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DISCUSSION

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The authors have done very well to solve the difficult problem of the envelope of a straight line attached to the coupler of a planar four-bar linkage. It is particularly interesting that the envelope is dual to the locus of a point attached to the coupler. This is not true for planar motion in general, as the example of Cardan motion shows: the locus of a point is a conic, whereas the envelope of a line is a much more complicated curve.

I have two points of detail. Firstly, the authors' result in Section 4 that the envelope has no more than four real asymptotes can be verified as follows. The circular points at infinity are double points, and the line at infinity is a bitangent, so eight points at infinity are accounted for. Since the order of the envelope is 12, there are four other points at infinity (not necessarily all real), and the tangents at these points are the asymptotes. Incidentally, the justification for the fact that the line at infinity is a bitangent should be that in equation (14) the terms of lowest degree in l and m are quadratic.

Secondly, in equation (7), the expression called Q^2 may be negative, because $s \le p + c + r$ does not imply that $s^2 \le p^2 + c^2 + r^2$. However, this does not affect the later results.

This paper takes an unusual look at planar four-bar linkages, and adds significantly to our understanding of them.

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Authors' Closure

We thank Dr. Primrose for his kind remarks. We agree with him about the duality between envelope and point-path, and remark that such exceptions can be inferred from our statements in Sections 7 and 9. In fact the matter is covered in some depth in reference [4] where it is, nevertheless, argued that, when geometrical elements at infinity and imaginary elements are taken into account, point-paths and line-envelopes do, in general, possess dual properties at least so far as their order and class (respectively) are concerned.

Dr. Primrose's second paragraph about asymptotes and imaginary circular points adds valuable material and confirms the substance of the paper. We admit inadequacy in our explanation of why the line at infinity is a bitangent and accept his final sentence in this paragraph.

Finally we agree entirely with his third paragraph and recognize our elementary error. True, since Q always appears raised to an even power, the results are unaffected, but it would have been better to abandon any semblance of dimensional consistency and write equation (7) as, say,

$$\frac{1}{2}(p^2 + c^2 + r^2 - s^2) = Z,$$

accepting that Z may be positive or negative. Then Z should be substituted for Q^2 and Z^2 for Q^4 throughout the paper.