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Discussion

The Elastic-Plastic Response of Thin Spherical Shells to Internal Blast Loading[1]

P. G. HODGE, JR.[2] There are certain questions which arise from the author's valuable treatment of this interesting problem. The first of these concerns the phenomenon of unloading. According to the analysis as given, the stress rises to a value σ_1 in an elastic and plastic response and then oscillates between $\pm\sigma_1$ elastically. For a perfectly plastic material, $\sigma_1 = \sigma_y$, and this analysis is theoretically correct. However, for a strain-hardening material, $\sigma_1 > \sigma_y$ and the unloading will be elastic only in the case of isotropic strain-hardening without any Bauschinger effect. If there is any Bauschinger effect, there would be plastic compressive behavior during the first unloading and possibly further plastic flow during subsequent oscillations. This phenomenon could easily be treated within the framework of the linear analysis and it would be interesting to compare the results with those of the author.

That this point may be of more than theoretical interest is suggested by the author's statement, "...maximum strains were recorded ... before failure..." This statement seems to imply that failure occurred subsequent to reaching the maximum stress. This result is difficult to explain on the basis of the author's analysis, but could be a natural consequence of the further plastic flow resulting from a Bauschinger effect.

In the interpretation of the data, an objection may be raised to the author's adoption of one standard deviation as a "confidence limit." The distribution of measurements is far from a normal one so that the theoretical significance of the standard deviation is questionable. Indeed, the scatter between the two plastic tests is so great that one should hesitate to draw any qualitative conclusions from either test.

As a final point, one might mention that standard methods may be used to reduce the nonlinear equation (15) to a solution by quadrature in the particular case where p = const. In particular, such a solution would be applicable for a step pulse or for the phase $p = 0$ of any pressure wave.

Author's Closure

The author appreciates the careful consideration given the paper by Dr. Hodge. He is quite correct in pointing out that the Bauschinger effect has been neglected in the analysis for a strain-hardening material. The analysis presented in the paper could be easily extended to a stress-strain law which included this effect, as long as the law was represented by linear segments. Although this effect could explain failure occurring after maximum strains were recorded at several locations on the shell, the author believes that the delayed failure can probably be attributed to the propagation of tears in the shell from localized weak points which failed at stresses considerably below the maximum values which were recorded at locations distant from these points.

The author does not agree with Dr. Hodge's statement that "the distribution of measurements is far from a normal one." If one plots the data of Table 3 in the form of histograms and fits normal distribution curves to these histograms, he can see that the fits are quite good. Although the data in Table 4 have a much larger spread, they also fit a normal curve reasonably well. So, the assumption of normal distribution, implied in the paper, appears to be reasonable. However, association of the words "confidence limits" with one standard deviation *is* questionable, as the author intended to convey when he used quotation marks.

As Dr. Hodge indicated in his comment on qualitative conclusions from the plastic response tests, one cannot decide which of the theories presented in the paper should be used to obtain close agreement with experiment. But, despite their shortcomings, the plastic experiments yield response measurements of the same order of magnitude as those predicted from either theory, assuring one that there are no gross disagreements with theory.

The author wishes to thank Dr. Hodge for noting that standard methods can be used to solve equation (15) for a step increase in pressure. This case is of some interest, although it does not apply to the blast loading problem. For example, one can show that the shell motion is unstable for a sufficiently intense step of pressure.

[1] By W. E. Baker, published in the March, 1960, issue of the Journal of Applied Mechanics, vol. 27, Trans. ASME, vol. 82, Series E, pp. 139–144.

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A Unified Criterion for the Degree of Constraint of Plane Kinematic Chains[1]

T. P. GOODMAN.[2] The clarification of the concept of constraint which this paper accomplishes is a welcome contribution to the literature of kinematics. The criterion of constraint based on velocities, as given in Section 2 of the paper, is particularly simple and easy to use.

Using the notation of the paper, this criterion may be stated as

$$x = (n - 1) - 2L \tag{1}$$

which is merely a rearrangement of equation (10) of the paper.

Section 4 of the paper may be interpreted as a demonstration of the equivalence between this constraint criterion and the conventional constraint criterion given by equation (9) of the paper. A simpler proof of this equivalence may be obtained by deriving equation (1) of this discussion directly from equation (9) of the paper in the following way:

1 Define the number of pairs in the mechanism as

$$p = \sum_i (i - 1)j_i \tag{2}$$

[1] By B. Paul, published in the March, 1960, issue of the Journal of Applied Mechanics, vol. 27, Trans. ASME, vol. 82, Series E, pp. 196–200.

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