3 Kececioglu, D. B., Lalli, V. R., and McConnell, J. B., "Design Features and Results From Fatigue-Reliability-Research Machines," *Experimental Mechanics,* Aug. 1972, pp. 345–351.

4 Kececioglu, D. B., and Cormier, D., "Designing a Specified Reliability Directly Into a Component," *Proceedings of the Third Annual SAE-ASME-AIAA Aerospace Reliability and Maintainability Conference,* Washington, D.C., 1964, pp. 546–565.

5 Kececioglu, D. B., and Smith, J. L., "Statistical Complex Fatigue Data for SAE 4340 Steel and Its Use in Design by Reliability," by The University of Arizona, submitted to NASA-Lewis Research Center, Cleveland, Ohio under NASA Grant NGR 03-002-044, Nov. 15, 1970, 175 pp.

6 Mood, A. M., and Dixon, W. K., "A Method for Obtaining and Analyzing Sensitivity Data," *Journal of the American Statistical Association,* Vol. 43, 1948, pp. 109–126.

7 Kececioglu, D. B., "Reliability Analysis of Mechanical Components and Systems," *Nuclear Engineering and Design,* North-Holland Publishing Co., 1972, pp. 259–290.

8 Shigley, J. E., *Mechanical Engineering Design,* McGraw-Hill, New York, 1963, 631 pp.

9 Juvinall, R. C., *Stress, Strain, and Strength,* McGraw-Hill, New York, 1967, 580 pp.

10 Peterson, R. E., *Stress Concentration Design Factors,* Wiley, New York, 1953, 155 pp.

11 Sines, G., and Weisman, J. L., *Metal Fatigue,* McGraw-Hill, New York, 1959, 415 pp.

12 Kececioglu, D. B., Smith, R. E., and Felsted, E. A., "Distributions of Cycles-to-Failure in Simple Fatigue and the Associated Reliabilities," *Annals of Assurance Sciences,* 1969, pp. 357–374.

DISCUSSION

H. O. Fuchs[5]

The data presented by the authors in Tables 2, 4, and 6 are a valuable addition to our small store of information about fatigue strength distributions and about the interaction of steady torsion with rotating bending.

The interpretation of the data raises serious questions and the claims made for the benefits obtained seem quite exaggerated.

In the strength diagram, Fig. 10, all lines are made to pass through the purely alternating strength S_a and through S_u, the point which corresponds to the ultimate tensile strength of unnotched bars. This approach is reasonable if only S_a and S_u are given and was still good practice fifteen years ago, but much better data and better ways of presenting them are available today—for instance, in the *Fatigue Design Handbook.*[6]

The authors show in Table 1 that the ultimate strength for their notched bars actually was 270 kpsi. Why treat 165 kpsi as a sacred point when 35 tests say 270?

The conversion of torsion to "equivalent" tension seems inappropriate and unnecessary. The Mises-Hencky criterion applies to yielding, and perhaps to fatigue crack initiation, but definitely not to fracture (S_u). Why not simply plot alternating bending stress over steady torsional stress and fit a line? If the Mises-Hencky conversion could be used for this condition we would not need special tests—we could use data from axial tests. The authors' results should be used to test the criterion.

The design data (Table 7) for low stress ratios are far too conservative because of the use of $S_u = 165$ kpsi. The least square fit and the very clever determination of a best fit exponent could not compensate for the disregard of physical reality.

As to the benefits of design by reliability methodology, there is no doubt that knowledge of the variability and analysis of its components is useful. But there is a lot of doubt about the statement that "on the average no more than one in 1000 such shafts will fail." There can be no confidence (in the statistical sense) in an estimate for 0.001 probability of failure in a population when the sample included only 35 specimens. It is an extrapolated guess presented as if it were a fact.

We could assume the 0.999 reliability to be true if we knew for sure that the tails of the stress distribution and of the strength distribution are normal. But we do not know the distribution in the tails and therefore can only say that the calculated reliability, based on the assumption of normal distributions, is 0.999. If lives depend on it, we had better design with some margin of safety to allow for our ignorance of the distribution in the tails.

The diameter of 0.435 calculated on the basis of the test data is compared to a diameter of 0.599 required, according to the authors, by the conventional deterministic design. But this comparison is misleading.

The probabilistic design is based on tests of 560 specimens of the same material, hardness, finish, and approximately the same stress concentration as the target design, tested with similar combinations of applied stresses. The "conventional" design is based only on the ultimate tensile strength, which is generally available. The authors are not comparing deterministic to probabilistic design but design based on few general rules with design based on a mass of specifically applicable test data. A fair comparison would have required use of the same data for both methods, excepting only ignorance of the standard deviations in the deterministic method.

The so-called deterministic design is not only forced to use estimates of fatigue strength, fatigue notch factor, surface effect, and size effect; it is also saddled with a stress concentration factor obtained by multiplying the k for bending and the k for torsion, which seems very unreasonable; in addition, the deterministic design is assigned a factor of 0.75 "to satisfy the reliability specification" and an additional margin of safety of 100 percent. No wonder that it comes out heavier than a design based on the data from 560 test pieces.

The worst part of the proposed approach is its utopian nature: We will never have enough time (or money) to run hundreds of tests for each of all the possible combinations of materials, load ratios, sizes, stress concentrations, and load histories. We must proceed by using estimates—based, of course, on all known data including variabilities—and, if the job warrants it, tests of the finished components, processed as in production, subjected to service loads. The variability of those tests will be the factor which helps determine the required margin of safety. Statistics and reliability theory will thus be used more economically and more effectively.

Authors' Closure

Dr. Fuch's first four specific criticisms all appear to be related to the use of the ultimate strength of ungrooved shafts and the use of the von Mises-Hencky failure criterion. These two questions were studied at the beginning of the research[7] and were found to be the most reasonable starting points. The experimental results given along the stress ratio lines in Fig. 9 and in Table 7, as well as similar results for the different groove radii [1] justify our basic assumptions. We used the ungrooved rather than the grooved static ultimate strength in the Goodman diagram so as not to dramatically alter currently accepted design convention for grooved shafts design. Had we used the grooved static ultimate tensile strength on

---

[5] Design Division, Mechanical Engineering Department, Stanford University, Stanford, Calif. Fellow, ASME.

[6] J. A. Graham, ed., *Fatigue Design Handbook,* Society of Automotive Engineers, Warrendale, Pa., 1968. See Fig. 3-17 and Fig. 3-20.

[7] Kececioglu, D. B., McKinley, J. W., and Saroni, M. J., "A Probabilistic Method of Designing Specified Reliabilities Into Mechanical Components With Time Dependent Stress and Strength Distributions," The University of Arizona, Tucson, Ariz., submitted to NASA-Lewis Research Center, Cleveland, Ohio, under NASA Grant NRG 03-002-044, Jan. 25, 1967, 331 pp.

the Goodman diagram, we would have come up with an even more favorable result for the Design-by-Reliability Methodology in the example shafts. This area deserves careful investigation.

It is acknowledged that confidence limits were not determined for the calculated reliability of 0.999, although this could have been done by the technique given by Kececioglu in reference [7]. However, the phrase "on the average" is believed to be valid. If lives depend upon it, a much higher reliability than 0.999 is required.

Dr. Fuch's observations regarding the Application Cases 2 and 3 express a valid concern, although his criticism that the probabilistic design is based on results obtained from 560 research specimens is not understood. We agree that the cost in time and money of testing all possible combinations of materials, sizes, stress concentrations, etc. would be very high. For that reason we recommend, and are conducting, further research to develop realistic models that incorporate strength and stress as functions of those factors. Until that can be accomplished, conventional design practices must incorporate modifying factors. Since the experimental research does include the effects of those factors, it appears reasonable to use them. The point attempted to be made in the cases is that if only average ultimate strength values for a material are available, the strengths and/or stresses must be modified by known factors to assure operational safety. Since this approach is conservative, overdesign usually results. The exclusive use of experimental distributional data to preclude such overdesign is currently utopian, but it is an end that should be striven for.