Large aspect ratio convection cells in the upper mantle

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Summary. Numerical experiments which model a possible large-scale flow in the upper mantle are described. The lithospheric plates are represented by a constant velocity boundary condition on the upper surface of a two-dimensional convecting layer. The transition from small-scale multiple-cell flow to single cell flow is described in terms of the Rayleigh number and the Péclet number. The mean flow field is successfully compared with a simple parameterized model in which flow is driven by the moving upper boundary and by a mean horizontal temperature gradient which must be present in any large aspect ratio cell. Bathymetry and gravity profiles are calculated for the long cells, and it is shown that in the absence of a low viscosity zone, a realistic variation of bathymetry and gravity is only obtained if the flow in the cell is being driven predominantly by the horizontal temperature gradient rather than the moving plate. Mean horizontal temperature gradients in the upper mantle of 0.025°C km\(^{-1}\) are suggested beneath a fast moving plate such as the Pacific. Transverse boundary layer instabilities in the large cells are shown to be governed by a critical boundary layer Rayleigh number, which linearly increases with the boundary layer Péclet number.

1 Introduction

Convection in the Earth's mantle is believed to be the principal form of heat transfer from the interior, where heat is generated by radioactive decay, to the surface. The flow of the mantle caused by the buoyancy forces is also linked with the observed motions of the lithospheric plates. However, it remains unclear how closely the shape, area and velocity of the plates reflect the planform of the convection beneath. The question of whether the convection extends in a single layer through the depth of the mantle (~3000 km), or is separated into two or more discrete layers (e.g. Richter & McKenzie 1981) is also undecided.

One view, whose ramifications are explored here, is that convection in the upper mantle is separated from that in the lower mantle by an increase in either density or viscosity at about 700 km depth. The two-layer model was originally proposed because of the evidence of earthquake mechanisms from the Benioff zones. These zones of seismicity, which extend to
depths of 700 km beneath island arcs, show that the subducted slabs are in a state of down-dip extension unless the bottom of the slab reaches a depth of 650–700 km (Isacks & Molnar 1971). When it does, the state of stress changes from extension to compression, and the compressive stress is apparently propagated up the slab. Although these observations are open to other interpretations, the simplest idea is that the phase change at 650 km acts as a barrier to the vertical flow of material.

Geochemical modelling of rare earth element isotope data has also led to the proposal of a two-layered model in which there is little mixing between an upper mantle depleted in the heat-producing elements and a lower mantle which is essentially of chondritic composition (Wasserburg & De Paolo 1979 and O'Nions, Evensen & Hamilton 1979). Consistent with the geochemical model, long-wavelength (2000 km) anomalies in the geoid have been interpreted as evidence of rising and sinking jets in an upper mantle layer which is primarily heated from below (McKenzie et al. 1980).

For a convecting layer only 700 km deep, the horizontal dimensions of the lithospheric plates imply that the aspect ratio of an upper mantle convection cell may be as large as 14 (e.g. the Pacific plate). Although the idea of long flat convection cells in the upper mantle has been in the literature, at least since plate tectonics emerged in the 1960s, most research has focused on convection in approximately equidimensional cells (e.g. Richter 1973a; McKenzie, Roberts & Weiss 1974; Busse & Whitehead 1971). The observation that large aspect ratio cells are unstable with the standard constant temperature boundary conditions has even been used as an argument in favour of convection cells extending through the depth of the mantle (Elsasser, Olson & Marsh 1979).

However, the precise boundary conditions for mantle convection are uncertain, and recent numerical (Hewitt, McKenzie & Weiss 1980; Houseman & McKenzie 1982; Rabinowicz, Lago & Froidevaux 1980) and laboratory (Nataf et al. 1981; Carrigan 1982) experiments have shown that stable large aspect ratio cells can be obtained for a range of geophysically relevant boundary conditions (although they may be associated with a small-scale flow). These experiments usually differ from the earlier ones, in which large aspect ratio cells were unstable, by what would appear to be only minor changes to the boundary conditions, e.g. changing a constant temperature boundary to a constant flux boundary (Hewitt et al. 1980). However, what appears to be the common factor in these experiments is that, in each case, the modified boundary conditions either permit or induce an increased horizontal temperature gradient across the layer, over the same length scale as the long convection cell.

The purpose of the numerical experiments described here is to investigate the importance of the horizontal temperature gradient in an example of stable large aspect ratio convection rolls, and to parameterize the resulting mean flow in terms of this temperature gradient. Previous numerical models of the large aspect ratio cells driven by a moving plate on the upper boundary (e.g. Richter 1973a; Parmentier & Turcotte 1978; Richter & McKenzie 1978; Lux, Davies & Thomas 1979) have ignored the effect of the horizontal temperature gradient in the parameterization of the flow. The results here show that the horizontal temperature gradient induced by the large-scale flow provides a significant contribution to the forces driving the flow.

The experiments here use a simplified model of a constant viscosity flow in a long flat box, with a rigid moving upper boundary representing the mechanical effect of a lithospheric plate. The fluid is heated from below, as for the upper layer of a two-layered earth model.

These boundary conditions are essentially the same as those used by Richter & Parsons (1975) in a laboratory experiment in which longitudinal rolls (axis parallel to the shear) were
the predominant planform for the convection. Because numerical experiments of the type presented here are necessarily two-dimensional, longitudinal rolls are precluded. Thus, although the planforms shown here are stable in two dimensions, they are unstable in three dimensions with the stated boundary conditions.

However, these boundary conditions are a convenient means of obtaining stable large aspect ratio cells over a wide parameter range, in order to test the proposed parameterization. If, as seems likely, large aspect ratio cells are important in upper mantle convection, because of different boundary conditions or fluid properties (e.g. temperature-dependent viscosity) it is hoped that the parameterization will be transferable to the new boundary conditions, with appropriate modifications. For some of the alternate boundary conditions for which long cells are stable, the induced horizontal temperature gradient is probably the main driving force for the flow in the cell.

A later section of the paper also discusses the stability of the 2-D boundary layer plumes described by Lux et al. (1979), using the critical boundary layer Rayleigh number criterion of Howard (1966).

2 Numerical solution of the convection equations

The formulation of the 2-D convection problem follows that of McKenzie et al. (1974). The velocity, \( v \), of the fluid in the \( x-z \)-plane is described in terms of a streamfunction, \( \psi \):

\[
v = \left( \frac{-\partial \psi}{\partial z}, 0, \frac{\partial \psi}{\partial x} \right).
\]  

(1)

Temperature, distance and time are made dimensionless by

\[
T = T'(Fd/k) + T_0, \quad (x, z) = (x', z')d \quad \text{and} \quad t = t'(d^2/k),
\]

(2)  

(3)  

(4)

where the prime (') denotes the dimensionless variable, \( d \) is the depth of the layer, \( F \) is the heat flux on the base of the layer and \( T_0 \) is the temperature at the top of the layer, \( k \) is the thermal conductivity and \( \kappa \) is the thermal diffusivity.

In terms of the dimensionless variables, the governing equations for infinite Prandtl number, Boussinesq, convection are:

\[
\nabla^2 \psi' = -\omega', \quad \text{(5)}
\]

\[
\nabla^2 \omega' = R \frac{\partial T'}{\partial x'} \quad \text{(6)}
\]

and

\[
\frac{\partial T'}{\partial t'} = -\nabla \cdot (T'\nu') + \nabla^2 T', \quad \text{(7)}
\]

where \( \omega \) is the component of the vorticity in the \( y \)-direction.

\[
R = \frac{g\alpha Fd^4}{k\kappa \nu} \quad \text{(8)}
\]
is the Rayleigh number, which indicates the relative importance of buoyancy forces to dissipative forces, i.e. a measure of the intensity of convection. $g$ is the acceleration of gravity, $\alpha$ is the coefficient of thermal expansion and $\nu$ is the kinematic viscosity. Internal heating and thermal dissipation are assumed negligible in the temperature equation (7).

The equations are solved in the rectangular region of width $\lambda d$, with the boundary conditions summarized in Fig. 1. Reflection boundary conditions are assumed on the lateral boundaries. On the lower boundary the heat flux is constant ($\partial T'/\partial z' = -1$) and both components of the velocity are zero. On the upper boundary the temperature is constant ($T' = 0$), as is the velocity $[v' = (Pe, 0)]$, where $\text{Pe} = u_0 d/\kappa$ (9) is the Péclet number, a measure of the relative importance of advected to conducted heat. The Rayleigh number and the Péclet number are the two principal external parameters in these experiments.

Equations (5–7) were solved using the finite difference methods described by McKenzie et al. (1974). The temperature is advanced on each half of a rectangular staggered mesh in two half timesteps. Because the boundary conditions here specify constant velocity rather than zero tangential stress, the boundary condition on vorticity is not explicit. The value of the vorticity on the boundary must be calculated, consistent with the prescribed surface velocity, by an iterative method described by Richter (1973a).

For all of the experiments $\lambda = 8$ was used, with 25 mesh points spanning the depth of the fluid layer. To check accuracy, the results of several runs were compared with those obtained from a different convection program, using the method of cyclic reduction (Sweet 1974) to solve the Poisson equations.

The experiments were run to large values of $t'$, so in general the initial conditions were unimportant. One run was usually started from the end condition of another run which was close in $R$ and $\text{Pe}$ values, so that steady state was attained as quickly as possible. The criterion used for steady state was that the mean temperature of the layer was only changing very slowly relative to the overturn time-scale of the fluid. Because of the boundary layer instabilities, few of the runs became time independent. In one or two runs, the initial conditions may have been significant in selecting one of two or more distinct final states, distinguished by the presence or absence of vertical sheets (Section 3).

When $\text{Pe} \neq 0$ there is a singularity in the vorticity field where the velocity is discontinuous (i.e. $z' = 1$ and $x' = 0$ or $\lambda$). Sufficiently close to the singularity, the buoyancy forces can be neglected and the flow field can be calculated easily for the simple analogue of non-buoyant viscous flow in a viscous half-space $z < 0$, with $u = -u_0$ for $x < 0$, $z = 0$ and $u = u_0$ for

![Figure 1. Diagram showing boundary conditions used in the calculations and the origin and orientation of the coordinate system. $\lambda$ is the aspect ratio of the box and $\text{Pe}$ is the non-dimensional horizontal velocity of the upper boundary. Reflection boundary conditions apply to both side boundaries.](https://academic.oup.com/gji/article-abstract/75/2/309/603120)
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$x > 0, z = 0$ (e.g. Skilbeck 1975). The solution for streamfunction and vorticity is:

\[ \psi = \frac{-2}{\pi} u_0 r \theta \cos \theta \]

and

\[ \omega = \frac{4}{\pi r} u_0 \sin \theta, \]

where $r$ and $\theta$ are the cylindrical coordinates centred on the discontinuity ($\theta = 0$ is the negative $z$-axis).

Clearly the model is unrealistic near the discontinuity since the tangential stress on the top surface is proportional to the vorticity, which goes as $(1/r)$. For Earth materials the behaviour at high stress is non-Newtonian, and in any case an upper limit to the stress is reached when the rock fractures. Because of the singularity, it is not possible to calculate the external force on the plate, without making further assumptions about the behaviour of the material in this region.

3 Observed flow fields

Table 1 shows the range of values of $\text{Pe}$ and $R$ used in the experiments, covering approximately one order of magnitude in each variable. The values of the physical constants in Table 2 were assumed. For these values of $\kappa$ and $\theta$ the range of Péclet numbers corresponds to plate velocities from 0.5 to 10.0 cm yr$^{-1}$. The Rayleigh number was changed by altering the heat flux. Values of $\mathcal{F} = 3, 10$ and 30 mW m$^{-2}$ were used (cf. a mean surface heat flow of 80 mW m$^{-2}$ as estimated by Sclater, Jaupart & Galson 1980). Calculations with higher Rayleigh number would have required a prohibitive amount of computer time.

As in the experiments of Lux et al. (1979), the transition from single cell laminar flow at large Péclet number, to time-dependent multi-cell flow at small $\text{Pe}$, is observed. The following distinct types of circulation (Fig. 2) are noted:

(A) At large Péclet number, the moving boundary dominates the flow, as in Fig. 2(A). The heat flux incident on the lower boundary is transported entirely by the forced flow. The flow is laminar, with a horizontal velocity profile given by equation (15), away from the side

<table>
<thead>
<tr>
<th>Identifier</th>
<th>$\text{Pe}$</th>
<th>$R$</th>
<th>$t'$</th>
<th>Flow type</th>
<th>$R_{bc}$</th>
<th>$\beta'$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1V1</td>
<td>1480</td>
<td>240 300</td>
<td>0.026</td>
<td>A</td>
<td>–</td>
<td>0.00539</td>
</tr>
<tr>
<td>A1V2</td>
<td>740</td>
<td>240 300</td>
<td>0.071</td>
<td>A</td>
<td>–</td>
<td>0.0105</td>
</tr>
<tr>
<td>A1V6</td>
<td>518</td>
<td>240 300</td>
<td>0.123</td>
<td>B</td>
<td>320</td>
<td>0.0136</td>
</tr>
<tr>
<td>A1V3</td>
<td>370</td>
<td>240 300</td>
<td>0.129</td>
<td>B</td>
<td>250</td>
<td>0.0162</td>
</tr>
<tr>
<td>A1V7</td>
<td>222</td>
<td>240 300</td>
<td>0.164</td>
<td>B</td>
<td>175</td>
<td>0.0211</td>
</tr>
<tr>
<td>A1V4</td>
<td>148</td>
<td>240 300</td>
<td>0.164</td>
<td>D</td>
<td>150</td>
<td>0.0143 (0.0281)</td>
</tr>
<tr>
<td>A1V5</td>
<td>74</td>
<td>240 300</td>
<td>0.032</td>
<td>D</td>
<td>–</td>
<td>0.0138 (0.0343)</td>
</tr>
<tr>
<td>A2V2</td>
<td>740</td>
<td>721 000</td>
<td>0.063</td>
<td>B</td>
<td>400</td>
<td>0.00902</td>
</tr>
<tr>
<td>A2V3</td>
<td>370</td>
<td>721 000</td>
<td>0.104</td>
<td>B</td>
<td>230</td>
<td>0.0118</td>
</tr>
<tr>
<td>A2V8</td>
<td>185</td>
<td>721 000</td>
<td>0.058</td>
<td>C</td>
<td>–</td>
<td>0.0112 (0.0124)</td>
</tr>
<tr>
<td>A3V3</td>
<td>370</td>
<td>72 100</td>
<td>0.151</td>
<td>A</td>
<td>–</td>
<td>0.0234</td>
</tr>
<tr>
<td>A3V7</td>
<td>222</td>
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<td>B</td>
<td>210</td>
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</table>
Figure 2. Temperature and streamfunction fields at large $t'$ for the experiments (see Table I): (A) A3V3, $F = 3 \text{ mW m}^{-2}$ and $u_o = -2.5 \text{ cm yr}^{-1}$; (B) A1V3, $F = 10 \text{ mW m}^{-2}$ and $u_o = -2.5 \text{ cm yr}^{-1}$; (C) A2V8, $F = 30 \text{ mW m}^{-2}$ and $u_o = -1.25 \text{ cm yr}^{-1}$, and (D) A1V5, $F = 10 \text{ mW m}^{-2}$ and $u_o = -0.5 \text{ cm yr}^{-1}$. The plate moves to the left from the ridge on the right side. Streamfunction is positive in the stippled region and negative elsewhere. The dimensionless contour interval is given below each drawing. The contour interval in $^\circ\text{C}$ is in parentheses.

boundaries. Both upper and lower boundary layers are stable and a time-independent solution is obtained.

(B) With the upper boundary being driven more slowly relative to the buoyancy forces, the boundary layers increase in thickness, and the laminar flow in the lower boundary layer becomes unstable (Fig. 2B). The lower boundary layer is thinned as the hot material rises in the form of plumes and is dispersed by the shear flow in the colder layers above. The
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Structure of each plume persists as it is swept along by the main flow into the rising region at the end of the box. A new plume is generated periodically, so the flow field does not become time independent. However, disturbances to the kinetic energy and mean temperature of the layer are reduced to small oscillations about a constant value.

(C) On further decreasing Pe relative to $R$, both boundaries have become unstable (Fig. 2C), and generate plumes, which mix material from the boundary layers into the isothermal region of the cell. The two sets of plumes slip past each other, carried by the main flow to the opposite ends of the box. The single large cell shows an internal structure of several smaller corotating cells. These cells are separated by narrow shear zones, rather than by rising or sinking regions, as in Rayleigh–Bénard convection. The streamfunction remains negative nearly everywhere, though small regions of positive (clockwise) circulation are appearing adjacent to the plumes on the lower boundary.

(D) For smaller values of the Péclet number, permanent counter-rotation cells are present, and appear in the temperature field as a pair of rising and sinking sheets which are not carried past each other by the main flow (Fig. 2D). The flow pattern is significantly changed by the appearance of these sheets, because they act as barriers to horizontal flow, to some extent isolating the fluid in one cell from the adjacent cells. In this domain, it may be more instructive to see the effect of the moving boundary as a perturbation to ordinary Rayleigh–Bénard convection, in which it increases the length of the corotating cells at the expense of the counter-rotating cells.

Fig. 3 shows the experiments on a stability map of the Pe-$R$ plane, which uses the preceding classification. The solid line below which flow type B is not observed is the

![Figure 3](https://academic.oup.com/gji/article-abstract/75/2/309/603120/309-fig3)

**Figure 3.** Stability map of the experiments on the Pe-$R$ plane for $\lambda = 8$. The solid line is the empirical relation between maximum velocity and Rayleigh number for free convection (Appendix A). The symbols denote the flow type: star for (A), triangle for (B), circle for (C) and square for (D).
relation between Rayleigh number and maximum convective velocity for free convection in square cells with no-slip boundary conditions (see Appendix A). The dashed line is drawn parallel to the solid line to partition the A and B data points. Clearly the ordinate of this line depends on λ, which was 8 in all the experiments.

Moving down the stability map on a trajectory normal to the free convection relation (i.e. decreasing the ratio of forcing velocity: free convection velocity), there is a progression from flow type A to B, to C and D. To determine precisely the position of the stability boundaries between flow types would require a large investment in computer time, which is probably not justified in this investigation.

Generally, when the boundary velocity is less than the free convective velocity, the circulation is of type C or D, with both boundaries unstable. When it is greater, type A or B is observed. However, there may be some overlap of the regions (i.e. hysteresis), in which the flow type would depend on the initial conditions of the run. In particular, a boundary between C and D has not been marked. The run labelled 'C' was strongly time dependent, and slow to reach steady state, showing both types of flow structure at different times.

For flow type D the stationary sheets always appear in pairs (i.e. one up and one down). Neither is advected by the main flow because the rising sheet blocks the path of the sinking one and vice versa. However, for the 3-D analogue of this flow, in which the instabilities occur as jets rather than sheets, it is probable that up and down jets would slip past each other by moving apart in the direction orthogonal to the main flow. In this case there would be no distinction between flow type C and D.

4 Flow parameterization

Depth averaged mean temperature profiles from the numerical experiments described in the previous section (Fig. 4) show that the average temperature in the long cells is, to first order, a linear function of the horizontal coordinate. Thus, a reasonable approximation to the mean velocity and temperature fields, away from the strong rising and sinking regions at the ends of the cell, is:

\[ v' = [U'(z'), 0], \]

and

\[ T' = \bar{T}'(z') + \beta'(x' - \lambda/2), \]  

where \( \bar{T}' \) is the horizontally averaged temperature profile, and \( \beta \) is the depth averaged horizontal temperature gradient.

The two components of the dimensionless momentum equation are then (e.g. Weber 1973):

\[ R \frac{\partial p'_1}{\partial x'} = \frac{d^2 U'}{dz'^2} \]

and

\[ \frac{\partial p'_1}{\partial z'} = \bar{T}' + \beta'(x' - \lambda/2), \]  

where \( p_1 \) is the non-hydrostatic pressure field, made dimensionless by:

\[ p_1 = p'_1 (\rho_0 g \alpha F d^2/k), \]  

where
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$T \, (^{\circ}C)$

![Graphs showing temperature profiles (a) to (f)]

**Figure 4.** Vertically averaged temperature profiles for the experiments (see Table 1): (a) A3V3 (type A), (b) A1V3 and (c) A1V7 (type B), (d) A2V8 (type C), (e) A1V4 and (f) A1V5 (type D). $\beta'$ was obtained from the least squares line of best fit in each case. For the three bottom curves, it was also read from the dashed line segments (not shown for d). The arrows indicate stable sinking sheets which separate these segments.

with $\rho_0$ the density of the fluid at temperature $T_0$. Eliminating the pressure,

$$\frac{d^3 U'}{dz'^3} = R \beta'.$$

(14)

$U'(z')$ is then obtained by integrating (14), subject to the two velocity boundary conditions
on the horizontal boundaries and the requirement that the net horizontal mass flux across any vertical plane be zero:

\[ U' = R \beta' \left[ \frac{z'^3}{6} - \frac{z'^2}{4} + \frac{z'}{12} \right] + \text{Pe} [3z'^2 - 2z'] \]  

This flow field can be used to predict the mean pressure field in the box, and hence the relative bathymetry and gravity anomalies above the box (Appendix B).

For the boundary conditions used here, the gradient \( \beta \) is not imposed directly, but results from the mean horizontal flow in the boundary layers, across which heat is transferred by vertical diffusion to the horizontal boundaries. Both boundary layers contribute to the horizontal temperature gradient in the direction opposite to \( u_0 \), though unequally because of the asymmetry between top and bottom boundary conditions.

The heat flux balance in steady state can be used to obtain a relation between \( \beta \) and the externally imposed parameters, \( \text{Pe} \) and \( R \). The temperature equation (7) is integrated over a segment of the cell bounded by \( 0 < z' < 1/2 \) and \( x_1 < x' < x_2 \), where \( x_1 \) and \( x_2 \) are chosen to exclude regions of strong vertical flow (e.g. at the ends of the cell) and the upper level, \( z' = 1/2 \), is chosen because the vertical temperature gradient in this part of the cell is negligible. Hence, for the fields defined by (11), with \( \beta' \) the mean temperature gradient between \( x_1 \) and \( x_2 \),

\[ \beta' \int_0^{1/2} U' \, dz' = 1. \]  

In physical terms there is an approximate balance between the heat flux incident on the base of the layer and the horizontal advection of heat.

Using (15) to evaluate the integral in (16), gives the positive definite quadratic equation:

\[ \frac{R \beta'^2}{32} - \frac{3 \text{Pe} \beta'}{2} - 12 = 0 \]  

and of the two zeros, the correct one has the opposite sign to \( \text{Pe} \). For the Péclet number sufficiently large,

\[ \beta' = -\frac{8}{\text{Pe}}, \]  

provided the assumptions used above remain valid.

Clearly the numerical coefficients of (17) are approximate. In addition to the other approximations used, the integration level \( z' = 1/2 \) was chosen arbitrarily. Any level outside of the upper and lower thermal boundary layers is consistent with the other approximations, and could be used in (16), resulting in minor changes to the numerical coefficients of (17). However, the form of the dependence on \( \text{Pe} \) and \( R \) is unaltered, and the resulting numerical values of \( \beta' \) only change slightly.

5 Comparison of numerical experiments and parameterization

To estimate \( \beta' \) for the large aspect ratio cells generated in the numerical experiments, the slope of the least squares line of best fit to the mean temperature curves (Fig. 4) was used. For the low Péclet number runs, the profiles are more irregular, but it still appears that there are regions of the layer where the profile is approximately linear, and these sections are offset at places where the vertical advection of heat by the plumes is important. In this case,
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... $eta'$ was also estimated graphically on the different subsections, as shown on the profiles (e.g. Fig. 4e, f).

The values of $eta'$ are listed in Table 1 and a comparison between the values of $eta'$ measured from the experiments and the prediction of equation 17 is shown in Fig. 5. For the three different Rayleigh numbers used in the experiments, $eta'$ is shown as a function of $Pe$. For the low $Pe$ runs, the vertical bars connect two estimates from the same run. The smaller value of $eta'$ is from the least mean squares slope for the whole profile. The larger is the graphical estimate from the linear subsections, as described above. With the latter values, agreement with equation (17) is good over the range of parameter values tested.

Vertical profiles of the horizontally averaged temperature are shown in Fig. 6. As the Péclet number is increased at constant $R$ (Fig. 6b) the most obvious effect is the marked decrease in the temperature of the relatively isothermal core, implying a significant decrease in the mean temperature of the entire cell. The asymmetry between upper and lower thermal boundary layers reflects the difference in velocity on the two boundaries. The upper thermal boundary layer is more effectively thinned by the increased shear as $Pe$ is increased. The mean temperature at the base of the box does not change significantly with $Pe$, though it clearly depends on $R$ (Fig. 6a). Note that the thinning of the upper boundary layer is in contrast to free convection with a temperature-dependent viscosity, for which the higher viscosity at low temperatures acts to thicken the upper boundary layer. In the mantle both effects may compete, so the upper boundary layer thickness calculated on the basis of variable viscosity (Torrance &Turcotte 1971) would be reduced for a fast moving plate.

The horizontally averaged horizontal velocity profiles are shown in Fig. 7(a). For large $Pe$, the profile is approximately the parabola given by (15), with $R\beta'/Pe$ increases. As $R\beta'/Pe$ increases the horizontal shear near the upper surface $(\partial u/\partial z)$ decreases because the internal buoyancy forces make an increasing contribution to the forces driving the flow. The maximum velocity of the return flow at depth $(u'/Pe)$ also increases. In Fig. 7(b) the solid line is from (15), and the points are taken from the experiments, using the least mean squares values of $\beta'$. The curve is not exactly linear, because the depth of the turning point increases from $z'=0.33$ to 0.25, as $-R\beta'/Pe$ increases from 0 to 50.

![Figure 5. $\beta'$ versus $-Pe$ for the experiments and for equation (17) ($Pe$ is negative in the experiments). The three lines are for $R = 721000$, 240300 and 72100 in order of $\beta'$ increasing. The symbols denote the Rayleigh number for the numerical experiments: $R = 721000$ (diamond), 240300 (triangle) and 72100 (circle). The vertical bars connect two points for the same experiment (see text).](https://academic.oup.com/gji/article-abstract/75/2/309/603120)
Figure 6. Horizontally averaged temperature profiles for: (a) $\text{Pe} = 370$ (2.5 cm yr$^{-1}$) and $R = 72100$, 240300 and 721000 in order of decreasing mean $T'$. (b) $R = 240300$ and $\text{Pe} = 740, 518, 370, 222$ and 148, in order of increasing mean $T'$. Temperature is non-dimensional.

The experimental velocities are generally slightly lower than the prediction because the averaging in the experiments includes the ends of the box, where the horizontal velocity is negligible. Even in flow examples of type D, where the flow is clearly broken into separate cells, the relation between the least mean squares temperature gradient and average velocity approximately holds. The horizontal velocity of the boundary layer plumes, observed from the time-dependent $x$ coordinate of the local Rayleigh number maximum (Section 6) is slightly less than $u'$, but shows a similar dependence on the parameters $R$ $\beta'$ and $\text{Pe}$.

A further test of the model is made by comparing the non-hydrostatic pressure field (Appendix B) for the numerical and the parameterized versions. In Fig. 8(a), $\partial p_1/\partial x'$ is shown as a function of depth for one experiment. The solid line is from equation (B1), using the values of $R$ and $\text{Pe}$, and the measured value of $\beta'$. The points show the numerical values
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Figure 7. (a) Horizontally averaged velocity profiles (normalized to the surface velocity). $u'_i$ is the maximum return flow velocity. $R = 240300$, and in order of increasing $u'_i$, $Pe = 1480, 518, 370, 222$ and 74, showing the effect of increasing $\beta'$. (b) $u'_i/Pe$ versus $R\beta'/Pe$, with the solid line from equation (15). For the experiments the symbols denote: $R = 72100$ (solid star), 240300 (circle) and 721000 (open star).

of $\partial p_i/\partial x'$ determined from the slope of the least squares line of best fit of the horizontal pressure profile at the given depth. The agreement is excellent, though the magnitude of $\partial p_i/\partial x'$ is slightly greater (3 per cent) than that given by (B1) at $z' = 0$, and slightly less at $z' = 1$. The discrepancy indicates a variation in $\beta'$ with depth, which is neglected in the parameterization. Note that the sign of the horizontal pressure gradient is reversed at $z' \approx 0.75$.

From equation (B1), the depth averaged pressure gradient $dp_i/\partial x'$ depends only on the Péclet number and the scaling factor $(\kappa v\rho_0/d^2)$, which is a constant in the numerical experiments. Fig. 8(b) shows the excellent agreement, over the range of Rayleigh numbers tested, 11
between the mean horizontal pressure gradient from equation (B1) and that measured from the slope of the average pressure profiles of the numerical experiments.

The agreement between the numerically averaged velocity and pressure profiles and those predicted by the parameterized model is a good test of the accuracy of the numerical experiments, and in particular, of the constant velocity boundary conditions on the horizontal boundaries. The least mean squares value of $\beta'$ gives a good estimate of the least mean squares horizontal pressure gradient and the average return velocity, even if the flow is of type D, where the respective profiles are not linear.

The 2-D bathymetry and gravity anomalies were calculated, as described by McKenzie...
Figure 9. Bathymetry and gravity profiles for the experiments: (a) A3V3 (type A), (b) A1V3 and (c) A1V7 (type B), (d) A1V4 and (e) A1V5 (type D) and (f) A2V8 (type C). The horizontal scale assumes $d = 700$ km, and the vertical scales are specifically for the physical parameters of Table 2. The gravity anomaly assumes a deformable lower boundary, and the bathymetry profiles were filtered to simulate the effect of a 30 km thick elastic plate.

(1977), for the numerical experiments (Fig. 9). From the above comparison of the pressure field parameterization, it follows that the mean slope of the bathymetry profile is quite accurately obtained from equation (B2). In addition, the details of the profile reflect the type of flow beneath. Rising plumes produce an upward deflection of the lithosphere and sinking plumes produce a downward deflection. For large Pe, the anomalies produced by
the plumes are small relative to the overall level difference between ridge and trench (e.g. Fig. 9b, c), although they dominate the profile when rising sheets are present (Fig. 9d, e). Note that in the parameterized equation (B2), \( \frac{db}{dx} \) changes sign when \( Pe = \frac{-R}{\beta/12} \). For smaller values of \( Pe \) the topography gets deeper moving from ridge to trench (as it should), whereas for larger values of \( Pe \), the topography shallows towards the trench. Note that the bathymetry calculated here has neglected the effect of thermal contraction in the lithosphere (e.g. Parsons & Sclater 1977), which should be added to the calculated profiles.

The 2-D calculation of the gravity field (McKenzie et al. 1974) assumes that the layer extends infinitely in the horizontal direction and the temperature field is reflected

![Diagram](https://academic.oup.com/gji/article-abstract/75/2/309/603120)

**Figure 10.** (a) Comparison of the slope of the gravity anomaly (for an undeformable lower boundary) from equation (B3) and (B4), with the values obtained from the numerical integrations of the experiments. Symbols as for Fig. 7. (b) The slope of the gravity anomaly for the experiments, assuming a deformable lower boundary (as in Fig. 9). The line of best fit to the points is also shown.
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ad infinitum in the side boundaries of the box. The approximate equations derived in Appendix B use a local calculation in which the lateral variation of topography is assumed negligible. As for the temperature and pressure measurements, the slope of the gravity anomalies was obtained for the experiments, from the slope of the least squares line of best fit to the calculated anomaly (e.g. Fig. 9). Figs 10(a and b) show respectively the comparison between experiment and parameterization for the cases of: (a) a perfectly rigid lower boundary, and (b) a lower boundary which is deflected in the vertical sense by the anomalous pressure field at the base of the box.

The comparison shows that the form of the parameterization (B3-B5) is appropriate for describing the slope of the gravity anomalies, in spite of a numerical difference which is probably due to the influence of the reflection boundaries in the 2-D calculation. For both cases (a) and (b) above, the points fall approximately on a straight line. For the case in which the lower boundary is allowed to deflect, note that the magnitudes of the anomaly slope are approximately an order of magnitude less than those with the rigid lower boundary. The three gravity components do not cancel completely, as suggested by equations (B3-B5), though the slope of the anomaly appears to retain its linear dependence on the parameter $R\beta/Pe$. For both assumptions (a) and (b), the gravity anomaly becomes more positive moving from ridge to trench, in all of the experiments.

6 Boundary layer plumes

An important feature of the flow in the large aspect ratio cells shown here (and in those described by Lux et al. 1979, Hewitt et al. 1980 and Nataf et al. 1981) is the periodic instability of the thermal boundary layers. Fig. 11 shows a time sequence in the temperature field which covers a complete cycle in the formation of a boundary layer plume, for $R = 240300$ and $Pe = 370$.

The plume first appears as a local maximum in the temperature contours of the boundary layer. It grows, as for a Rayleigh–Bénard instability, but is advected towards the rising region by the main shear flow. The boundary layer is then rapidly thinned to approximately

![Figure 11](https://academic.oup.com/gji/article-abstract/75/2/309/60320/18-April-2019)
half its maximum thickness as the plume moves up into the relatively isothermal layer above. As the plumes move in a train towards the ridge, they continue to channel hot fluid upwards, so the boundary layer thickness is kept at a fairly stable value. The lower thermal boundary layer is thicker, and therefore less stable, than the upper thermal boundary layer, because it does not move relative to the sides of the box.

Because the important length scale for the instability of a boundary layer is \( \delta \), the thickness of the boundary layer, rather than \( d \), the modified Rayleigh and Péclet numbers for the boundary layers are first defined:

\[
R_b = \frac{g \alpha T_b \delta^3}{\kappa \nu},
\]

\[
Pe_b = \frac{u_0 \delta}{\kappa},
\]

where \( T_b \) is the temperature contrast across the thermal boundary layer. At each \( x \) coordinate, \( T_b \) and \( \delta \) are obtained from the temperature–depth profile. \( T_b \) is the difference between the temperature on the boundary and the depth averaged temperature, and \( \delta \) is the vertical coordinate at which the linear extension of the boundary layer geotherm (based on \( \partial T/\partial z \) at the first interior mesh point) intersects the mean temperature of the layer. These definitions of \( T_b \) and \( \delta \) are not unique but they are representative of the characteristic temperature and depth scales of the boundary layer. Different definitions should lead to similar results, though numerical factors may differ slightly.

For temperature fields such as those in Fig. 11, \( R_b \) can be calculated as a function of the \( x \) coordinate, and the resulting profiles are in fact similar to the isotherms in the lower boundary layer. At a given time the plume appears as a local maximum in \( R_b \). From the point at which it first appears, the local maximum moves in the direction of flow and its value increases (though once the plume has grown to the extent of disrupting the boundary layer, \( R_b \) is meaningless).

To follow the development in time of the plume, this maximum value, \( R_{bm} \), is recorded from the point at which the plume first appears, until another one appears in front of it. Fig. 12 shows the record of \( R_{bm} \) for two different experiments over several periods of plume formation.

For smaller Rayleigh numbers (Fig. 12a) the cycle is repeated with almost constant amplitude and a periodicity which is also observed in small fluctuations of the kinetic energy of the layer. There is a well-defined minimum local Rayleigh number, at which the plumes first appear, and this number is precisely determined after a few periods. In defining a critical Rayleigh number, \( R_{bc} \), for the boundary layer, the appearance of a new plume is interpreted to mean that the local Rayleigh number of the boundary layer has exceeded this critical value. In Fig. 12(a) there is little variation to the value \( R_{bc} = 250 \), though there is a very small systematic increase, indicating that the layer is still slowly adjusting to an equilibrium temperature.

For larger Rayleigh numbers (Fig. 12b) the flow is more irregular and shows a range of periods (and therefore amplitudes) in plume generation. In this example, \( R_{bc} \) is taken to be \( 230 \pm 20 \), with the time variation expressed as the uncertainty in the value of \( R_{bc} \). The critical Rayleigh number was thus obtained for each experiment (where possible), and is listed in Table 1.

As shown in Fig. 13 there is an approximately linear relation between the critical boundary layer Rayleigh number, \( R_{bc} \), and the boundary layer Péclet number, \( Pe_{bc} \), defined
Convection cells in the upper mantle

Figure 12. The value of the first local maximum of the boundary layer Rayleigh number as a function of time, for the experiments: (a) A1V3 and (b) A2V3 (see Table 1). The horizontal lines show the estimated critical value $R_{bc}$:

$$R_{bc} = 88 + 2.6 \text{Pe}_{bc}.$$  \(20\)

Thus, convective disturbances with axes perpendicular to the shear are suppressed as the shear is increased, as found by Richter (1973b), for the convective instability of a uniformly sheared layer.

The most obvious, and probably the most important, effect of the strong temperature dependence of viscosity for the mantle is the rigid lithospheric plate. The plate is incorporated into the model in the example of Fig. 14, to show qualitatively how the solid layer on top affects the large-scale circulation discussed above.
Figure 13. $R_{bc}$ versus $Pe_{bc}$ for the experiments. The symbols show the Rayleigh number, $R = 721\,000$ (diamond), $240\,300$ (triangle) and $72\,100$ (circle). The error bars show time variation due to either short time-scale variation, or long time-scale drift.

Figure 14. Temperature and streamfunction for $R = 240\,300$ and $Pe = 370$, with a solid plate of non-dimensional thickness $1/8$ moving over the fluid layer. Contour intervals are: for $T$, $50^\circ C$ and for $\psi'$, $11.4$ (cf. Fig. 2B). Streamfunction contours are omitted from the solid layer, but they are parallel, evenly spaced lines which join the vertical contours at the ends of the box.

A conducting layer of non-dimensional thickness $1/8$, moving with the same velocity as the upper boundary, is placed above the fluid. Except for viscosity, the physical parameters are identical in both fluid and solid. Velocity, temperature and heat flux are continuous across the boundary. Fluid is pumped out of the box at the ridge by fixing the streamfunction on the boundary. It emerges from a jet of width equal to the plate thickness, and at the same speed as the plate moves horizontally. A similar jet with the opposite velocity is at the trench. Within the plate, an error function solution for the temperature is patched on to the region of discontinuous velocity near the ridge, and the horizontal temperature gradient is set to zero before the cold plate is subducted.
The experiment shown in Fig. 14 has the same values of $R$ and $Pe$ (with $d$ taken as the depth of the fluid layer, excluding the solid plate) as the example of Fig. 2(B). The addition of the conducting layer representing the lithosphere has changed the flow field from type B to type A. Boundary layer plumes which were initially present in the flow field were suppressed as the run became time independent. Calculation of the local Rayleigh number for the lower boundary layer shows a local maximum of $\sim 315$ (near the ridge), which is significantly greater than the critical value of 250 for the corresponding experiment without the solid layer (Table 1).

Thus, the presence of the lithosphere appears to stabilize the flow in large aspect ratio cells for the following reasons: the upper boundary layer (including the plate) is thicker and colder. As this material returns to the ridge after subduction, it requires a longer time to heat up and go unstable. Secondly, because the return flow in the convecting layer must now balance the mass flux in the solid layer, the velocity gradient (shear) across the layer is generally greater, thus suppressing convective disturbances. And finally, the thermal boundary conditions effectively permit a greater horizontal temperature gradient in the convecting layer, which stabilizes the large aspect ratio cell (Houseman & McKenzie 1982).

7 Discussion

The success of the parameterized model in determining the mean properties of the large-scale flow has been demonstrated here for a parameter range which is relevant to convection in the upper mantle, and a specific set of boundary conditions. The key parameter is the horizontal temperature gradient, which is determined by the interaction of the large-scale flow with the given thermal boundary conditions.

The parameterization can be modified simply to accommodate changes in the boundary conditions (e.g. stress free or constant flux upper boundary), provided that the large aspect ratio cells remain stable. Even in cases where the planform is unstable on the long timescale, the parameterization is useful as long as the long cells persist, because the flow field responds immediately to the existing temperature structure, and the time required for an existing horizontal temperature gradient to decay is probably of the order of several overturn times of the cell ($\sim 10^9$ yr for the Earth). A sufficiently large horizontal temperature gradient will stabilize the long cells against the small-scale instabilities such as boundary layer plumes, which are an important component of the flow field.

The major difficulty in applying the results of these experiments to convection in the upper mantle is the uncertainty surrounding the planform of the circulation and the precise boundary conditions. Only a very limited range of possible boundary conditions has been explored by experiment, yet even for 2-D numerical experiments, apparently minor changes in the boundary conditions may result in major changes to the planform (e.g. Hewitt et al. 1980).

The first requirement of a model of upper mantle convection is that the flow is consistent with the large-scale overturn representing the steady formation and subduction of lithospheric plates on the upper surface. Previous attempts to satisfy this broad requirement and permit a 3-D planform in a laboratory experiment have required some method of externally forcing the large-scale flow.

Richter & Parsons (1975) used a moving upper boundary to simulate the mechanical (though not the thermal) effects of the lithosphere. Beneath a stationary plate an irregular 3-D planform (the spoke pattern) is established for Rayleigh numbers typical of the upper mantle. However, with the plate moving horizontally, rolls commence to form, aligned in the direction of shear. For a fast moving plate, such as the Pacific, the alignment should be
fairly complete after a period of 20–50 Ma. While there may still be some contribution to
the flow in these experiments from a horizontal temperature gradient, most of the heat
appears to be transferred by the small-scale longitudinal rolls.

An alternative experiment was constructed by Nataf et al. (1981). In their experiments
the large-scale flow is forced by cooling one of the side boundaries. Thus the thermal effects
of the subducted lithosphere are modelled, though the mechanical effects on the upper
surface are ignored. Fluid cooled by the cold lateral boundary sinks, and moves out across
the bottom of the cell before being heated up by the hot lower boundary. The resulting
planform is more nearly 2-D than that of Richter & Parsons, and a single large aspect ratio
cell forms with axis parallel to the cold side boundary. 3-D boundary layer instabilities are
observed as small blobs of cold fluid sinking from the upper boundary layer, as they are
swept along in the large-scale flow.

The boundary conditions used by Nataf et al. (1981) are significantly different from
those used in the numerical experiments described here. However, the basic assumption of
2-D flow is valid for both. It is clear in the laboratory experiment that a horizontal
temperature gradient is driving the large cell. If the gradient is uniform across the cell, as in
the 2-D experiments, then (15) with Pe = 0 should describe the mean horizontal flow in the
large cell.

The two laboratory experiments described above have each partially modelled the effect
of the lithosphere on the planform, one emphasizing the mechanical effect and one empha-
sizing the thermal effect. The planform inferred for the mantle clearly depends on which of
these effects is deemed more important. However, surface observations on the Earth appear
to rule out the possibility of longitudinal rolls. An irregular pattern of highs and lows in the
geoid appear to indicate instead rising and sinking jets in the convecting mantle (McKenzie
et al. 1980), superimposed on the evident large-scale circulation.

From comparison with the two experiments it thus seems likely that the thermal bound-
dary conditions may be more important in determining the planform for the upper mantle.
An alternative explanation is that a low viscosity layer beneath the lithosphere would reduce
the effective shear sufficiently to prevent alignment of longitudinal rolls occurring on a
reasonable time-scale (Richter & McKenzie 1978).

One of the important constraints on the 2-D models is the observed variation of bathy-
metry and gravity from trench to ridge. If there is no horizontal temperature gradient
driving the flow, the pressure gradient produced by the moving plate predicts that the sea-
floor shallows towards the trench by as much as several kilometres. Thus, Richter &
McKenzie (1978) argued that a narrow low viscosity zone was necessary to decouple the
plate from the convecting layer. Most of the return flow would then occur in the low
viscosity layer, reducing the pressure gradient to an acceptable value, and effectively
decoupling the plate from the convecting layer beneath. Although there would still be a
shallowing towards the trench, the effect would be small and masked by the increase in
depth due to the thermal contraction of the plate as it moves away from the ridge.

As is clear from the results in Section 5, the presence of a horizontal temperature
gradient, \( \beta \), bypasses this argument, because there is a component of the surface pressure
gradient which is proportional to \( \beta \), and thus predicts seafloor deepening toward the trench.
The low viscosity zone is not needed if

\[
|\text{Pe}| \leq \left( \frac{R \beta'}{12} \right), \tag{21}
\]

i.e. it is the horizontal temperature gradient rather than the plate which is driving the flow in
the large aspect ratio cell.

Using the parameters in Table 2, with a plate velocity of 10 cm yr\(^{-1}\) and a heat flux of
Convection cells in the upper mantle

Table 2. Nominal values for the physical constants for convection in the upper mantle (after McKenzie et al. 1974). Uncertainty in the Rayleigh number is largely due to uncertainty in the viscosity. The other parameters are relatively well known.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>g</td>
<td>$10 \text{ m s}^{-2}$</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>$2 \times 10^{-4} \text{ K}^{-1}$</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>$1.5 \times 10^{-2} \text{ m}^3\text{s}^{-1}$</td>
</tr>
<tr>
<td>$\nu$</td>
<td>$2 \times 10^{17} \text{ m}^2\text{s}^{-1}$</td>
</tr>
<tr>
<td>$d$</td>
<td>$7 \times 10^4 \text{ m}$</td>
</tr>
<tr>
<td>$\rho$</td>
<td>$3.7 \times 10^3 \text{ kg m}^{-3}$</td>
</tr>
<tr>
<td>$C_p$</td>
<td>$1.2 \times 10^3 \text{ J kg}^{-1} \text{ K}^{-1}$</td>
</tr>
</tbody>
</table>

80 mW m$^{-2}$, the minimum $\beta$ for the Pacific plate would be approximately 0.1°C km$^{-1}$ in (21). Over a distance of 10,000 km this estimate clearly leads to an unrealistic temperature contrast across the cell. However, the viscosity in Table 2 is probably over-estimated. Estimates of the viscosity based on the onset of small-scale convection beneath the cooling plate require the smaller value of $5 \times 10^{16} \text{ m}^2\text{s}^{-1}$ (Parsons & McKenzie 1978; Houseman & McKenzie 1982), for which $\beta = 0.025 \text{°C km}^{-1}$ is sufficient to drive the flow in the long cell. A mean horizontal temperature contrast of 250°C across the 10,000 km width of the Pacific plate convection cell is physically plausible, and might be detected by seismic velocity anomalies. The difference in travel time for two rays vertically traversing the 700 km depth at the two ends of the convection cell would be of the order of 2 s for P-waves and 4 s for S-waves.

Calculated gravity anomalies for a perfectly rigid lower boundary are far too large. Thus, the main conclusion to be drawn from the calculated gravity profiles (Figs 9 and 10), is that the lower boundary (at 700 km) is pushed down beneath the cold end of the convection cell, and moves up beneath the hot end of the cell. The magnitude of the deflection is inversely proportional to the density difference at this level. While the observed gravity anomalies should show the presence of mantle plumes (e.g. Fig. 9), the contributions to the anomalies from thermal contraction of the lithosphere and small-scale bathymetric features are of comparable or greater magnitude.

Boundary layer instabilities such as those observed in the experiments of Nataf et al. (1981) are similar to those seen in the numerical experiments, but the instabilities appear as jets or blobs, rather than sheets or furrows parallel to the cold boundary. On the basis of the stability analysis of Section 6, the blobs should be elongated in the direction of the flow, because growth in the perpendicular direction is inhibited by the shear flow.

There is an interesting analogy between the boundary layer plumes described here and the mantle plumes which are though to underlie hot spots. The traces of the different hot spots on the Pacific plate show that the plate is moving relative to several localized sources of hot material, which do not move significantly with respect to each other. They form a local reference frame, and are sometimes used as a global reference frame for the measurement of plate velocities. However, Molnar & Atwater (1973) showed that the hot spots on different plates do move relative to each other. These observations are consistent with the idea of boundary layer instabilities, which are advected by the flow beneath the plates. If the mean flow is approximately 2-D, the plumes beneath one plate move at the same velocity, advected by the return flow. Relative to plumes beneath another plate, the velocity is determined by the relative plate velocities.
Acknowledgments

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References


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**Appendix A: steady state convection in square cells with no-slip boundaries**

For convection in square cells with stress-free boundaries, McKenzie *et al.* (1974) calculated the dependence on Rayleigh number for parameters such as mean temperature, boundary layer thickness and maximum flow velocity. The corresponding results for convection with no-slip horizontal boundary conditions are given here. The boundary conditions shown in Fig. 1 were used with $\text{Pe} = 0$ and $\lambda = 1$. Each experiment was run to steady state for five different Rayleigh numbers, using the parameters of Table 2 and values of $F = 1, 3, 10, 30$ and $100 \text{mW m}^{-2}$.

For each solution, the following parameters were measured: The mean temperature of the cell, $\bar{T}'$, the maximum $x$ and $y$ components of the velocity $u_{\text{rms}}$ and $v_{\text{rms}}$, the root mean square velocity $u_{\text{rms}}$, and the thickness of the top and bottom boundary layers, $\delta_T$ and $\delta_B$. The least squares line of best fit, of the form

$$\log_{10}P = A \log_{10}R + B,$$

was calculated for each parameter, $P$. The values of $A$ and $B$ are given in Table A1; $v_{\text{rms}}$ is in fact the maximum velocity, since it is measured where the fluid is moving vertically along the stress-free sides of the box. $v_{\text{rms}}$ scales as the square root of the total kinetic energy of the box. The boundary layer thicknesses are obtained, as described in Section 6, using the horizontally averaged temperature profile. Note that the constant flux condition on the lower boundary results in a slightly thinner boundary layer than constant temperature on the upper boundary.

<table>
<thead>
<tr>
<th>$P$</th>
<th>$A$</th>
<th>$B$</th>
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<tbody>
<tr>
<td>$\bar{T}'$</td>
<td>-0.194</td>
<td>0.123</td>
</tr>
<tr>
<td>$u_{\text{rms}}$</td>
<td>0.552</td>
<td>-0.855</td>
</tr>
<tr>
<td>$v_{\text{rms}}$</td>
<td>0.438</td>
<td>-0.487</td>
</tr>
<tr>
<td>$u_{\text{rms}}$</td>
<td>0.477</td>
<td>-0.793</td>
</tr>
<tr>
<td>$\delta_T$</td>
<td>-0.147</td>
<td>-0.083</td>
</tr>
<tr>
<td>$\delta_B$</td>
<td>-0.144</td>
<td>-0.116</td>
</tr>
</tbody>
</table>

**Appendix B: elevation and gravity anomalies for the long cell**

The non-hydrostatic pressure field is obtained for the velocity—temperature distribution of equation (11) by integrating equations (12), using (15) and the arbitrary boundary condition
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\[ p_1' = 0 \text{ at } (\lambda/2, 1): \]

\[ p_1' = \left[ \frac{6 \text{Pe}}{R} + \beta'(z' - \frac{x'}{2}) \right] \left( x' - \frac{\lambda}{2} \right) - \int_{z'}^{1} T' \, dz'. \]  

(B1)

The zero vertical velocity boundary condition on the surface \( z' = 1 \) is thus satisfied if the excess pressure is balanced by excess topography, \( b \), which in dimensional terms is:

\[ b(x) = \left( \frac{\rho \nu k}{g d^3 \Delta \rho} \right) \left[ 6 \text{Pe} + \frac{5}{2} R \beta' \right] \left( x - \frac{\lambda d}{2} \right), \]  

(B2)

where \( \Delta \rho \) is the density difference between mantle and seawater. The bathymetry is arbitrarily zeroed at \( x = \lambda d/2 \).

The variation in density across the cell, and the deflection of the upper and lower boundaries of the cell all contribute to the gravity anomaly above the cell. For an infinite plane layer the contribution to the gravity anomaly from the linear (in \( x \)) density variation within the layer is

\[ \Delta G_1(x) = -2\pi G \left( \frac{\rho \nu k}{g d^3} \right) R \beta'(x - \lambda d/2), \]

(B3)

where \( G \) is the gravitational constant. The component of the gravity anomaly from the elevation \( b(x) \) of the seafloor is approximated at \( x \) by the anomaly for an infinite layer of thickness \( b(x) \) and density \( \Delta \rho \):

\[ \Delta G_2(x) = 2\pi G \left( \frac{\rho \nu k}{g d^3} \right) \left[ 6 \text{Pe} + \frac{5}{2} R \beta' \right] (x - \lambda d/2). \]

(B4)

If the lower boundary of the convection cell is not perfectly rigid, but is also deformed by the pressure perturbation on \( z' = 0 \), there is a third contribution to the gravity anomaly:

\[ \Delta G_3(x) = 2\pi G \left( \frac{\rho \nu k}{g d^3} \right) \left[ -6 \text{Pe} + \frac{5}{2} R \beta' \right] (x - \lambda d/2). \]

(B5)

The sum of the three components of the gravity anomaly thus cancels in this idealized model.