Effects of Left-Right Scalar Quark Flavor Mixings
on CP Violation in Supersymmetry

Takeshi KURIMOTO

Institute of Physics, College of General Education
Osaka University, Toyonaka 560

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Studies of flavor mixings among left squarks (superpartners of left-handed components of quarks) and right squarks are made in the minimal low energy supergravity model, where all one-loop corrections are taken into account from renormalization group equations. So far, their effects on physics have been presumed to be negligible. It is found, contrary to that presumption, that they play major roles in two phenomena, the electric dipole moment of neutron and the asymmetry of muon polarization in $K_L \rightarrow \mu^+ \mu^-$ decay.

§ 1. Introduction

Since the discovery of supersymmetry (SUSY), many studies on the phenomenological aspect of SUSY theories have been made. Some of them argued direct processes where SUSY particles are really produced and decay in accelerator experiments. Others dealt with the phenomena, which we call indirect processes, where SUSY particles contribute as virtual fields in Feynman diagrams.

At present, the effort of observing direct processes is in progress. Here we focus on the analysis of indirect processes. Among various indirect processes, CP violation through SUSY particle exchange is one of the most interesting subjects. Some phenomenologically successful SUSY models have new kinds of CP violating interactions which are not present in the standard model. It is likely that their predictions on physics differ from those by the standard model. In fact, the recently measured long $B$-meson life-time ($\sim 10^{-12}$ sec) may be a threat for the standard model to realize the experimental value of $\varepsilon$-parameter if top quark is light ($m_t \lesssim 50$ GeV), while some SUSY models are free from this difficulty. We think that a detailed study of CP violation in SUSY models is useful for the test of SUSY in low energy physics.

As a model, we adopt the minimal low energy supergravity model. In this model there are two kinds of sources of CP violation phases in general. One is the Kobayashi-Maskawa (KM) phase and the other is complex parameters of soft SUSY breaking terms. The latter gives large contributions to the electric dipole moment (EDM) of neutron, so that there is a severe constraint on them.

The left (right)-handed component of a quark has a superpartner, scalar quark, which is called the left (right) squark in this paper. In three generation models, a mass matrix of squarks is a $6 \times 6$ matrix consisting of three $3 \times 3$ matrices, $(L-L)$, $(L-R)$ and $(R-R)$, as follows:

\[
\begin{pmatrix}
(L-L) & (L-R)^\dagger \\
(L-R) & (R-R)
\end{pmatrix}
\]

(1.1)
Left-Right Squark Flavor Mixings

where $(L-L)$ is a part of mass matrix among three left squarks, and so on. The $(R-R)$ part shall be chosen to be diagonal, then in general both $(L-L)$ and $(L-R)$ parts contain flavor mixings among squarks, which become origins of $CP$ violation. The effects of flavor mixings in the $(L-R)$ part have not been taken into account seriously so far in the studies of $CP$ violation in SUSY models, since the $(L-R)$ part is suppressed by (quark mass)/(mass scale of SUSY particles). However, flavor mixings in the $(L-R)$ part may give significant contributions in some cases where left-right squark mixings play an important role. In this work we investigate the $CP$ violating effects of left-right squark flavor mixings, and show that they give contributions larger than those by flavor mixings in the $(L-L)$ part to EDM of neutron and to the longitudinal polarization of muons in $K_L \to \mu^+\mu^-$ decay.

The next two sections are devoted to the description of the model and our notation. Quantum effects on squark flavor mixings are investigated in detail with the use of renormalization group equations (RGE). We present interaction Lagrangian and squark mass matrices by taking into account full one-loop corrections. With these results we study in §§ 4 and 5 $CP$ violation phenomena where left-right squark flavor mixings play more important roles than those among left squarks only do. Conclusions are given in the final section.

§ 2. Interaction Lagrangian

Our model is a minimal supersymmetric extension of the standard model, where SUSY is softly broken by super-Higgs effect. Chiral supermultiplets which appear in this work are those of quarks, leptons and two Higgs doublets. One of the Higgs scalars gives masses to charged leptons and $d$-type quarks, another to $u$-type quarks. To explain our notation we show the chiral superfields with their transformation properties under $SU(3) \times SU(2) \times U(1)$ in Table I.

<table>
<thead>
<tr>
<th>Superfields</th>
<th>components (fermion boson)</th>
<th>under $SU_3 \times SU_2 \times U_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Q_n = \left[ \begin{array}{cc} Q_n^e \ Q_n^d \end{array} \right]$</td>
<td>$[u_L \bar{u}_L \bar{d}_L \bar{d}_L]^n$</td>
<td>$(3, 2, 1/3)$</td>
</tr>
<tr>
<td>$U_n^c$</td>
<td>$[u^c_R \bar{u}^c_R]^n$</td>
<td>$(3^*, 1, -4/3)$</td>
</tr>
<tr>
<td>$D_n^c$</td>
<td>$[d^c_R \bar{d}^c_R]^n$</td>
<td>$(3^*, 1, 2/3)$</td>
</tr>
<tr>
<td>$L_n = \left[ \begin{array}{c} L^e_n \ L^d_n \end{array} \right]$</td>
<td>$[\nu_L \bar{\nu}_L \bar{e}_L \bar{e}_L]^n$</td>
<td>$(1, 2, -1)$</td>
</tr>
<tr>
<td>$E_n^c$</td>
<td>$[e^c_R \bar{e}^c_R]^n$</td>
<td>$(1, 1, 2)$</td>
</tr>
<tr>
<td>$H = \left[ \begin{array}{c} H_0 \ H^- \end{array} \right]$</td>
<td>$[\tilde{h}<em>0 \tilde{h}<em>0 \tilde{h}</em>- \tilde{h}</em>-]^n$</td>
<td>$(1, 2, -1)$</td>
</tr>
<tr>
<td>$H' = \left[ \begin{array}{c} H'<em>0 \ H'</em>- \end{array} \right]$</td>
<td>$[\tilde{h}'<em>0 \tilde{h}'<em>0 \tilde{h}'</em>- \tilde{h}'</em>-]^n$</td>
<td>$(1, 2, 1)$</td>
</tr>
</tbody>
</table>

The Lagrangian relevant to our study is given as follows:

$\mathcal{L} = [(Q^u, Q^d)_n^* e^V (Q^u, Q^d)_n^T + U^c_n e^V U_n^c + D^c_n e^V D_n^c]_{PD} + [y^mn D^c_n Q_n H + y^mn U^c_n Q_n H'] + \mu H H' + \bar{\xi}^m \tilde{u}^m \tilde{d}^m (\tilde{u}_L h^- - \tilde{d}_L h^-) + (h.c.) + (m^p)_{mn} (\tilde{u}_L, \tilde{d}_L) m^* (\tilde{u}_L, \tilde{d}_L) n^T - (m^q)_{mn} (\tilde{u}_L, \tilde{d}_L) m^* (\tilde{u}_L, \tilde{d}_L) n^T - (m^q)_{mn} (\tilde{u}^m \tilde{d}^m \tilde{d}^m \tilde{u}^m \tilde{d}^m - (m^q)_{mn} (\tilde{u}_L, \tilde{d}_L) n^T$}

where the field $V$ is the vector superfield of

---

* There are many papers on $CP$ violation in SUSY models. As comprehensive ones we refer the reader to Refs. 3) and 7).
At the GUT scale \((M_G)\), the parameters of soft SUSY breaking terms appearing in Eq. (2.1) have the following relations\(^4\) in matrix form:

\[
\begin{align*}
\xi_{\nu} &= A m_\sigma y_{\nu}, \\
m^2_{\nu} &= A m_\nu = m^2_{\nu} = m_\sigma^2 \cdot 1,
\end{align*}
\] (2.2)

where the parameters \(m_\sigma\) and \(A\) are the gravitino mass and a constant of order 1, respectively. Development of parameters in the theory at lower scale is obtained with the aid of RGE which are given in several references.\(^4,8\) We use the most general one by Derendinger and Savoy.\(^8\) To solve RGE exactly is a hard task, so that we solve them perturbatively following Dugan et al.\(^7\). The results up to (coupling)\(^5\) corrections are given below,

\[
\begin{align*}
\xi_{\nu} &= A m_\sigma y_{\nu} \left[ 1 - \frac{3}{2} \text{Tr}(y_{\nu}^\dagger y_{\nu}) - 2 g^2 (M/A m_\sigma) + \frac{3}{2} y_{\nu} y_{\nu} + y_{\nu} y_{\nu} \right], \\
m^2_{\nu} &= m_\sigma^2 \left[ 1 + \frac{2d^2 g^2 (M/m_\sigma)^2 - (3+|A|^2) (y_{\nu} y_{\nu} + y_{\nu} y_{\nu})}{t} \right], \\
m^2_{\nu} &= m_\sigma^2 \left[ 1 + \frac{2d^2 g^2 (M/m_\sigma)^2 - 2(3+|A|^2) y_{\nu} y_{\nu}}{t} \right],
\end{align*}
\] (2.4)

where \(M\) and \(g\) are the gaugino mass and the gauge coupling constant, respectively, at the GUT scale, \(t = (1/8\pi^2) \ln(M_G/M_W)\). The values of other constants are calculated as follows:

\[
c = 22/5, \quad d^o = 21/5, \quad d^p = 14/5, \quad d^u = 16/5.
\] (2.9)

Our results look different from those of Ref. 7). It is because the Yukawa coupling constants \(y_{\nu}\) and \(y_{\nu}\) on the right-hand side of Eqs. (2.4) \sim (2.8) are those at \(M_W\), while the authors of Ref. 7) used those at \(M_G\). The parameters \(A\), \(m_\sigma\) and \(M\) depend on our choice of SUSY breaking hidden sector.\(^1,4\) On the other hand, the values of Yukawa couplings are measured at low energy \((\lesssim M_W)\). As for the gauge couplings, they are measured at low energy, but the expressions (2.4) \sim (2.8) become simpler when we use the gauge couplings at \(M_G\) since we assume grand unification of gauge interactions. A calculation of gauge coupling constant at \(M_G\) has been made by Inoue et al. in this kind of model.\(^9\) They found \((4\pi/g^2) = 24.1\) at \(M_G\). We believe our expressions are most convenient for the analysis of low energy phenomena.

In the Lagrangian (2.1) at \(M_W\), we rotate the chiral superfields so that the fermion (quark) components of them may be transformed into mass eigenstates. We define unitary matrices as

\[
\begin{align*}
\gamma_{\nu} &= - U_{\nu}^\dagger \tilde{y}_D U_D \left( \tilde{\nu}_D = M_D \langle h_0 \rangle = \text{diag}(m_d, m_s, m_b) / \langle h_0 \rangle \right), \\
\gamma_{\nu} &= U_{\nu}^\dagger \tilde{y}_U U_U \left( \tilde{\nu}_U = M_U \langle h_0 \rangle = \text{diag}(m_u, m_c, m_t) / \langle h_0 \rangle \right).
\end{align*}
\] (2.10)

By these unitary matrices we have

\[
Q^u = U_{\nu}^\dagger X_{\nu} Q^u, \quad Q^d = U_{\nu}^\dagger X_{\nu} Q^d.
\]
where the underlined fields are the rotated ones. Hereafter we use the transformed fields expressing them by the same letters as before. The matrices $X_U$, $X_D$, $Y_U$ and $Y_D$ are diagonal matrices of the form $\text{diag}(e^{ia}, e^{ib}, e^{ic})$ corresponding to the phase freedom of the fields. The values of the components of $X_U$, $X_D$ are chosen so that we may have KM matrix $\Omega$ as

$$X_U^\dagger U_U U_D^\dagger X_D = \Omega.$$  \hfill (2.13)

As for the remaining matrices $Y_U$ and $Y_D$, we set

$$X_U = Y_U, \quad X_D = Y_D.$$  \hfill (2.14)

With these relations and Eqs. (2.4)~(2.8), our Lagrangian becomes as follows:

$$\mathcal{L} = \left[(Q^u)^* Q^d Q^u + U^c e^V U^c + D^c e^V D^c\right] + \left[D^c \{\bar{y}^u Q^d H_0 - \bar{y}^u Q^u H_+\} + U^c \{\bar{y}^u Q^u H_0 - \bar{y}^u Q^d H_+\} + \mu HH^\prime\right]_F$$

$$- \tilde{d}_R^* \tilde{y}_D \{(a_d + \beta \tilde{y}_D^2) Q^d + \gamma Q^d \tilde{y}_D^2\} \tilde{u}_L h_0 + \tilde{d}_R^* \tilde{y}_D \{(a_d + \beta \tilde{y}_D^2) + \gamma \tilde{y}_D^2\} \tilde{u}_L h_0$$

$$+ \tilde{u}_R^* \tilde{y}_U \{(a_d + \beta \tilde{y}_U^2) + \gamma \tilde{y}_U^2\} \tilde{u}_L h_0 - \tilde{u}_R^* \tilde{y}_U \{(a_d + \beta \tilde{y}_U^2) + \gamma \tilde{y}_U^2\} \tilde{u}_L h_0$$

$$+ \tilde{u}_R^* \tilde{y}_U \{(a_d + \beta \tilde{y}_U^2) + \gamma \tilde{y}_U^2\} \tilde{u}_L h_0$$

$$- \tilde{d}_R^* \{a_d + b_d \tilde{y}_D^2\} \tilde{d}_R - \tilde{u}_R^* \{a_d + b_d \tilde{y}_U^2\} \tilde{u}_R,$$  \hfill (2.15)

where generation indices are implicit. The parameters in the above expression are defined as

$$a_d = A m_\phi [1 + \{2 g^2 (M/A m_\phi) - 3 \text{Tr} (\tilde{y}_D^2)\} t],$$

$$\beta/3 = \gamma = A m_\phi t,$$

$$a_{Q,D,U,V} = m_\phi^2 \{1 + 2 d^{Q,D,U,V}g^2 (M/m_\phi)^2\},$$

$$b_d = b_d / 2 = b_U / 2 = -(3 + |A|^2) m_\phi^2 t.$$  \hfill (2.16)

Now the parameters in the Lagrangian (2.15) are expressed in terms of quark masses, KM matrix, some calculable quantities ($c$, $d^D$, $d^U$, $d^d$ and $t$) and a few model dependent parameters ($A$, $m_\phi$ and $M$). This Lagrangian is used for Feynman diagram calculations by mass insertion method. In the later section (§5) we use this method to estimate the degree of longitudinal polarization of muons in $K_L \rightarrow \mu^+ \mu^-$ decay.

§ 3. Scalar quark mass matrices

Suppose the electrically neutral components of Higgs scalars, $h_0$ and $h^\prime_0$, develop vacuum expectation values (VEV), $v$ and $v^\prime$, respectively. Then the $W$ boson gets mass, $M_w^2 = (g_2^2/2) (v^2 + v^\prime^2)$, where $g_2$ is $SU(2)$ gauge coupling constant. Both VEVs are supposed to be of order $10^2$ GeV.

There are three kinds of contributions to squark mass matrices, one contribution from soft SUSY breaking terms and the others from supersymmetric $D$ and $F$ terms. The scalar parts of $D$ terms are
where \( g_N \) is the gauge coupling constant of \( SU(N) \) (\( U(1) \) for \( N = 1 \)) interaction, \( T^a \) is a generator of the gauge group and \( \varphi \) stands for all the scalar fields in the theory. On weak symmetry breaking, \( D \) term leads to

\[
(v^2 - v'^2) \left( \left( \frac{g_2^2}{4} - \frac{g_1^2}{12} \right) |\bar{u}_L|^2 - \left( \frac{g_2^2}{4} - \frac{g_3^2}{12} \right) |\bar{d}_L|^2 + \left( \frac{g_2^2}{3} \right) |\bar{u}_R|^2 - \left( \frac{g_3^2}{6} \right) |\bar{d}_R|^2 \right). \tag{3·2}
\]

These terms depend on transformation properties under gauge groups only, so that they have no generation dependence.

The contributions from \( F \) term are obtained with the aid of the well-known formula, \( V_F = \sum |\partial W/\partial \phi|^2 \), where \( W \) is a superpotential. The Lagrangian (2·15) gives

\[
\bar{d}_L^* \tilde{M}_d^2 \bar{d}_L + \bar{d}_R^* \tilde{M}_u^2 \bar{u}_L + \bar{u}_R^* \tilde{M}_u^2 \bar{u}_R \\
+ \bar{d}_R^* \mu \tilde{M}_u (v/v') \bar{d}_L \bar{u}_L + \bar{u}_R^* \mu \tilde{M}_u (v/v') \bar{u}_L + (\text{h.c.}). \tag{3·3}
\]

Finally, replacing \( h_0 \) and \( h_0' \) by \( v \) and \( v' \), respectively, in the soft SUSY breaking terms of Eq. (2·15), we have

\[
\bar{d}_L^* \tilde{M}_d^2 \left( a_D + \beta (\tilde{M}_d/v)^2 + \gamma \tilde{Q} (\tilde{M}_u/v')^2 \Omega \right) \bar{d}_L \\
+ \bar{u}_R^* \tilde{M}_u \left( a_U + \beta (\tilde{M}_u/v)^2 + \gamma \tilde{Q} (\tilde{M}_d/v')^2 \Omega \right) \bar{u}_R + (\text{h.c.}) \\
+ \bar{d}_L^* \left( a_R + b_R (\tilde{M}_d/v)^2 + \Omega (\tilde{M}_u/v'^2) \right) \bar{d}_L + \bar{d}_R^* \left( a_D + b_D (\tilde{M}_d/v)^2 \right) \bar{d}_R \\
+ \bar{u}_L^* \left( a_U + b_U (\tilde{M}_u/v'^2 + \Omega (\tilde{M}_d/v'^2) \right) \bar{u}_L + \bar{u}_R^* \left( a_U + b_U (\tilde{M}_u/v'^2) \right) \bar{u}_R. \tag{3·4}
\]

We sum up these contributions, then the mass matrix of \( d \)-type squarks are given as follows:

\[
(\bar{d}_L, \bar{d}_R)^* M_d^2 (\bar{d}_L, \bar{d}_R)^T \equiv (\bar{d}_L, \bar{d}_R)^* \begin{bmatrix} (L-L)_D & (L-R)_D^* \\ (L-R)_D & (R-R)_D \end{bmatrix} \begin{bmatrix} \bar{d}_L \\ \bar{d}_R \end{bmatrix}, \tag{3·5}
\]

where

\[
(L-L)_D = \left( a_D - g_2^2/4 - g_1^2/12 \right) (v^2 - v'^2) 1 \\
+ \left( 1 + b_d/v^2 \right) \tilde{M}_d^2 + b_d \tilde{Q} (\tilde{M}_u/v)^2 \Omega, \tag{3·6}
\]

\[
(L-R)_D = a_D + \mu (v'/v) + \beta (\tilde{M}_d/v)^2 \tilde{M}_d + \gamma \tilde{M}_d \tilde{Q} (\tilde{M}_u/v')^2 \Omega, \tag{3·7}
\]

\[
(R-R)_D = a_D - g_3^2/6 (v^2 - v'^2) 1 + \left( 1 + b_d/v^2 \right) \tilde{M}_d^2. \tag{3·8}
\]

For convenience we reparametrise Eqs. (3·6) ~ (3·8) by dividing them into three parts; (a part proportional to unit matrix) + (generation diagonal part) + (flavor mixing part). Then we have

\[
(L-L)_D = \mu_L^2 (D) + \rho_D \tilde{M}_d^2 + q_d \tilde{Q} (\tilde{M}_u^2 \Omega, \tag{3·6'}
\]

\[
(L-R)_D = A_D m_d \tilde{M}_d + (r_d/m_d) \tilde{M}_d^3 + (s_d/m_d) \tilde{M}_d \tilde{Q} (\tilde{M}_u^2 \Omega, \tag{3·7'}
\]

\[
(R-R)_D = \mu_R^2 (D) + k_D \tilde{M}_d^2, \tag{3·8'}
\]
where the coefficients are defined as

\[
\begin{align*}
\mu_L^2(D) &= m_\sigma^2 - (g_2^2 / 4 + g_1^2 / 12) (v^2 - v'^2) + 2d_\phi g^2 M^2 t, \\
p_D &= 1 - (3 + |A|^2)(m_\sigma/v)^2 t, \\
q_D &= -(3 + |A|^2)(m_\sigma/v')^2 t, \\
A_D &= A + (\mu v / m_\sigma v) - \{3ATr(M_\sigma^2) / v^2 - 2cg^2(M/m_\sigma)\} t, \\
r_D &= 3A(m_\sigma/v)^2 t, \\
s_D &= A(m_\sigma/v')^2 t, \\
\mu_R^2(D) &= m_\sigma^2 - (g_1^2 / 6)(v^2 - v'^2) + 2d_\phi g^2 M^2 t, \\
k_D &= 1 - 2(3 + |A|^2)(m_\sigma/v')^2 t.
\end{align*}
\]

(3.9)

In the same way we have \( u \)-type squark mass matrix:

\[
\begin{align*}
(L-L)u &= \left(\begin{array}{c}
\mu_L^2(U) + p_u \bar{M}_u^2 + q_u \Omega \bar{M}_d^2 \Omega^\dagger \\
(L-R)u = A_u m_\sigma \bar{M}_u^3 + (r_u / m_\sigma) \bar{M}_d^3 \Omega \bar{M}_d^3 \Omega^\dagger \\
(R-R)u &= \mu_R^2(U) + k_u \bar{M}_u^2.
\end{array}\right)
\]

(3.10)

The definitions of the coefficients in the above equations are

\[
\begin{align*}
\mu_L^2(U) &= m_\sigma^2 + (g_2^2 / 4 - g_1^2 / 12)(v^2 - v'^2) + 2d_\phi g^2 M^2 t, \\
p_u &= 1 - (3 + |A|^2)(m_\sigma/v)^2 t, \\
q_u &= -(3 + |A|^2)(m_\sigma/v')^2 t, \\
A_u &= A + (\mu v / m_\sigma v) - \{3ATr(M_\sigma^2) / v^2 - 2cg^2(M/m_\sigma)\} t, \\
r_u &= 3A(m_\sigma/v)^2 t, \\
s_u &= A(m_\sigma/v')^2 t, \\
\mu_R^2(U) &= m_\sigma^2 + (g_1^2 / 3)(v^2 - v'^2) + 2d_\phi g^2 M^2 t, \\
k_u &= 1 - 2(3 + |A|^2)(m_\sigma/v')^2 t.
\end{align*}
\]

(3.11)

(3.12)

(3.13)

(3.14)

Let us transform squarks into their mass eigenstates. The squark mass matrices, \( M_D^2 \) and \( M_U^2 \), are Hermitian, so that we need two \( 6 \times 6 \) unitary matrices, \( X_D \) and \( X_U \), to diagonalize them. Let

\[
\begin{align*}
M_D^2 &= X_D \bar{M}_D^2 X_D^\dagger, \\
M_U^2 &= X_U \bar{M}_U^2 X_U^\dagger,
\end{align*}
\]

(3.15)

where \( \bar{M}_D \) and \( \bar{M}_U \) are diagonal matrices. The unitary matrix \( X_D \) (\( X_U \)) is further decomposed into two \( 3 \times 6 \) matrices as follows:

\[
X_D(U) = \left[ \begin{array}{c}
\Gamma_L^{D(U)} \\
\Gamma_R^{D(U)}
\end{array} \right].
\]

(3.16)

We express the mass eigenstate scalar quarks by \( S_D \) and \( S_U \), then

\[
X_D^\dagger(\bar{d}_L, \bar{d}_R)^\dagger = S_D, \\
X_U^\dagger(\bar{d}_L, \bar{d}_R)^\dagger = S_U,
\]

(3.17)
that is
\[
\begin{align*}
\tilde{d}_L &= \Gamma^{\rho}_L S_a, \\
\tilde{d}_R &= \Gamma^{\rho}_R S_a, \\
\tilde{u}_L &= \Gamma^{\nu}_L S_u \\
\tilde{u}_R &= \Gamma^{\nu}_R S_u.
\end{align*}
\tag{3.18}
\]
Applying these relations to Eq. (2.15), we have a Lagrangian in which both quarks and squarks are mass eigenstates. The Lagrangian obtained in this way will be so complicated (especially in trilinear scalar couplings) that we do not show here the complete one, but the part of \(SU(3)\) interaction among quark, squark and gluino will be given in the next section. There we will discuss how the flavor mixings in the \((L-R)\) part affect the supersymmetric contributions to EDM of neutron.

§ 4. Electric dipole moment of neutron

It has been known that one-loop diagrams which include gluino propagation give major supersymmetric contributions to the electric dipole moment (EDM) of neutron in softly broken SUSY models.\(^6\) The interaction Lagrangian among quark, squark and gluino in Eq. (2.15) reads as follows:

\[
\mathcal{L}_{\text{int}} = i\sqrt{2} g_3 [\tilde{u}_L^* \lambda_3 u_L + \tilde{d}_L^* \lambda_3 d_L - \tilde{u}_R \lambda_3 s_R - \tilde{d}_R \lambda_3 s_R] + (\text{h.c.}).
\tag{4.1}
\]

The squarks are rotated into mass eigenstates as Eq. (3.18), then we have

\[
\mathcal{L}_{\text{int}} = i\sqrt{2} g_3 [S_u^* (\Gamma^{\nu}_L)^{-1} \lambda_3 u_L + S_d^* (\Gamma^{\rho}_L)^{-1} \lambda_3 d_L - \tilde{u}_R \lambda_3 s_R - \tilde{d}_R \lambda_3 s_R] + (\text{h.c.})
\tag{4.2}
\]

The EDM of neutron \(\mathcal{D}_N\) is related to EDM of \(u\)-quark \(\mathcal{D}_u\) and that of \(d\)-quark \(\mathcal{D}_d\),

\[
\mathcal{D}_N = (4\mathcal{D}_u - \mathcal{D}_d)/3
\tag{4.3}
\]
Figure 1 shows the dominant one-loop SUSY contributions to EDM of \(d\)-quark \(\mathcal{D}_d^{\text{SUSY}}\), which gives

\[
\mathcal{D}_d = 2g_3^2 (-e/3) \text{Tr}[(T^a/2)^2]/3 \sum_j \text{Im}[(16\pi^2 M_3)^{-1}(\Gamma^{\rho}_R)^{-1}(Z_j)^{-1}(\Gamma^{\nu}_L)^{-1}],
\tag{4.4}
\]

where \(T^a\) is an \(SU(3)\) Gell-Mann matrix, \(Z^a_j = \tilde{M}^2_{d^j}/M_3^2(\tilde{M}^2_{d^j}; \ j\)-th eigenvalue of \(M^2_{d^j}, M_3^2\) gluino mass) and the function \(f(Z)\) is defined as follows:

\[
f(Z) = (1-Z)^{-2}[1 + Z + 2Z(1-Z)^{-1} \times \ln Z]/2.
\tag{4.5}
\]

To evaluate the imaginary part in Eq. (4.4) we use Eq. (3.16),

\[
(\Gamma^{\rho}_R)^{-1} f(Z_j)(\Gamma^{\nu}_L)^{-1}
= (X_3 X_3^* f(Z_j)(X_3 X_3^*),
\tag{4.6}
\]

Fig. 1. Dominant one-loop supersymmetric contribution to EDM of \(d\)-quark. The field \(\lambda^a_3\) is gluino.
and the identity derived in Appendix A. The final expression of $\mathcal{D}_d^{SS}$ is given as follows:\(^{\ast})

$$\mathcal{D}_d^{SS}/e = -2\alpha_s/(9\pi M_3)^{-1} \times \left(\sum_n (1/n!) f^{(n)}(Z_0^d) \text{Im}\left[\left( (M^2_d - \bar{M}^2_d \mathbf{1})^n \right)_{41}/M_3^{2n}\right]\right)$$

(4.7)

where $\bar{M}^2_d$ is the average of eigenvalues of $M^2_d$. In the same manner we have for $u$-quark,

$$\mathcal{D}_u^{SS}/e = 4\alpha_s/(9\pi M_3)^{-1} \sum_n (1/n!) f^{(n)}(Z_0^u) \text{Im}\left[\left( (M^2_u - \bar{M}^2_u \mathbf{1})^n \right)_{41}/M_3^{2n}\right].$$

(4.8)

Let us discuss the imaginary part of Eq. (4.7). If the soft SUSY breaking parameter $A$ has a complex phase, the $n=1$ term dominates and reproduces the result of Ref. 6. Then that phase has to be so small ($\approx 10^{-2}$) as not to overreach the experimental upper bound for expected values ($\sim 10^9$ GeV) of superparticle masses. Hereafter in this section we assume that the soft SUSY breaking parameters are real, and study the effects of flavor mixings in the $(L-R)$ parts of squark mass matrices on EDM of neutron. Under this assumption, the quantity $\text{Im}[\left( (M^2_d - \bar{M}^2_d \mathbf{1})^n \right)_{41}]$ vanishes for any $n$ if there are no $(L-R)$ flavor mixings in $M^2_d$, which can be proved by induction\(^3\) as shown in Appendix B. With $(L-R)$ flavor mixings, the first non-zero value appears in $n=3$ term and is calculated from Eqs. (3\'·6)~(3\'·8) as

$$\text{Im}\left[\left( (M^2_d - \bar{M}^2_d \mathbf{1})^3 \right)_{41}\right] = (s_d/m_\theta)^3 m_d (m_b^2 - m_s^2) (m_c^2 - m_u^2) (m_t^2 - m_d^2) \times (m_t^2 - m_c^2) s_1 s_2 c_2 s_3 s_8. \quad (4.9)$$

Similarly for $u$-type squark mass matrix, the first nontrivial value is

$$\text{Im}\left[\left( (M^2_u - \bar{M}^2_u \mathbf{1})^3 \right)_{41}\right] = (s_u/m_\theta)^3 m_u (m_t^2 - m_c^2) (m_s^2 - m_d^2) (m_b^2 - m_d^2) (m_c^2 - m_s^2) s_1 s_2 c_2 s_3 s_8. \quad (4.10)$$

The quantities $s_d$ and $s_u$ are supposed to be of the same order, while we know $m_t \gg m_b$ and $m_c \gg m_s$. Thus substituting Eqs. (4.9) and (4.10) into Eqs. (4.7) and (4.8), respectively, we find $\mathcal{D}_d^{SS}$ dominates $\mathcal{D}_u^{SS}$.

Finally, SUSY contribution to EDM of neutron is calculated as follows:

$$|\mathcal{D}_N^{SS}/e| \approx (4\alpha_s/81\pi) M_3^{-7} f^{(3)}(Z_0^d) (s_d/m_\theta)^3 \times m_d (m_b^2 - m_s^2) (m_c^2 - m_u^2) (m_t^2 - m_c^2) s_1 s_2 c_2 s_3 s_8$$

$$\approx (4\alpha_s/81\pi) M_3^{-7} f^{(3)}(Z_0^d) (s_d/m_\theta)^3 m_d m_b^2 m_c^2 m_t^4 s_1 s_2 c_2 s_3 s_8$$

$$\approx s_d^3 \eta^{-7} f^{(3)}(Z_0^d) (m_t/10^3) (m_d/10^3)^{-19} s_8 \times 10^{-32} \text{(cm)},$$

(4.11)

where $\eta \equiv M_3/m_\theta$, $Z_0^d \equiv \bar{M}^2_d/M_3^2$ and the masses $m_\theta$ and $m_t$ should be measured in GeV unit. To estimate this value we first study the quantity $\eta^{-7} f^{(3)}(Z_0^d)$. According to our results given in the previous sections, $\bar{M}^2_d = \text{Tr}(M^2_d)/6 \approx (11/4 - (m_t/m_\theta)^2) m_\theta^2$ under the assumptions $M = m_\theta \sim v \sim v'$ and $A \sim 3$. We write $Z_0^d = \bar{M}^2_d/M_3^2 = w^{-2}$ by using a constant $w = \bar{M}^2_d/m_\theta^2$ which is supposed to take a value from 1 to 3. The gluino mass $M_3$ is known to be proportional to the gauge coupling constant squared as $M_3 = (g_3/g)^2 M$,\(^{\ast})$ so we

\(^{\ast})\) The gaugino mass is taken to be real through a suitable phase transformation. (See the previous work on EDM of neutron by the author.\(^{\ast})\)
Table II. Numerical values of $\eta^{-7}f^{(3)}(Z)$.

<table>
<thead>
<tr>
<th>$\eta$</th>
<th>$w$</th>
<th>$Z = \eta^{-2}w$</th>
<th>$-\eta^{-7}f^{(3)}(Z)$</th>
</tr>
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<tr>
<td>2.0</td>
<td>1.0</td>
<td>0.25</td>
<td>6.2($\times 10^{-2}$)</td>
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<td></td>
<td>1.5</td>
<td>0.375</td>
<td>2.2</td>
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<td>2.0</td>
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<td>1.1</td>
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<td>2.5</td>
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<td></td>
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<td>0.40</td>
<td>0.40</td>
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<td>0.17</td>
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<tr>
<td></td>
<td>3.0</td>
<td>0.33</td>
<td>0.17</td>
</tr>
</tbody>
</table>

As found in Ref. 12), there are two kinds of SUSY particle involving Feynman diagrams which contribute to $H_{\text{eff}}$, one is the box type diagram and the other is the triangle type one. The former gives less contributions than the latter as long as the mass of neutral Higgs scalar is reasonable ($\leq 10^2$ GeV). 13)

We consider here triangle type diagrams shown in Figs. 2(a) and (b). For a range of parameters, we find

$$P \approx 1.8 \times 10^{-6} \text{Re}(g_{\text{SP}}).$$

As found in Ref. 12), there are two kinds of SUSY particle involving Feynman diagrams which contribute to $H_{\text{eff}}$, one is the box type diagram and the other is the triangle type one. The former gives less contributions than the latter as long as the mass of neutral Higgs scalar is reasonable ($\leq 10^2$ GeV). 13)
Left-Right Squark Flavor Mixings

Fig. 2. Major contributions to $g_{SP}$. The cross in Fig. 2(b) is a mass insertion expressing a flavor mixing between left squarks, $\tilde{d}_L$ and $\tilde{s}_L$.

Involves a flavor mixing in the $(L-R)$ part, while Fig. 2(b) involves that in the $(L-L)$ part. Contributions to $g_{SP}$ in Eq. (5·2) can be calculated from these diagrams. For simplicity of calculations we use Lagrangian (2·15) and mass insertion method.

The contribution to $(G/\sqrt{2})\text{Re}(g_{SP})$ from Fig. 2(a) is

$$
\frac{g_3^2}{3} \text{Tr}[ (T^a/2)^2 ] (g_2^2/2 + g_1^2/6) v \{ \text{Im}[\gamma(O \bar{M}_u^2 \Omega)_{21}] m_\mu/v^2 m_H^2 \} \\
\times (16\pi^2 x m_\eta)^{-1} \\
\simeq (a_3/3)x) (m_\mu m_\eta/v^2 m_H^2) [m_t^2] \\
\times [\text{Im}(A) (s_1 c_1 s_2^2 c_3 - s_1 c_2 s_2 s_3 c_2) + \text{Re}(A) s_1 c_2 s_2 s_3 s_3],
$$

where we have expressed loop integral by a constant $x$ and $m_\eta$ which is a mass scale of SUSY particles. In the estimation of the imaginary part we have kept only major terms reminding $m_t \approx 20 m_c$ and $s_2, s_3 \approx 0.05$. In the same manner Fig. 2(b) gives

$$
\frac{g_3^2}{3} \text{Tr}[ (T^a/2)^2 ] (g_2^2/2 + g_1^2/6) v \{ \text{Im}[\gamma(O \bar{M}_u^2 \Omega)_{21}] m_\mu/v^2 m_H^2 \} \\
\times (16\pi^2 x m_\eta)^{-1} \\
\simeq (a_3/3) x) (m_\mu m_\eta/v^2 m_H^2) [m_t^2] \\
\times [\text{Im}(A) (s_1 c_1 s_2^2 c_3 - s_1 c_2 s_2 s_3 c_2) + \text{Re}(A) s_1 c_2 s_2 s_3 s_3],
$$

where the parameter $y$ is a constant which parametrize loop integral. The definitions of $x$ and $y$ are given in Appendix C.

We divide the contribution (5·4) from $(L-R)$ flavor mixing by the contributim (5·5) from $(L-L)$ mixing to see which is larger. The answer is

$$
(2\pi a_2)^{-1} (v^2 m_\eta^2/v^4) (y/x) (3 + |A|^2)^{-1} [\text{Im}(A) (c_1 s_2 c_3 / c_2 s_2 s_3 s_3 - \text{Re}(A)].
$$

Now let us discuss the ratio $y/x$. Assuming $m_\eta \approx M$, Eqs. (c·3) ~ (c·5) and (3·9) tell us that $M^2_{dl} \approx M^2_{sl} \approx 3.1 m_\eta^2$ and $M^2_{sr} \approx 2.4 m_\eta^2$. For these values we numerically calculated $1/x, 1/y$ and $y/x$ as functions of $\eta = M_3/m_\eta$. The results are given in Table III, which informs us that the factor $y/x$ is of the order 10. Then for $v \sim v' \sim m_\eta$ and $|A| \sim 3$, the ratio (5·6) becomes of the order 10. We calculated $y/x$ also for other values of $M^2_{dl}(\approx M^2_{sl})$ and $M^2_{sr}$, but $y/x$ kept being of the order 10 as long as $M^2_{dl} > M^2_{sr} > m_\eta^2$. Even in the limiting case $M^2_{dl} = M^2_{sl} = M^2_{sr} = M_3^2$, we have $y/x = 3$ and the ratio is larger than 1. Thus we conclude that for reasonable values of parameters the contribution by $(L-R)$ part is superior to that by $(L-L)$ part.

Lastly we make a numerical estimation of major SUSY contribution (5·4) to the asymmetry $P$. Table III tells us $1/x \sim 1/3$ for reasonable range of parameters. We take $v \sim 10^2$ GeV and $|A| \sim 3$. Leaving the masses $m_t$ and $m_H$ as inputs, we have

$$
|P| \sim (m_t/m_H)^2 s_6 \times 10^{-4}.
$$
We find that even for a light neutral Higgs ($m_H \lesssim 10 \text{ GeV}$) the top quark mass has to be as large as about 100 GeV for this asymmetry to show the existence of physics beyond the standard model.

§ 6. Conclusion

Flavor mixings in the left-right part of the squark mass matrix do give large contributions to EDM of neutron and to the longitudinal polarization of muons in $K_L \rightarrow \mu^+\mu^-$ decay in comparison with those by flavor mixings among left squarks. With left-right squark flavor mixings we have non-zero supersymmetric contribution to EDM of neutron even if the parameters of soft SUSY breaking terms are real. Also to the longitudinal polarization of muons in $K_L \rightarrow \mu^+\mu^-$ decay, the contribution through left-right squark flavor mixing excels that through left-left squark mixing.

To derive these conclusions we have used the interaction Lagrangian and squark mass matrices where all one-loop corrections are taken into account with the use of RGE. We have included corrections by small Yukawa interactions. Our only approximation at this stage was to use perturbative solutions of RGE up to (coupling)$^5$. We believe these results are more precise than those so far obtained in other works. Our interaction Lagrangian will be useful for other studies of SUSY phenomenology at low energy.

Although we have discussed only two phenomena in this paper, there may be another phenomenon where left-right squark flavor mixings play important roles. It would be interesting to study the effects of left-right squark flavor mixings for other processes.

Acknowledgements

The author thanks Professor T. Kotani and Professor E. Takasugi so much for reading the manuscript and helpful advice.

Appendix A

Suppose that a Feynman diagram calculation gives a term $\sum_i \{ X_{a_1} F(M^2_i) X^*_{b_1} \}$, where $F(x)$ is an analytic function, then using the Taylor expansion of $F(x)$,

$$ F(\bar{M}^2) = \sum_n F(M^2) (\bar{M}^2 - \bar{M}^2)^n / n! , \tag{A·1} $$

where $\bar{M}^2$ is an average of $\bar{M}^2_i$, we obtain the following identity:

$$ \sum_i X^* F(\bar{M}^2) X_{a_1} = \sum_n F^{(n)}(\bar{M}^2) \left[ (\bar{M}^2 - \bar{M}^2)^n \right]_{a_1 b_1} / n! $n \left[ (\bar{M}^2 - \bar{M}^2)^n \right]_{a_1 b_1} / n! \tag{A·2} $$

To derive the second equality we have used Eq. (3·15).
Appendix B

Let us consider a $2n \times 2n$ Hermite matrix $M$ of the following form:

$$
M = \begin{bmatrix}
X & a \\
a & b
\end{bmatrix} \in \begin{bmatrix}
X : a n \times n \text{ Hermite matrix} \\
a, b: \text{real diagonal matrices}
\end{bmatrix}
$$

(B.1)

The matrix $M^p (p \text{ an integer})$ is also Hermitian. The $(n+1, 1)$ component of $M$, i.e., $(M)_{n+1,1} = a_{11}$, is real. Assume that $(M^k)_{n+1,1}$ is real, and let us write $M^k$ as

$$
M^k = \begin{bmatrix}
X_k & Y_k^t \\
Y_k & Z_k
\end{bmatrix}
$$

(B.2)

where $X_k$, $Y_k$ and $Z_k$ are $n \times n$ matrices, then Hermiteness of $M^k$ implies that $X_k$ is also Hermitian, so that $(X_k)_{11}$ is real. In addition, $(Y_k)_{11} = (M^k)_{n+1,1}$ is also real by assumption. Now we calculate $M^{k+1}$,

$$
M^{k+1} = M^k \cdot M
$$

(B.3)

Then using Hermiteness of $M^{k+1}$, we find

$$
(M^{k+1})_{n+1,1} = (M^{k+1})^*_{n+1,1} = (X_k \cdot a + Y_k^t \cdot b)_{11}^* = (X_k)_{11} \cdot a_{11} + (Y_k)_{11}^* \cdot b_{11}.
$$

(B.4)

As noted before, the quantities appearing in the last equality are all real, so that $(M^{k+1})_{n+1,1}$ is real. By induction we have proved $(M^p)_{n+1,1}$ is real for any $p$.

Without flavor mixings in the $(L-R)$ part, the mass matrix $M^2_D$ is expressed in the form of Eq. (B.1), so also the matrix $(M^2_D - \tilde{M}^2_D)$ is. Therefore $\text{Im}[(M^2_D - \tilde{M}^2_D)^n]_{41}$ vanishes in that case.

Appendix C

The definitions of parameters $x$ and $y$ in Eqs. (5·4) and (5·5) are given below,

$$
1/x = m_b m_3 (M^2_{sR} - M^2_{sL})^{-1} \ln M^2_{sR} + M^2_s (M^2_{sR} - M^2_s)^{-1} \ln (M^2_{sR}/M^2_s) \\
- (M^2_{sR} - M^2_{sL}),
$$

(C.1)

$$
1/y = m_d^4 (M^2_s - M^2_{dL})^{-1} [(M^2_s - M^2_{dL})^{-1} (1 - M^2_s (M^2_s - M^2_{dL})^{-1} \ln (M^2_s/M^2_{dL})) \\
- (M^2_s - M^2_{dL})],
$$

(C.2)

where $M^2_{dL}$, $M^2_{sL}$ and $M^2_{sR}$ are the (mass)$^2$s of fields $d_L$, $s_L$ and $s_R$, respectively, appearing in Figs. 2(a) and (b). Values of them are obtained from the diagonal parts of mass matrix $M^2_D$ in Eq. (3·5),

$$
M^2_{dL} = \mu^2_L (D) + p\mu m_d^2,
$$

(C.3)
\[ M^2_{2L} = \mu L^2(D) + p_0 m_\theta^2, \]  
\[ M^2_{2R} = \mu R^2(D) + k_D m_\theta^2. \]  

\textbf{References}