



Fig. 17 Complex truck adhesion influence: (a) contact forces, (b) power dissipation

based on a qualitative comparison of certain results with reports describing the behavior of real vehicles, can be made.

The results found here for very low values of adhesion correlate quite well with reported performance under such conditions. Matsudaira [3] and others have reported that hunting of rail vehicles ceased on rainy days. As shown here, low adhesion due to the wet rails probably allowed the vehicles to oscillate without flange contact and with creep forces so low that the usual characteristics of hunting could not be distinguished from the motions due to roadbed irregularities.

It is reasonably well established that the wear and damage to wheels and rails can be reduced by closing the clearance between rail and flange [11]. Davies [12] conducted several experiments with hunting railway vehicles and found that the "violence of the oscillation increases markedly with the flange clearance." The results reported here are in agreement with these observations. The flange forces were found to increase with approximately the square of the clearance.

Finally, Langer and Shamberger [13] found, in experiments on locomotives with cylindrical wheel treads, that above certain critical speeds flange impacts alone were sufficient to maintain a hunting oscillation. This gives some support to the findings that the flange contact nonlinearity reduces the critical speed at which hunting commences.

### Conclusions

The results discussed here demonstrate that the nonlinear flange contact effect can induce hunting motion at speeds well below the critical speed computed from a linear analysis. The nonlinear wheel-rail adhesion relationship does not affect the critical hunting speed, but low adhesion values do reduce the violence of the hunting oscillations. Very low adhesion restricts the oscillations enough that the flanges do not contact. This gives the impression that hunting has ceased.

The simple truck model provides a good qualitative understanding of the hunting behavior. The frequency of the hunting oscillations can be approximated by the frequency found from a linear analysis. The violence of the hunting motion measured by the wheel-rail contact forces and the power dissipated between the wheel and rail increases rapidly as the forward speed, flange clearance, adhesion, or wheel load increase. Flange clearance has the strongest influence of these. The rail stiffness has a relatively small influence on hunting violence in a wide range between about  $10^6$  and  $5 \times 10^6$  lb/ft. The violence of hunting increases rapidly as the wheels wear.

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### DISCUSSION

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I would like to congratulate Professor Cooperrider for his fine paper. Those familiar with the "real life" effects on truck hunting of such things as flange clearance and adhesion will appreciate this study.

Recent analytical results which I have obtained for a single axle exhibit properties qualitatively similar to Cooperrider's results for a simple truck. My results also indicate that stable limit cycle oscillations (with flange contact) may occur for certain configurations at speeds below the critical speed as predicted by a linear analysis. If such is the case, then "...at speeds below the critical value from the linear analysis, hunting will not begin unless the vehicle initial conditions are large enough... to cause the flanges to contact. If the flanges do not contact, the

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vehicle response will decay. . .” Cooperrider also states that for the configurations he analyzed “. . . the existence of a non-zero speed below which limit cycles cannot be induced for any initial conditions was not established . . .” However, my results indicate the theoretical possibility that for certain configurations, no stationary oscillations are possible for speeds below the linear critical speed even if flange contact occurs.

One aspect of wheel/rail kinematics which can lead to results qualitatively different from those obtained by Professor Cooperrider is the non-linear change in wheel rolling radius with lateral

displacement. If the parameter  $\lambda$  (the effective taper ratio of the wheels) is considered as a non-linear function of the lateral displacement, it may be shown that for certain configurations an unstable limit cycle exists for speeds below the linear critical speed. As speed increases up to the linear critical speed, the amplitude of this unstable limit cycle decreases. This implies that below the linear critical speed sufficiently large initial conditions, that do not cause the flanges to contact, may cause the amplitude of the motion to increase until the flanges do contact.