

$$\gamma_{2\rho} = \frac{b_M \bar{\mu}_0}{2h_0 \bar{\rho}_0} [u_{20}(\gamma_J + \gamma_B)] - \frac{h_0}{\bar{\rho}_0} \frac{\partial p_0}{\partial x_2} \quad (\text{A.7})$$

$$\gamma_{1\mu} = \frac{-b_M}{2h_0} [u_{10}(\gamma_J + \gamma_B) - \Lambda_1 \gamma_J] \quad (\text{A.8})$$

$$\gamma_{2\mu} = \frac{-b_M}{2h_0} [u_{20}(\gamma_J + \gamma_B)] \quad (\text{A.9})$$

where:

$$k_{10} = k_{20} = 0.5(k_{J0} + k_{B0}); k_{J0} = R_{J0} f_{J0}, k_{B0} = R_{B0} f_{B0} \quad (\text{A.10})$$

$$C_{CJ} = (1/2)[R_{J0} C_J + b_M] \gamma_J; C_{CB} = (1/2)[R_{B0} C_B + b_M] \gamma_B \quad (\text{A.11})$$

$$f_{J1} = \frac{(\text{Re}_{\rho} \bar{\rho}_0 / \bar{\mu}_0)^2 h_0}{2R_{J0}} [f_{J0} + \beta_J \gamma_J] \quad (\text{A.12})$$

$$f_{B1} = \frac{(\text{Re}_{\rho} \bar{\rho}_0 / \bar{\mu}_0)^2 h_0}{2R_{B0}} [f_{B0} + \beta_B \gamma_B]$$

$$\gamma_J = \frac{-a_M \cdot e_M}{[f_{J0} / a_M - 1]^{(1/\epsilon_M - 1)}}; \quad (\text{A.13})$$

$$\gamma_B = \frac{-a_M \cdot e_M}{[f_{B0} / a_M - 1]^{(1/\epsilon_M - 1)}}$$

$$\beta_J = b_M / R_{J0}; \beta_B = b_M / R_{B0} \quad (\text{A.14})$$

$$C_J = c_M \frac{r_J}{c_* h_0}; C_B = c_M \frac{r_B}{c_* h_0} \quad (\text{A.15})$$

and,

$$R_{J0} = \text{Re}_{\rho}(\bar{\rho}_0 / \bar{\mu}_0) h_0 [(u_{10} - \Lambda_1)^2 + u_{20}^2]^{1/2}$$

$$R_{B0} = \text{Re}_{\rho}(\bar{\rho}_0 / \bar{\mu}_0) h_0 [(u_{10}^2 + u_{20}^2)^{1/2}] \quad (\text{A.16})$$

are the zeroth-order flow field Reynolds numbers relative to the journal and bearing surfaces, respectively.

For laminar flows, (inertialless):

$$\gamma_{11} = \gamma_{22} = 12\bar{\mu}_0 / h_0, \gamma_{12} = \gamma_{21} = \gamma_{1\rho} = \gamma_{2\rho} = 0$$

$$\gamma_{20} = -24\bar{\mu}_0 u_{20} / h_0^2, \gamma_{10} = (-24u_{10} + 12\Lambda_1)\bar{\mu}_0 / h_0^2$$

$$\gamma_{1\mu} = 12(u_{10} - \Lambda_1/2) / h_0, \gamma_{2\mu} = 12u_{20} / h_0 \quad (\text{A.17})$$

## DISCUSSION

### J. K. Scharrer<sup>1</sup>

The author addresses some of the issues of compressibility of cryogenic fluids with respect to hydrostatic bearing performance. However, he has neglected to consider the possibility of choked flow in the bearing. This condition can occur in liquid hydrogen bearings, especially if they are located near the turbine section of a turbopump. Has the author investigated this? If so, what were the results?

In the derivation of the first-order equations, the author shows a term for the derivative of density with respect to pressure for a barotropic fluid. However, no mention is made as to how this term is obtained in the solution. Is this obtained numerically and if so, what increments of pressure were used to determine this derivative? What were the sensitivities of the results to this increment? If it was obtained analytically, what equations were used?

In this paper, comparisons are made to liquid hydrogen test data. For these comparisons did the authors obtain leakage results by blindly applying orifice discharge coefficients and recess edge loss coefficients based on pipe flow models or did they vary these parameters to match the data? Regardless of the method, what values were used to match the data? Are these values realistic?

### A. F. Artiles<sup>2</sup>

Professor San Andres is to be commended for accomplishing the difficult task of extending the theory of HBJ's to include barotropic fluids. It is interesting that the use of constant properties over-predicts the flow and torque relative to the full use of the variable barotropic properties, while use of linear properties has the opposite effect. One might expect that using constant properties, separately evaluated at the supply and at the discharge pressures would bracket the two variable-properties model.

It might be interesting to repeat the same comparison of the three models already done in the paper of the force coefficients

on the stability indicators (threshold speed and critical mass), in order to determine whether stability predictions with constant properties are conservative or optimistic.

Now I would like to make offer some remarks concerning the pressure variation at the recess boundary. The pressure drop from the recess to the film lands is given by Eq. (7) of San Andres' paper, which is derived from Eq. (22) in the work of Constantinescu and Galetuse (1975). It consists of two terms: the pressure rise due to viscous shear and the entrance pressure drop due to the Bernoulli effect. However, Constantinescu's equation is based on an incompressible, isoviscous turbulent flow through an infinitely-long step bearing with smooth surfaces. One might question whether this equation could be generally applicable to a two-dimensional flow which is really three-dimensional at the pocket boundaries and where both the viscosity and density are strong functions of the local pressure, the variation of which is significant at the pocket boundaries. Besides the restrictions inherent in Constantinescu's formula (such as  $1/\eta < 20$ ), it might be elucidating to know what additional restrictions apply when extending it to more general applications.

Now some clarifications concerning the implementation of Eq. (7):

1. The paper states that the viscous shear term is only present downstream of the orifice. Is this term set to zero for parts of the boundary upstream of the orifice?
2. The restriction that the component of velocity normal to the pocket boundary be positive ( $\mathbf{U} \cdot \mathbf{n} > 0$ ) only applies to the Bernoulli pressure drop term.
3. A distinction is made between the axial and circumferential pressure drop coefficients,  $\xi_1$  and  $\xi_2$ , respectively. Which is used at the pocket corner? Is the pressure drop allowed to be discontinuous there?

### Author's Closure

The author thanks gratefully the valuable comments presented by Dr. Antonio Artiles and Dr. Joseph Scharrer.

In regard to Dr. Scharrer discussion, we have not introduced in our model the possibilities of choked flow. The hydrostatic bearing (HJBs) geometries studied, in especial for LH2, show

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local Mach numbers as large as 0.60 at the bearing discharge end. As the Mach number increases towards one, the discharge pressure boundary condition can not be imposed in the current analysis. In the future, and due to the importance of large Mach number flows in a practical situation, we will account in the analysis and numerical algorithm for the possibility of choked flow. The first order density and viscosity derivatives relative to pressure are calculated numerically by setting a variation of 1 percent in the local pressure. Larger percentile variations to 10 percent of the local pressure are permissible without appreciable error in the calculated results. In regard to the comparison with the experimental results of Butner and Murphy (1986), the discharge orifice coefficient,  $C_d = 0.90$ , was obtained from the same reference and calculated from measurements of flow rate and pressure drop across the orifice. The recess edge coefficients used in the analysis are equal to  $\xi = 0$  in all directions and are based on the experience of the author. In HJBs, and unlike the case of annular seals, direct force coefficients are not greatly affected by this recess-edge coefficients since the orifice resistance coefficient is the one of primary importance to provide the hydrostatic load support.

The comments presented by Dr. Antonio Artiles are most important and do help greatly in the understanding of the present work. Equation (7) is indeed valid for an incompressible fluid; however, the material properties variation in the recess volume have been calculated to be small and the use of

(7) does not affect the results (less than 0.5 percent difference) when compared to a more accurate solution based on the equations shown below. Due to Dr. Artiles suggestion, we have implemented a new recess pressure rise and drop equation valid for a barotropic fluid. The analysis performed is largely based on that presented by Constantinescu et al. (1987). The improved recess edge downstream pressure rise ( $P_e^-$ ) and edge pressure drop ( $P_e^+$ ) equations are given by:

$$P_e^- = P_r - \mu_r k_r \frac{b}{2} \frac{1}{H_r^2} \bar{\mathbf{n}} \cdot \left( \bar{\mathbf{U}} (\rho_e^- / \rho_r) \eta - \frac{\bar{\mathbf{V}}}{2} \right) \frac{1}{(1 - M^2)} \quad (\text{A.1})$$

$$P_e^+ = P_e^- - \frac{\rho_e^+}{2} (1 + \xi) \left\{ 1 - (\rho_e^+ / \rho_e^-)^2 \eta^2 \right\} (\bar{\mathbf{U}} \cdot \bar{\mathbf{n}})^2, \text{ if } (\bar{\mathbf{U}} \cdot \bar{\mathbf{n}}) > 0 \quad (\text{A.2})$$

and where, M is the local Mach number at the orifice inlet.

The clarifications presented by Dr. Artiles at the end of his discussion are most welcome. The issues under scrutiny were brought up in earlier papers of the author (San Andres, 1990a, b).

#### Additional Reference

Constantinescu, V. N., and Fl. Dimofte, 1987, "On the Influence of the Mach Number on Pressure Distribution in Gas Lubricated Step Bearings," *Rev. Roum. Sci. Techn.-Mech. Appl.*, Tome 32, No. 1, Bucarest, pp. 51-56.