Nordic Hydrology, 9, 1978, 75-94

Wind Induced Circulation in Lakes

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In most lakes the wind is the most important flow generating mechanism. In this paper the problem of wind generated circulation – directly wind induced currents and seiches – in small lakes is reviewed. Many field observations are presented and discussed. In the thermocline and the hypolimnion forced seiche currents are shown to dominate the directly induced wind currents. Different kind of non-convective mathematical lake models are discussed and applied to different small lakes. Comparisons of observed and calculated currents show that lake models can be used to reproduce the currents of the upper 3-4 metres in a lake. The interaction between large-scale flow and turbulent flow is yet unknown, and therefore it is not possible to explain the physical current pattern and density anomalies at greater depth. In respect to the limited knowledge on turbulent processes in lakes, it is acceptable to apply a quadric relationship between wind stress and wind speed with a drag coefficient of about $1.0 . 10^{-3}$.

Introduction

The quality of the water of a lake is dependent on the circulation and diffusion processes in the lake. The water movements in a lake are induced by 1) the wind, from which momentum is transferred to the water, 2) by through-flow, and 3) by horizontal density differences caused by surface cooling or heat flow from the sediments. Tides and uneven distribution of atmospheric pressure are of no importance. The circulation is dependent on a number of factors such as lake morphometry, shore configuration, bottom conditions, wind exposition, general climatic conditions, relative position of inlet and outlet, the rotation of the earth, density stratification etc. The wind also generates waves-ordinary wind waves from the transfer of energy from wind to waves as well as standing waves with wavelengths determined by the dimensions of the lake and speed by the average depth.

In most lakes the wind is the most important flow generating mechanism. The directly induced wind-driven currents as well as the seiche currents are at least of the order centimetres/second. In this paper these two types of wind generated circulation in lakes of dimension 5-100 km² are discussed.

Momentum Transfer at the Water Surface

When the wind blows over a lake, it exerts a shearing stress on the water surface. Momentum is transferred from the wind to the surface and due to the turbulence transported further down into the water. Accurate knowledge of the force exerted by the wind on the surface of the water is a prerequisite for every attempt to analyse the movements of the water. The two factors, which, apart from the wind speed at high levels, affect the motion of the air near the water surface the most, are the characteristic roughness of the water, z_0, and the restoring force due to thermal stability. Over water with weak winds the water surface may be considered as aerodynamically smooth, which means that also molecular properties strongly influence not only the structure of temperature but also the structure of momentum. The air current flows over a moving surface, and the water surface velocity influences the wind drag. The formula used for determining the wind stress from a given wind speed is

$$\tau = \rho_{air} C_D (W - u_0)^2 \tag{1}$$

where

τ = wind stress
ρ_{air} = density of the air
C_D = drag coefficient
W = wind speed of a given height above the water surface
u_0 = water surface velocity

For ground surfaces the characteristic roughness is a truly characteristic of the surface. The water surface, however, changes character with increasing wind speed and becomes more corrugated. Also the dimensions of a lake influences the characteristic roughness of the water. The longer the fetch is the higher waves may be generated. It is clear that there exists no such thing as a true characteristic roughness parameter of water surfaces. The surface velocity, u_0, is of very little importance, since it is only about two percent of the wind speed.

Literature Survey – Although it may not be possible to truly define a roughness parameter of a lake, this parameter or rather the drag coefficient has often been determined from wind measurements and from observations of wind set up. Wilson (1960) has analysed the literature on drag coefficients over oceans and computed average values of the drag coefficient referred to an elevation of 10 metres as

$$C_D = 1.5\cdot 10^{-3} \pm 0.8\cdot 10^{-3} \quad \text{for} \quad W < 6 \text{ m/sec}$$

$$C_D = 2.4\cdot 10^{-3} \pm 0.6\cdot 10^{-3} \quad \text{for} \quad W > 10 \text{ m/sec}$$

From the Great Lakes of North America much lower values have been reported, cf. Ryznar and Portman (1964) and Bruce et al. (1961). Recent observations indicate than an appropiate value for the Great Lakes is $1.2\cdot 10^{-3}$. For example Donelan, Elder and Hamblin (1974) estimated the wind stress from the steady-state water set-up of Lake Ontario and found the drag coefficient to be $1.3\cdot 10^{-3}$ for stable and neutral conditions and $1.6\cdot 10^{-3}$ for unstable conditions.

For enclosed water bodys and fetches of the order 10 kilometres the reported values on drag coefficients are scarce. Fleagle, Deardorff and Badgley (1958) found a value of $1.2\cdot 10^{-3}$ with no variation over the range 3-9 m/sec. For even smaller fetches (0.5-10 kilometres) some results from Lake Pääjärvi in Finland have been analysed. From the results given by Huovila (1971) the drag coefficient in neutral stability can be determined to $0.9\cdot 10^{-3}$ referred to 10 metres elevation. More than 150 daily hours of measurements of wind- and temperature profiles over Lake Vomb (3×5 km^2) in southern Sweden have been described by Bengtsson (1973a). It was found possible to describe the wind stress by means of the formula (1) with a drag coefficient

$$C_{10} = 1.2\cdot 10^{-3} \quad (W > 5.5 \text{ m/sec})$$

$$C_{10} = 0.9\cdot 10^{-3} \quad (W < 4.5 \text{ m/sec})$$

Smooth Surfaces – For at least light winds over small lakes the water surface may be regarded as smooth and a laminar boundary layer next to the surface may be assumed to exist. Then the wind velocity distribution above this layer should obey the law established by von Kármán for flow over smooth plates. Eq. (1) can still be used in order to determine the wind stress, provided the drag coefficient once for all is determined for different wind speeds. The drag coefficient will be $0.85\text{-}0.9 \cdot 10^{-3}$. Bengtsson (1973a) found that for neutral air stratification over Lake Vomb the von Kármán profile fitted observed wind profiles for wind speeds below 5 m/sec. better than the logarithmic profile did. However, a drag coefficient of $0.9 \cdot 10^{-3}$ was also found from the logarithmic profile.

Non Neutral Stratification – In neutrally stable air mechanical turbulence predominates. When unstable stratification prevails mechanical turbulence is augmented by thermally induced turbulence, while in the stable case turbulence is suppressed by thermal stratification. At heights which are small compared with the Obukhov length scale mechanical turbulence is still dominant and the logarithmic wind profile is approached. Following the similarity hypothesis developed by Obukhov and Monin, cf. Monin and Yaglom (1973), the wind profile for small departures from neutrality is the familiar log-linear profile

$$W = \frac{W_F}{k} \ln \left(\frac{z}{z_0}\right) + \beta \frac{z}{L} \tag{2}$$

where

L = Obukhov length

β = constant, which may vary depending on stratification.

Only few analysis of measured wind profiles in non-neutral stratification over lakes have been made. Some investigations by Bengtsson (1973b), Paulsson (1967) and Hoeber (1968) imply that β in stable stratified air should be somewhat smaller than 10. This is consistent with observations over land. For weak winds (W<5 m/sec.) Bengtsson, however, found the coefficient to be as small as 1.5.

In the case of unstable stratification a transition from the logarithmic law to a free convection profile takes place in a very thin layer. Hence, it is for unstable air not advisable to apply the logarithmic – linear approximation widely. Still,many investigators have found observed wind profiles to fit the log-linear law also for unstable stratified air over lakes. Bengtsson (1973b) found β to be 0.6-1.0, but Deacon (1962) and Webb (1960) found β to be considerably higher 3-10.

It is clear that the drag coefficient is larger for unstable cases than for neutral and stable conditions. This was also, as previously mentioned found by Donelan et al. (1974). However, from Bengtssons measurements over Lake Vomb the drag coefficient referred to 10 metre elevation was found to be rather insensitive to air stratification, although the wind profiles deviated from the logarithmic profile. When the Obukhov length is 10 metres or more, it seems possible to use drag coefficients referred to 10 metre found from neutral stratification.

Distribution of Wind Stress – Once knowing the wind speed over a lake it is not a big problem to determine the stress. There is still, however, a problem of determining the distribution of wind stress over the lake. When air moves from rough terrain to smooth water surface, the surface stress drops. The wind profile adjusts to the new surface very quickly at low elevations but more slowly a little higher up. Bengtsson (1973a) investigated the distribution of wind speed and wind stress over the ice covered Lake Velen, which is surrounded by woods. The wind stress was found to be evenly distributed over the lake. Already 25 metres out from the shores the wind stress agreed with the wind stress on the central part of the lake except for onshore winds, for which the stress increased about 10%, although the wind speed dropped about 10% at this distance from the shore. When analysing currents it should thus be enough to use wind measurements obtained in the central part of a lake for the entire lake. This is of course not true for very large lakes.

Relationship Land-Lake Winds – Often in dealing with winddriven circulation in lakes one has to transform land-winds to winds over lakes. Empirical studies have shown (Hunt 1958) that the ratio of wind speed over water to that over land is representative at 1.3. Still, from a theoretical point of view it is evident that the ratio must vary with wind speed and stability, such that the ratio increases with decreasing wind speeds and when the water surface is warmer than the air. Resio and Vincent (1977) compared empirical results obtained from oceans and the Great Lakes of North America. For wind speeds below 5 m/sec. the ratio exceeded 1.6. However, when the wind blows off the land it takes tens of kilometres to alter the wind profile from land to lake profile. Thus for small lakes of 5-50 km² the ratio of lake wind speed to land wind speed must be considerably less.

Conclusions – From my review of the literature I conclude that for small lakes the drag coefficient referred to 10 metres should be

$$C_D = 0.9\cdot10^{-3} \quad \text{for} \quad W < 5 \text{ m/sec}$$

$$C_D = 1.1\cdot10^{-3} \quad \text{for} \quad W = 5\text{-}10 \text{ m/sec}$$

These coefficients can be used even for such low values of $W^2/|T_{water}-T_{air}|$ as 5 m²/sec², °C. For large lakes *and* very high wind speeds of about 20 m/sec the dragcoefficient is considerably higher, about $2\cdot10^{-3}$.

For small lakes the wind stress can be considered evenly distributed over the lake. It is not possible to postulate a relationship between land-lake winds.

Empirical Estimations of Currents

Many researchers and engineers suggest that there exists a simple relationship between surface currents and wind speed, the ratio being 2-3%. At least for small lakes this is certainly not true. From investigations in a great number of Swedish lakes the author has found firstly the ratio to be smaller than that proposed above (1-2%), secondly that it decreases with increasing wind speed, and thirdly that it increases with increasing lake dimension. Also, in most small lakes the surface current pattern is so complex that it is hard to define some mean surface current velocity. This is clearly recognised in Fig. 1, which shows the surface current pattern generated by a northerly wind of 6 m/sec. blowing over Lake Ivö in southern Sweden.

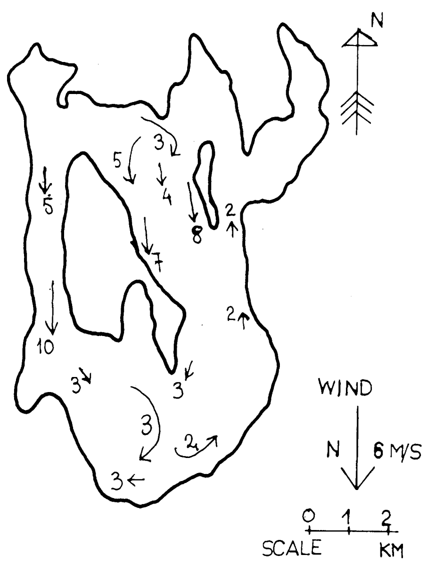

Fig.1. Observed current pattern at depth 1 m in Lake Ivö, Sweden, for wind N 6 m/sec. Depth of thermocline 12 m. The numbers show the stream velocities in cm/sec.

The vertical velocity gradients of wind generated currents in a closed channel must for continuity reasons be quite sharp. This does not need to be true for lakes, and for large lakes it is often claimed that velocity gradients are very small except for the top metre. However, once again referring to his measurements in small Swedish lakes the author has found the velocity gradients in these lakes to be pronounced also at greater depths. Down to 5-6 metres the velocity gradient was found to be about (z being the distance downward from the surface)

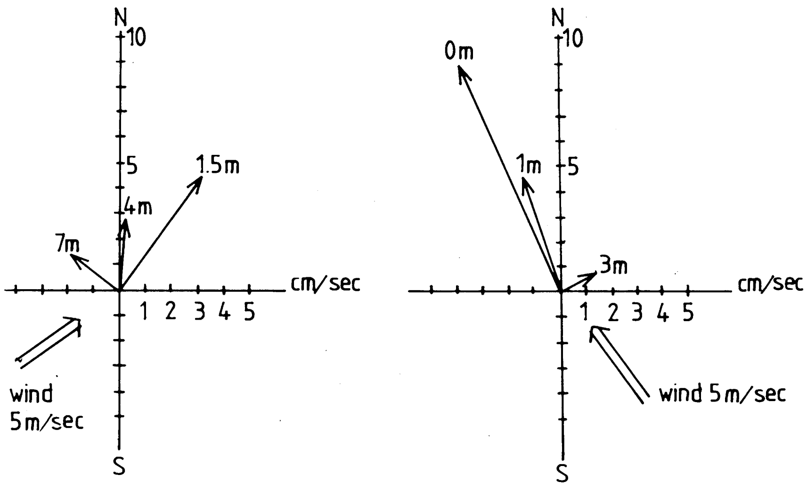

Fig.2. Measured currents in the central part of Lake Velen (left) and Lake Möckeln (right). The measuring depth is indicated at the arrows.

$$\frac{\partial u}{\partial z} = (0.5\text{-}1)\,\frac{u_0}{z}$$

The complex velocity gradients in the top layer are shown in Fig. 2, which shows measured velocity »spirals« from Lake Velen and Lake Möckeln.

Most Swedish lakes are, unless they are very shallow and wind exposed, stratified during the summer. Warmer epilimnion water is separated from colder hypolimnion water by a more or less pronounced thermocline. Some typical examples are shown in Fig. 3. From Fig. 3 it is noted that the thermocline is rather sharp. It is well known that strong stratification inhibits mixing of momentum. The turbulence level at the thermocline is thus very low, and the thermocline can often be regarded as a frictionless boundary. This is confirmed by field observations, which show that the velocity profile often has a maximum just above the thermocline and also that currents above and below the thermocline are oppositely directed. Momentum is probably transported through the thermocline where it touches the bottom, but this fact has not yet been observed in the field. The bottom currents are much more dependent on the density stratification than the surface currents are. Droques released below the thermocline have often been observed to move slowly back and fourth or in horizontal circulation cells as suggested by Eriksson and Bengtsson (Falkenmark ch. 7, 1973).

Investigations carried out by the Swedish IHP (Bengtsson 1973b, 1978 and Falkenmark 1973) showed that steady-state epilimnion currents generally were obtained after 3-5 hours of fairly steady winds, but in the hypolimnion steady-

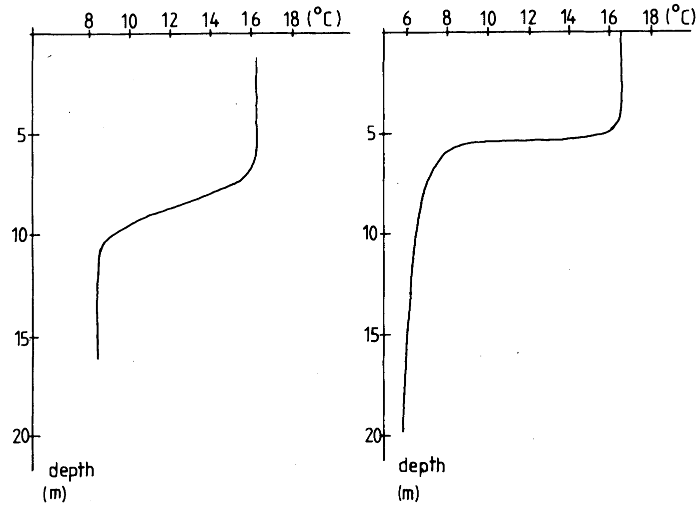

Fig.3. Summer stratification 1969 of Lake Velen (left) and Lake Tivsjön (right).

state conditions were never found to exist. In the hypolimnion and also in the thermocline seiche currents may very well dominate the directly wind induced currents. By analysing temperature data from Lake Velen, Bengtsson (1973b) calculated the currents of the epilimnion, the metalimnion (the thermocline) and the hypolimnion. The rather interesting result from June 5, 1970, is shown in Fig. 4.

The common explanation for the occurrence of these seiches is the following mechanisms. When winds act on the surface the warm surface water forms a wedge-like structure downwind of increasing thickness, while the cooler water

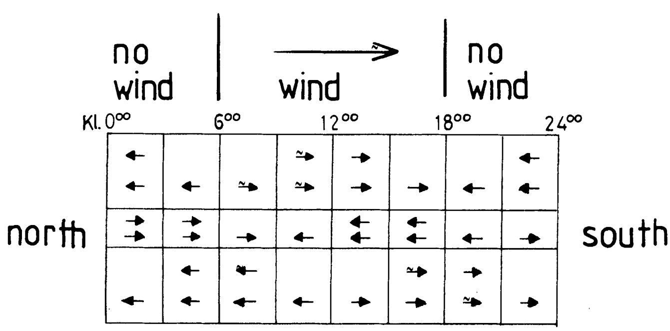

Fig.4. Seiche currents in Lake Velen June 5, 1970, calculated from Temperature observations. Two arrows in a layer indicate strong currents (3 cm/sec) and one arrow indicates small currents (1 cm/sec).

accumulates at the upwind end of the lake. The thermocline, hence, is strongly inclined. When the wind stops blowing the readjustment of water takes place as an oscillation, i.e. a standing wave. The amplitude of the internal seiche is comparatively large so that the mass movements of water connected to it become appreciable. In Lake Velen the theoretical oscillation period and also the observed periods are somewhere about 20 hours. However, since this is very close to the diurnal period of wind speeds, the oscillations of the thermocline must not be free oscillations but may be forced oscillations. In fact, spectrum analysis made on vertical positions of the thermocline as time series do not support the idea of free oscillations. In his work for the Swedish IHP (1975) Edenman analysed the relationship between »heat displacements« determined from temperature measurements and current pattern in the narrow basin of Lake Velen. During three days in mid July 1970 he used drogues to measure currents, and he also analysed temperature data for this period. On July 9 the wind was blowing from the south and increased from zero velocity in the forenoon to 5 m/sec. in the afternoon and weakened again in the evening. The temperature profile and the measured velocity profile of the afternoon is shown in Fig. 5. It is interesting to note the strong compensation current just above the thermocline and also the homogeneity of the hypolimnion currents.

From the observed displacements of the isopleths and knowing the morphometry of the southern basin Edenman was able to estimate velocities associated with horizontal heat displacements. He found the velocity distribution in the thermocline to be very complex, but generally with low velocities (1 cm/sec) in the upper part of the metalimnion and higher velocities (5 cm/sec) in the lower part. The

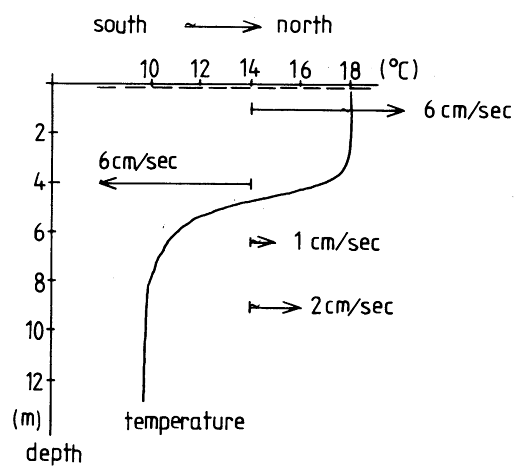

Fig.5. Measured temperature profile and measured velocities in the southern part of Lake Velen, afternoon July 9, 1970.

currents in the thermocline and in the bottom layer were due to the oscillations of the isotherms and were not directely induced by the wind. The insignificance of epilimnion currents on hypolimnion currents was shown by Bengtsson (1978) for winter reversed stratification.

The vertical motions of the temperature pattern naturally imply large horizontal displacements of water masses. From observations of the diurnal vertical temperature variation across a section of the Bay Habelsbolsviken of Lake Velen, Bengtsson and Eriksson (Falkenmark 1973) found that during the period 20-24 May, 1971, all the water of the bay was exchanged every day. This investigation has been extended further by Andersson (Bengtsson 1978), who showed that due to vertical temperature displacements unevenly distributed over a lake, the water exchange between a shallow bay and its main basin is usually quite intense.

Mathematical Models of Wind Induced Circulation

From experiences obtained from field observations one can state that the general problem of wind induced lake circulation is very complex and remains unsolved. However, in recent years wide use has been made of mathematical models of lake circulation. Such models are based on hydrodynamical and thermodynamical principles expressed by mathematical equations. In principle the motions of lake waters can be calculated from numerical solutions of the hydrodynamical differential equations. However, the field of motion in a lake is composed of a range of scales of motion all the way down to scales associated with turbulence. In fact, the interaction between large-scale flow and turbulent flow is completely unsolved, although some promising approaches have been made by for example Svensson (Bengtsson 1978). In lake modelling this interaction is approximated by parameterization of the small-scale phenomena. Vertical and horizontal exchange coefficients are introduced.

Already in 1956 Hansen (1956) applied a one layer storm surge model to the North Sea. In such one-layer models the lake is represented by one layer of fluid with no vertical gradients. Apart from knowing the topography of the basin and in-outflow conditions a bottom friction coefficient must be assigned an appropriate value when utilizing this model.

The two horizontal momentum equations are vertically integrated over the depth and divided by the depth to give

$$\frac{du}{dt} - \frac{(\tau_0 - \tau_B)_x}{\rho h} - fv - A_H \Delta u + g\frac{\partial n}{\partial x} = 0$$

$$\frac{dv}{dt} - \frac{(\tau_0 - \tau_B)_y}{\rho h} + fu - A_H \Delta v + g\frac{\partial \eta}{\partial y} = 0 \qquad (3)$$

where

u, v	= stream velocities in x and y-directions respectively
$(\tau_0, \tau_B)_{x,y}$	= wind and bottom shear stress in x and y-directions
f	= Coriolis' parameter
g	= acceleration of gravity
A_H	= horizontal exchange coefficient
Δ	= horizontal Laplace operator
η	= water elevation
h	= depth
ρ	= density of water

A quadratic relationship between bottom stress and velocity is assumed such as

$$\begin{aligned} \tau_{Bx} &= - r\rho\, v\,(u^2+v^2)^{\frac{1}{2}} \\ \tau_{By} &= - r\rho\, v\,(u^2+v^2)^{\frac{1}{2}} \end{aligned} \qquad (4)$$

where

r = bottom friction coefficient

The mathematical system is closed by the vertically integrated continuity equation, which takes the form

$$\frac{\partial \eta}{\partial t} + \frac{\partial}{\partial x}\,(u(h+\eta)) + \frac{\partial}{\partial y}\,(v(h+\eta)) = 0 \qquad (5)$$

The boundary values to be used are zero normal stream velocities at the shores and known conditions at inlets and outlets. This model should describe circulation pattern generated by through-flow in thermally homogeneous lakes. Since the model does not allow for vertical velocity gradients it is not appropriate for wind induced circulation, although the wind stress τ_0 can be incorporated.

In a series of papers Simons (1973) extended the approach made by Hansen and developed a three-dimensional multi-layered type of lake model. He assumed the water layers to be rigid, horizontal and permeable. Hydrostatic equilibrium is assumed, and therefore vertical accelerations due to buoyancy effects are excluded. Although the model is developed for stratified lakes, the experiences of the Swedish IHP-Group on Lake Hydrology is that our knowledge about physical processes in small lakes is so limited that the model should only be applied to homogeneous or two-layered summer stratified lakes. The layer thickness in the epilimnion should not exceed two metres and the top layer should not be thicker than one metre.

In a multi-layered model the equations for the layered density homogeneous system are obtained by vertical integration over each layer. No vertical gradients are assumed to exist within a layer. Excluding the non-linear terms, one obtains for each layer, i,

$$\begin{aligned} \frac{\partial u_i}{\partial t} + g\frac{\partial \eta}{\partial x} - fv_i - \frac{(\tau_+ - \tau_-)_x}{\rho\, h_i} - A_H \Delta u_i = 0 \\ \frac{\partial v_i}{\partial t} + g\frac{\partial \eta}{\partial y} + fu_i - \frac{(\tau_+ - \tau_-)_y}{\rho\, h_i} - A_H \Delta v_i = 0 \end{aligned} \qquad (6)$$

where

h_i = thickness of layer
$(\tau_{+-})_{xy}$ = shear stress in x and y-directions at the upper (+) and lower boundary (-) of the layer.

The integrated continuity equation is

$$\frac{\partial u_i}{\partial x} + \frac{\partial v_i}{\partial y} = w_+ - w_- \qquad (7)$$

where

w = vertical velocity through upper (+) and lower (-) boundary of the layer

The shear between two layers is in principle determined from the velocity difference between the two layers as

$$(\tau_+)_x = \nu_+ \frac{u_{i-1} - u_i}{0.5(h_{i-1} + h_i)} \qquad (8)$$

where index i-1 refers to the layer immediately above i and ν_+ is the vertical eddy viscosity at the upper boundary of layer i. The vertical eddy viscosity is thus a very important and crucial parameter. In practice the shear between two layers is obtained by subtracting the momentum equations for the layers, and the difference in velocities between successive layers is determined from this equation. For a more general treatment see Simons (1973).

In principle it is possible to solve the multilayer system by solving the equations for one layer at the time. However, almost equivalent information is obtained by solving one system of equations integrated over the total depth and one system of equations for the velocity differences. The equations integrated over the total depth are those used by Hansen Eqs. (3-5) for average velocities in a vertical only that the bottom friction is determined from the velocity in the bottom layer and that the non-linear terms, if used, must be determined from known velocities within each layer.

Another type of lake models, which may be called vertical profile models, are those steady-state models suggested by Ligget and Hadjitheodorou (1969), Bengtsson (1973b), Gedney and Lick (1972), Gallagher et al. (1973). Hydrostatic equilibrium is assumed. Most authors consider the lake to be thermally homogeneous, but Bengtsson extended the theory to two-layered lakes assuming the thermocline to be frictionless. Lindh and Bengtsson (1971) used a step-method to solve the non steady-state circulation in a lake. Even though the method is conceptually straightforward, it is very time consuming on the computer. As shown by Young and Ligget (1977) the number of solutions using the Laplace transform technique is an order of magnitude less. For the algebraic development see Young and Ligget.

Integrating the linear horizontal momentum equations one obtains for steady-state conditions

$$\begin{aligned} u(z) &= -g_1(z)\frac{g}{f}\frac{\partial\eta}{\partial y} + g_2(z)\frac{g}{f}\frac{\partial\eta}{\partial x} + g_5(z)\frac{1}{\rho}\tau_{0x} + g_6(z)\frac{1}{\rho}\tau_{0y} \\ v(z) &\equiv g_2(z)\frac{g}{f}\frac{\partial\eta}{\partial y} + g_1(z)\frac{g}{f}\frac{\partial\eta}{\partial x} - g_6(z)\frac{1}{\rho}\tau_{0x} + g_5(z)\frac{1}{\rho}\tau_{0y} \end{aligned} \tag{9}$$

where the g-functions are determined analytically. For a detailed analysis see Bengtsson (1973b) or a manual prepared by the Swedish IHP-Group on Lake Hydrology. For the vertical coordinate (directed upwards with zero at the water surface) z is used.

By introducing vertically integrated stream functions which satisfy the continuity equation

$$\frac{\partial\Psi}{\partial y} = \int_{-h}^{0} u\,dz \qquad \frac{\partial\Psi}{\partial x} = -\int_{-h}^{0} v\,dz \tag{10}$$

and integrating the equations for the stream velocities, one obtains for the pressure gradients

$$\begin{aligned} \frac{g}{f}\frac{\partial\eta}{\partial y} &= -A\frac{\partial\Psi}{\partial y} - B\frac{\partial\Psi}{\partial x} + \frac{1}{\rho}D\tau_{0x} + \frac{1}{\rho}C\tau_{0y} \\ \frac{g}{f}\frac{\partial\eta}{\partial x} &= B\frac{\partial\Psi}{\partial y} - A\frac{\partial\Psi}{\partial x} + \frac{1}{\rho}C\tau_{0x} - \frac{1}{\rho}D\tau_{0y} \end{aligned} \tag{11}$$

where A,B,C,D are determined from the g-functions. By differentiating the equations for the pressure and subtracting one from the other the pressure terms are eliminated. This yields

$$\Delta\Psi + a\frac{\partial\Psi}{\partial x} + b\frac{\partial\Psi}{\partial y} - \frac{1}{\rho}T = 0 \tag{12}$$

where a,b are functions of the eddy viscosity and of the depth, and T is also a function of the wind stress. The boundary conditions are constant value of the stream function along the shores except at inlet and outlet, where the stream function is determined from the inflow and the outflow. The stream function of each grid point is found from the partial differential equation (12). Once the stream functions are known the pressure gradients and the three-dimensional stream velocities are easily found.

Applications of Lake Models

Both vertical profile models and multi-layered models have been applied by the Swedish Group on Lake Hydrology to several small lakes, cf. Bengtsson (1973b,d,1978), Falkenmark (1973). It has been found that it is with a satisfactory degree of accuracy possible to reproduce actual currents in the top five metres of a homogeneous or two-layered lake using a profile model and in the top three metres using a multi-layered model. Some examples are shown below. In the afternoon of May 18, 1972, drogues were released at 1.5, 3, and 6 m depth in two areas of Lake Velen. Using the model proposed by Bengtsson the paths of the drogues were simulated and compared with the observed trajectories. The results are shown in Fig. 6. Although the wind speed is very low, the agreement is very good except maybe for 6 m depth. Generally one can state that the model gives reliable results at greater depths the higher the wind speed is.

Applications to Lake Ivö (profile model) and Lake Kösen (multilayered model) are presented by Bengtsson (1978). Measured and calculated currents at 1 m depth for Lake Ivö are shown in Fig. 7. It is seen that the agreement is good. Although the observed circulation pattern is very complex, it is very well reproduced in the model.

Measured and calculated currents at 1 m depth for Lake Kösen are shown in Fig. 8. A multi-layered model with a layer thickness of 2 metres was used. The agreement is good. However, at depths 3 and 5 metres measured and calculated currents did not agree. Wittmiss from the Swedish IHP-Group (personal communication) also tested a profile model on Lake Kösen and found that observed currents were reproduced satisfactory well.

Influences of Physical Parameters on the Circulation

The wind stress, the »turbulence« as vertical and horizontal eddy viscosities, the earth's rotation, the stratification and the bottom topography all influence the wind induced circulation in a lake. Parameters of mathematical models are affected by the horizontal and vertical resolution of the model and are often designated values, which are not physically relevant. Non-linear effects and horizontal dispersion effects may be incorporated in the vertical eddy viscosity and numerical dispersion may be incorporated in the horizontal exchange coefficient. By comparing model currents calculated with different parameters and also using observed currents the author investigated the effect of different parameters. For a more detailed analysis see Bengtsson (1973b).

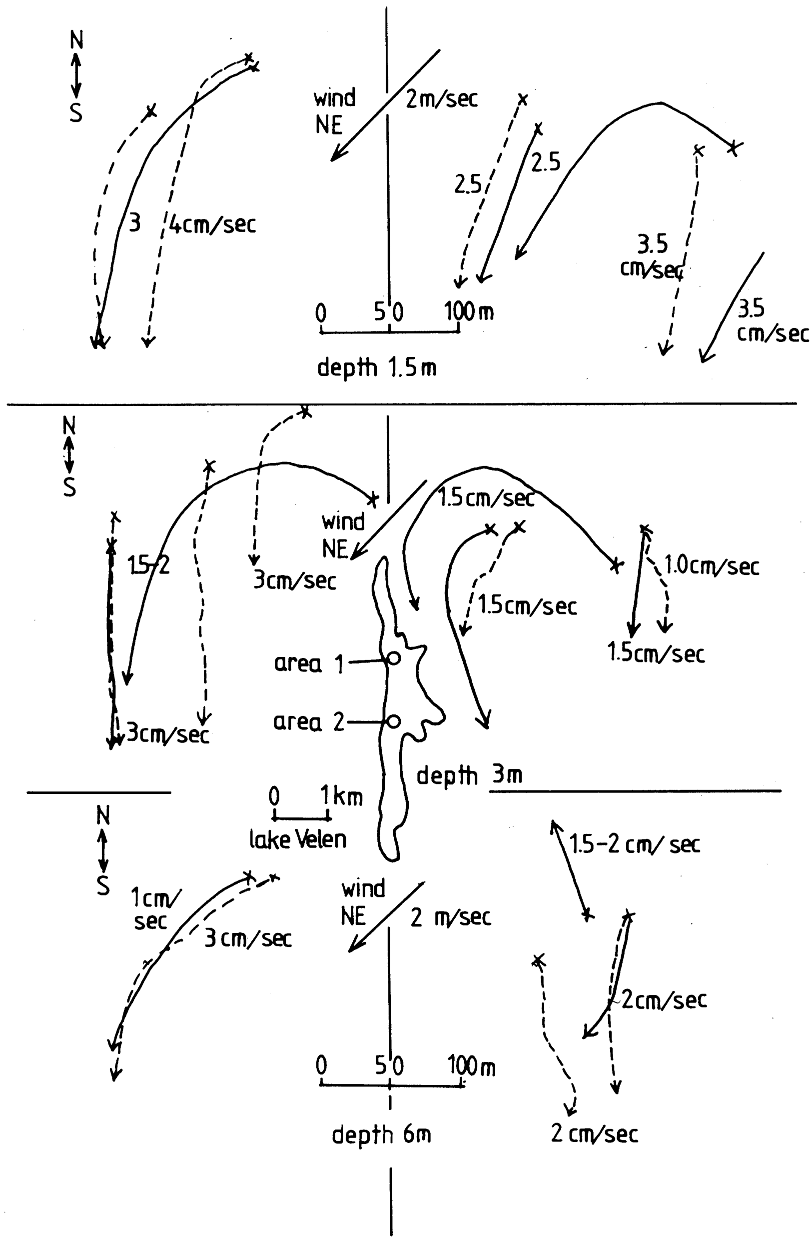

Fig.6. Calculated (solid lines) and observed (dashed lines) drogues trajectories in Lake Velen. Observation area 1 (left) and observation area 2 (right).

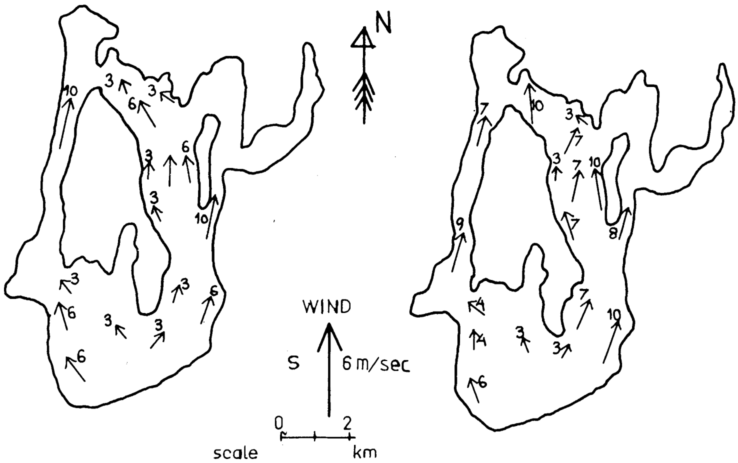

Fig.7. Measured (left) and calculated (right) currents at the 1 m level in Lake Ivö. Wind S 6 m/sec. Depth of thermocline 12 m. Stream velocities in cm/sec.

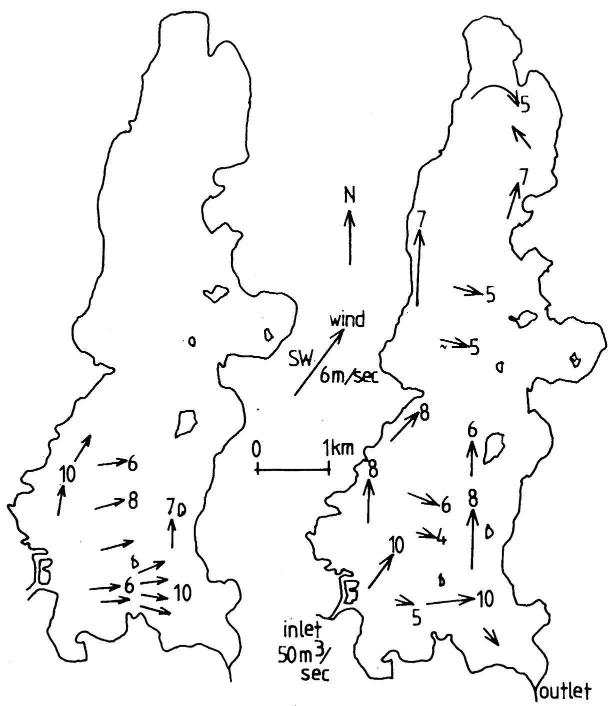

Fig.8. Measured (left) and calculated (right) currents at depth 1 m in Lake Kösen. Wind NE 6 m/sec. Homothermal water masses. Stream velocities in cm/sec.

Wind Stress - The wind stress does not directly affect the circulation pattern but only the magnitude of the currents. However, strong winds generate high vertical turbulence, the water feels the bottom in shallow areas, horizontal eddies are produced, and already at very small depths the circulation pattern changes with increasing winds.

Horizontal Turbulence - By running lake models with different values of the horizontal exchange coefficient Bengtsson (1973d) found that the influence of the horizontal turbulence terms is reduced with increasing depth and is of importance only close to the shore and when a steady-state is almost approached. The horizontal dispersion of momentum acts as to smooth out strong local currents. In the multi-layered lake model the numerical dispersion is included in the horizontal turbulence terms, and the horizontal exchange coefficient is dependent on the grid size, see for example the simulations carried out by Wittmiss (Bengtsson 1978).

Non-linear Effects - The non-linear terms of the momentum equation may or may not be included in layered numerical lake models. By comparing results obtained from lake model simulations it can be shown that the non-linear effects are small except maybe close to the shores.

Coriolis' Acceleration - The rotation of the earth influences the circulation pattern also in small lakes. This fact is especially notable in the central part of summer-stratified lakes and for light winds. For shallow ponds of depth less than 3-4 metres and strong winds the earth rotation is insignificant.

Stratification - In summer stratified lakes the bottom currents are due to free and forced internal seiches, but in such a complex way that the currents cannot be predicted.

Shore Currents - Although strong currents often has been observed along the shores of big lakes, there seems to be no such tendency in small lakes. When running lake models not including the horizontal turbulence terms or the nonlinear terms and using a constant value of the vertical eddy viscosity all over the lake, the computed currents close to the shore may exceed the measured ones, see Bengtsson (1973b, 1978). The explanation for this may be underestimations of the effects of horizontal shear and dispersion, or it may be so that vertical turbulence is very intense close to the shores.

Vertical Turbulence - The vertical turbulence has a profound influence on the magnitude of the flow and on the whole circulation. When the turbulence is strong, the currents »feel« the bottom, and large horizontal eddies will be created. On the other hand a small value of the vertical eddy viscosity gives rise to strong surface currents, but with depth fast decreasing currents. It also produces strong currents along the shores with return currents rather close to the surface in the central part of the lake.

For channel flow many investigators suggest that the vertical exchange coefficient is proportional to the depth and the bottom friction velocity. Bengtsson (1973b) applied the same approach to wind driven currents, and putting the friction velocity at the water surface proportional to the wind speed he obtained

$$\nu = ch\,W \tag{13}$$

where

ν = vertical eddy viscosity
W = wind speed
h = depth
c = non-dimensional coefficient

The formula above is correct also for dimensional reasons. The depth may be either the thermocline depth (for stratified lakes) or the average depth of the lake (for non-stratified lakes). Model simulations as well as dye experiments, described by Bengtsson (1973c) and Bengtsson and Eriksson (Falkenmark 1973), confirm the above formula. The coefficient should be about $1.5 \cdot 10^{-5}$ for stratified lakes and $2.0 \cdot 10^{-5}$ for non-stratified lakes, but increases somewhat with increasing wind speeds and decreases with increasing depth. At depths greater than 15 metres the depth was found not to influence the value of the eddy viscosity. From Eq. (13) it seems reasonable that the dddy viscosity should vary from one point to another in a lake. If, however, $\nu \sim h$, there will, according to model computations, arise jetlike currents along the shores. On the contrary by choosing high eddy viscosity values next to the shores, computed currents agree very well with observed currents.

The vertical eddy viscosity should vary through a vertical. At the very surface and at the bottom the viscosity must for physical reasons tend to molecular values. Also in the thermocline the turbulence is almost diminishing. Bengtsson (1973c) compared computed and in the field observed velocity profiles. He found that the vertical eddy viscosity was parabolically distributed with a maximum value some metres below the surface. This is consistent with field observations that show strong surface and bottom currents but low stream velocities in intermediate layers. Svensson from the Swedish IHP-Group (personal communication) obtained identical distribution by applying a turbulence model.

Conclusions

The problem of wind induced lake circulation is fairly well understood concerning surface currents in thermally homogeneous or two-layered stratified lakes. By the aid of mathematical lake models currents of the upper 3-5 metres in a lake can be satisfactory reproduced. At greater depth even very small density anomalies influence the circulation. Some kind of free or more likely forced irregular »seiche currents« are produced. The interaction between large-scale flow and turbulent flow is yet unknown. This problem must be solved before there is any hope of being able to reproduce currents at great depth using mathematical models.

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Received: 3 March, 1978

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