DISCUSSION

A. G. Fabula[2]

The authors are to be complimented for another of their many excellent papers on polymeric drag reduction. Two minor points may be worth discussing. First, in the Discussion, it is said that ". . . the mass transfer between the viscous sublayer and its surroundings involves diffusive rather than convective processes." If this were the case, wouldn't the additive wall concentration be expected to decrease from the ejected concentration with increasing distance downstream much slower than it does in fact? In the paper on additive diffusion in these same experiments (J. Wu, *Journal of Hydronautics*, Vol. 6, Jan. 1972, pp. 46–50), the additive wall concentrations near the end of the plate are shown to be in the range of 1–2 percent of the ejected concentrations.

Second, in their conclusions, it is recommended that for maximum economy (of additive) the solution ejection rate should be comparable to the normal viscous-sublayer discharge rate, with ejected concentration governed by the length of boundary and wall roughness. Data in Figs. 3–7 support and contradict this recommendation and its emphasis on solution ejection rate. For appreciably less than maximum drag reduction, the data for discharge rates of Q_s, $2Q_s$, and $4Q_s$ are nearly coincident, supporting the recommendation. However, except in Fig. 3 for the narrowest slot gap and smooth wall, the data for maximum and near maximum drag reduction (a regime of primary interest) show typically (but not always) that the points for 2Qs and 4Qs fall about at $^1/_2$ and $^1/_4$, respectively, times the ejected concentrations of the Q_s points for the same drag reduction. Thus, in 4 out of the 5 cases, performance near the maximum seemed to depend mainly on additive supply rate, rather than separately on ejection rate and ejected concentration.

G. K. Patterson[3]

The research described in this paper was directed at determining the effects of the following conditions on boundary layer reduction:

1 additive solution ejection rate,
2 additive solution concentration,
3 ejection slot size, and
4 plate surface roughness

for a constant hydrodynamic condition, i.e., velocity, plate orientation, channel size, etc. In general the wealth of data presented seems to meet the above objective and was well discussed.

Further interpretation of the data, however, is complicated by the lack of certain explicit information about the experimental apparatus. An important aspect of drag reduction in boundary layers is whether the layer is developing as in flow over a flat plate suspended in the stream, or already fully developed, as in flow through pipes or channels after a suitable entrance region. The authors did not mention this aspect of the problem and did not give enough information about the placement of the drag plate in the stream for readers to later make their own judgments. In other words, it is not clear whether the plate was suspended in the channel with a free leading edge or was simply part of the channel floor or top. It is also not clear how long the entry region to the plate for boundary layer development was if it was part of the floor or top.

The reason for the concern for the type of boundary layer is that the growth of an undeveloped boundary layer causes a decreasing value of the shear strain rate (and so shear stress). This means that the longer is the development region, the lower is the overall drag reduction, the quantitative relationship depending on the sensitivity of the drag reduction causing property of the additive on shear strain rate (or shear stress). A case where this was shown to be very important by calculations based on pipe drag reduction data and a boundary layer drag reduction model was illustrated by Seyer [13].[4] He showed for a 0.01 percent solution of Separan AP 30 in water that overall drag reduction in a developing boundary layer was dependent on plate length for Reynolds number 10^9 as follows: for plate lengths of 100, 500, and 1000 ft, overall drag reduction was 59, 23, and 10 percent.[5] For a higher concentration, 0.1 percent, the drag reduction was shown to be the maximum possible, about 60 percent, for all these lengths.

Such results are very dependent on the properties of the additive solution, as shown above. Based on drag reduction measurements in pipes with Polyox WSR-301 solutions [14] at peak drag reduction concentration, the discussion author has calculated, using simply equal values of wall shear strain rate for 30 percent local drag reduction, that a plate many miles long would still be drag reducing at 88 ft/sec. The problem of the great sensitivity of drag reduction in developing boundary layers to additive solution properties must be deeply and thoroughly investigated. The data that Wu and Tulin have obtained for enjection drag reduction may be a valuable part of this investigation, *if* the experimental conditions mentioned above are reported. Also helpful for correlation purposes are measurements of basic solution rheological properties at the time of drag reduction measurement.

In order to accurately determine the amount of drag reduction expected on a very long plate for a given additive, one must know the dependence of drag reduction on additive concentration in the sublayer (not as ejected) and on sublayer conditions (i.e. shear strain rate or shear stress, thickness, etc.) and, if ejected solution is involved, the rate of diffusion under the boundary layer conditions involved. With such information it should be possible to numerically determine overall drag reduction values as a function of plate length. This can already be done for the case of known polymer concentration using the method of Seyer, if the dependence of the required parameter from pipe flow drag reduction measurements is known. Since this involves the use of a model for boundary layer drag reduction and transposition of measurement results for pipe flow, direct confirmation on large plate facilities seems desireable. The remaining unstudied variable, additive solution diffusion rate, could also be studied in such a facility. An empirical method of approach which might yield the needed information would be to measure local drag reduction at several distances downstream of additive ejection, then to compare these results with the same local values with homogeneous (known) additive concentration. This would yield effective values of the sublayer additive concentrations as a function of distance from ejection point.

Additional References

13 Seyer, F. A., "Friction Reduction in Turbulent Flow of Polymer Solution," *Journal of Fluid Mechanics*, Vol. 40, 1970, p. 807.

14 Patterson, G. K., Zakin, J. L., and Rodriguez, J. M., "Drag Reduction," *Industrial Engineering Chemistry*, Vol. 61, 1969, p. 22.

---

[2] Naval Undersea Center, San Diego, Calif.

[3] Department of Chemical Engineering, University of Missouri-Rolla, Rolla, Mo.

[4] Numbers in brackets designate Additional References at end of discussion.

[5] Of course, if the velocity (~100-ft/sec) for the 100-ft plate at a Reynolds number of 10^9 is maintained for the longer lengths, the amount of drag reduction decreases much more slowly.

Authors' Closure

We are grateful to Dr. Fabula and Professor Patterson for their kind words and appreciate their helpful discussions. The questions raised by them are answered in the following:

As shown in the text, the flow rate within the viscous sublayer is independent of the distance from the leading edge of the boundary. This was interpreted to show that the viscous sublayer is enclosed by a streamline, or that the viscous sublayer flows inside a stream tube. Therefore, the mass transfer between the viscous sublayer and its surroundings does not involve a *mean-flow* convective process. Certainly, it is through a *turbulence* convective process that material is lost from the viscous sublayer.

One of the primary purposes of our experiment was to verify the basic consideration that the polymer solution need only fill the viscous sublayer to cause near-maximum drag reduction. This is well supported by our data; a large drag reduction is obtained by ejecting the additive solution at the viscous-sublayer discharge, and relatively little gain of drag reduction is recorded as the ejection rate increases to two and four times the viscous-sublayer discharge. We agree with Dr. Fabula that for practical interest of estimating additive consumption, the drag-reduction data should be plotted versus the additive supply rate. Such a plot is meaningful, however, only when the diffusion losses of additives to the outer region of the boundary layer are approximately the same for different ejections, because the diffusion rate depends on the ejection rate and the ejected concentration.

As stated in the text, the drag measuring plate is a part of the cover plate (top) at the test section. The boundary layer is developing along the drag-measuring plate, as the boundary-layer thickness at the end of the test section is only about one tenth of the channel depth. The effective origin of the boundary layer is 4.5 in. upstream from the leading edge of the drag-measuring plate [15];[6] we are sorry for not making this information available. More recently, we have proceeded exactly along the line considered by Professor Patterson in the rest of his discussion. We have performed drag-reduction as well as wall additive concentration measurements [16, 17] with a long plate (10 ft) at a high channel speed (35 ft/sec). The results along with the data obtained here at a low Reynolds number provide the basis for determining additive requirement for plates of various lengths.

Additional References

15 Wu, Jin, "Suppressed Diffusion of Drag-Reducing Polymer in a Turbulent Boundary Layer," *Journal of Hydronautics*, Vol. 6, No. 1, 1972, pp. 46–50.

16 Wu, Jin, Wiggert, D. C., and Tulin, M. P., "Drag Reduction Studies With a Long Plate at High Reynolds Number," HYDRONAUTICS, Incorporated Technical Report (in preparation).

17 Fruman, D. H., and Tulin, M. P., "Drag Reduction and Diffusion Accompanying Thin Slit Injections of a Drag-Reducing Polymer cn a Flat Plate at High Reynolds Number," HYDRONAUTICS, Incorporated Technical Report 7101-3, 1972.

---

[6] Numbers in brackets designate Additional References at end of Closure.