From the Orifice," *Revista Matematica Hispano Americanas*, Vol. 14, Nos. 4 and 5, 1954, pp. 143-178.
- Steiger, M. H., and Bloom, M. H., "Axially Symmetric Laminar Free Mixing With Large Swirl," *Journal of Heat Transfer*, TRANS. ASME, Series C, Vol. 84, No. 4, Nov. 1962, pp. 370-374.
- Lee, Shao-Lin, "Axisymmetric Turbulent Swirling Jet," *Journal of Applied Mechanics*, Vol. 32, TRANS. ASME, Vol. 87, Series E, No. 2, June 1965, pp. 258-262.
- Chigier, N. A., and Chervinsky, A., "Experimental Investigation of Swirling Vortex Motion in Jets," *Journal of Applied Mechanics*, Vol. 34, TRANS. ASME, Vol. 89, No. 3, Sept. 1967, pp. 443-451.
- Rose, W. G., "A Swirling Round Turbulent Jet, I. Mean Flow Measurements," *Journal of Applied Mechanics*, Vol. 29, TRANS. ASME, Vol. 84, No. 3, Sept. 1962, pp. 615-625.
- Traugott, S. C., "Influence of Solid-Body Rotation on Screen-Produced Turbulence," NACA TN 4135, 1958.
- Townsend, A. A., *The Structure of Turbulent Shear Flow*, Cambridge University Press, London, 1956.

DISCUSSION

J. A. Brighton² and C. J. Hendricks²

The authors are to be congratulated for their thorough study of the flow of a swirling free turbulent jet. They have considered one set of flow conditions (a single swirl and jet velocities) where they measure the mean axial velocity, mean tangential velocity, jet spread rate and turbulence quantities. Their work extends and compliments the very fine work previously done by Chigier and Chervinsky.

Thus, the remarks that follow are not meant as a criticism of the paper, but are made primarily as suggestions for further work.

The work presented here was carried out for only one flow condition and the constants established apply only for that particular condition. Additional data are needed to establish the effect of swirl over the complete range from zero to very strong swirl strength.

The authors point out the limitations of their analytical approach. By using the integral equations they must assume that the various velocity profiles can be represented as self similar functions. This means that they cannot describe the region close to the jet source or describe flows involving strong swirl conditions. The integral approach would also not be used for the swirling jet within confining walls.

After having made the previous statement we should also recognize that the similarity-integral approach works very well even in rather severe conditions where they are "not supposed" to give a good description of the flow. One example of this is the flow of a confined jet (without swirl) which we have modeled [9, 10]³ using the integral equations and yet obtained good agreement with measurements even for conditions involving strong adverse pressure gradients and near the jet source.

However, what we would like to suggest is the consideration of solving the differential equations directly for this problem. The approach which might be considered is to use the turbulence kinetic energy equation along with the dissipation equation to model the turbulence. The differential equations of momentum and continuity would also be used. This approach has been suggested by Reynolds [11] and others as having promise for calculating turbulent boundary layers. It seems to us that it would be advantageous to pursue this method for the swirling jet, particularly for the initial region, for strong swirl and confined jets where similarity may not give good results. The other advantage is that more physics is contained in the model and are a part of the calculated results.

We would like to see a comparison between the results of the present paper and the results obtained in [12]. We would also like to know how the turbulence data were obtained. Meaningful turbulence data are difficult to obtain in swirling flows especially at high turbulence levels. A short note on experimental procedure and experimental difficulties would be helpful to other investigators.

In conclusion we would like to repeat that these comments are not meant as criticism of the present paper which we feel is a valuable addition to the literature. What we are suggesting, however is, that (1) more data be obtained for a wide range of swirl strength and (2) a completely different approach be pursued in the modeling of the problem which involves a numerical solution of the differential equations directly as has been suggested for turbulent boundary layers.

Additional References

- Exley, J. T., and Brighton, J. A., "Flow Separation and Reattachment in Confined Jet Mixing," *JOURNAL OF BASIC ENGINEERING*, TRANS. ASME, Series D, Vol. 92, No. 2, June 1971, pp. 192-198.

² Associate Professor of Mechanical Engineering and Graduate Student-Dupont Fellow in Mechanical Engineering, respectively, The Pennsylvania State University, University Park, Pa.

³ Numbers in brackets designate Additional References at end of discussion.

10 Razinsky, E., and Brighton, J. A., "Confined Jet Mixing-Analysis," *JOURNAL OF BASIC ENGINEERING*, TRANS. ASME, Series D, Vol. 92, No. 3, Sept. 1971, pp. 333-349.

11 Reynolds, W. C., "A Morphology of the Prediction Methods," Proceedings, *Computation of Turbulent Boundary Layers*, S. J. Kline, et al., eds, Stanford University, 1968.

12 Lilley, D. A., Chigier, N. A., "Non-Isotropic Turbulent Stress Distribution in Swirling Flows from Mean Value Distributions," *International Journal of Heat and Mass Transfer*, Vol. 14, 1971, pp. 573-585.

Authors' Closure

The authors wish to report several draughting errors in the figures. In Fig. 8, the triangles should be for $w_0/(w_0)_m$ instead of $v_0/(u_0)_m$. The line through the dots is equation (17), and that through the triangles, equation (18). The open triangles in Fig. 12 are for $\eta = 0.125$ and not 0.135.

The discussers are to be commended for making several important points: the suggestion by Reynolds [11] of directly solving the differential equations for turbulence kinetic energy, continuity, momentum and dissipation, is becoming more feasible. Closure of the terms in the turbulence energy equation up to now has required assumptions to be made about the Reynolds stresses, pressure-velocity correlation, and dissipation. However, the present measurements of Reynolds stress may help to replace at least one of these assumptions. In the excellent work by Lilley and Chigier [12], Reynolds stresses $\overline{wv}/(u_0)_m^2$ and $\overline{vw}/(u_0)_m^2$ and also effective viscosities, were deduced from mean velocity u , v , w , and pressure measurements in jets with swirl numbers S from 0 to 0.6. Comparison of their predicted dis-

tributions of Reynolds stresses with our experimental measurements, shows a reasonably good agreement in distribution shape, but in magnitude their peak stresses are 3 to 4 times larger than ours several diameters downstream of the jet exit. However by x/D of 12 to 15, the $\overline{vw}/(u_0)_m^2$ stresses are in good agreement and yet their maximum $\overline{uv}/(u_0)_m^2$ is still over 4 times larger. Since the swirling jets were produced by entirely different means, the initial jet conditions of radial pressure and turbulence may account for some of the disagreement. In addition, experimental inaccuracies in measurement of our turbulence stresses, and also assumptions made in their theory are contributing factors. Further work in resolving these differences is needed, for the acceptance of their rather powerful technique should prove of considerable use in the solution of practical mixing problems with swirl.

Most of our tests were made at only one swirl number due to mechanical limitations in the swirling pipe apparatus. As is obvious now, the most satisfactory way of producing swirls of various intensities is the vortex chamber or pipe of Chigier et al. even though it does not produce quite the same exit velocity and pressure profiles as through-flow from a spinning pipe. Our turbulence measurements were made with a single miniature normal DISA hot-wire probe, the stem of which was aligned along the normals to the mean velocity vector directions to obtain the turbulence components by appropriate rotations of the wire itself. Repositioning via lead-screws to within 0.001 in. in the small flow field was necessary for each probe orientation and could introduce some error in measurement.