A General Solution for the Elastoplastic Thermal Stresses in a Strain-Hardening Plate With Arbitrary Material Properties

E. A. DAVIS. The authors of this paper, together with their co-workers at the Lewis Research Center, are to be commended for their approach to the elastic-plastic, strain-distribution problem. The writer believes that the unique effective-stress versus effective-strain diagram provides the most reasonable approach to this type of problem. There is one point in the fundamental equations upon which this method is based, however, that this writer does not understand and which he wishes to bring up for discussion. Unfortunately, this discussion is aimed more at reference [10] than at the paper itself, but since this paper is based on that reference and because there seems to be no other way of discussing it, the writer must use this indirect means.

This discussion is concerned chiefly with the definition of total effective strain and whether or not total effective strain can or should be expressed in terms of the total-strain components $e_{p}$ and $e_{e}$. The total effective strain is defined in Appendix B of the reference as

$$
e_{t} = \sqrt{\frac{2}{3}} [(e_{p} - e_{e})^2 + (e_{e} - e_{o})^2 + (e_{o} - e_{o})^2]^{1/3}.$$

There is no question or dispute about the definitions given for effective stress $\sigma_{e}$ and effective plastic strain $e_{p}$,

$$\sigma_{e} = \frac{1}{\sqrt{2}} (\sigma_{x} - \sigma_{s})^2 + (\sigma_{y} - \sigma_{s})^2 + (\sigma_{z} - \sigma_{s})^2)^{1/2},$$

$$e_{p} = \frac{1}{\sqrt{2}} [(e_{x} - e_{x})^2 + (e_{y} - e_{y})^2 + (e_{z} - e_{z})^2]^{1/2},$$

but if Hooke's law is applied to the equation for effective stress, an elastic effective strain $e_{el}$ is obtained which is

$$e_{el} = \frac{\sigma_{e}}{E} = \frac{1}{\sqrt{2(1 + v)}} [(e_{x} - e_{x})^2 + (e_{y} - e_{y})^2 + (e_{z} - e_{z})^2]^{1/2}.$$

It is the feeling of the writer that the total effective strain should be the sum of the elastic and the plastic parts of the strain

$$e_{t} = e_{el} + e_{p}.$$

This expression precludes any simple relationship between the total effective strain and the total-strain components. The relationship exists and is fixed by the foregoing expression, but it certainly is not simple.


2 Advisory Engineer. Westinghouse Research Laboratories, Pittsburgh, Pa. Mem. ASME.