Many Body Pion Absorption Processes in the Delta Resonance Region

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We study the many body pion absorption processes microscopically for pions in the delta-isobar energy region. It is found that the three body pion absorption process is as large as the two body process, while the four body process (the $2\Delta$ mechanism) turns out to be quite small.

§ 1. Introduction

Pion absorption in the resonance energy region by a nucleus had been discussed in terms of the two body mechanism (Fig. 1(a)). This belief was, however, questioned by the experimentalists, who took the proton spectra after pion absorption. Although a peak is expected at a proper kinematics for a proton coming out directly after sharing the absorbed pion energy with another nucleon, they did not observe it clearly even for a nucleus as small as $^{12}$C. In addition, the angular distribution was found nearly isotropic around a certain rapidity in the rapidity plane. Arguing then that the final state interaction for the out-going proton is small, they claimed that there should be many body pion absorption processes. This discussion was further strengthened by the coincidence experiments on out-going protons, which indicated that the two body absorption process would be a fraction of the total pion absorption cross sections.

Brown et al. then suggested a $2\Delta$ mechanism shown in Fig. 1(b), which leads to the emission of four nucleons directly after pion absorption. This $2\Delta$ process seemed to reproduce the experimental features without the introduction of the final state interaction for out-going protons. The estimate of the magnitudes was, however, very crude.

Recently, Oset et al. calculated explicitly the three body pion absorption processes (Figs. 1(c) and (d)). The question asked there was simply, 'if the $2\Delta$ process is large, then why aren't the three body processes also large?' With the use of the vertices and propagators provided by the deuteron pionic disintegration, they found that the three body processes were large. At low energies the two body process dominates, but as the pion energy approaches the resonance energy, the three body processes would become as large as the two body process. Oset et al. then applied the three body processes to other channels and found that all the total cross sections (elastic, inelastic charge exchange and absorption) were reproduced very well within the model. They also found that the delta isobar width obtained from the analyses of pion elastic scattering could also be explained by the three body processes and the higher order quasi-elastic processes.

In the work by Oset et al., however, one of the three body processes (Fig. 1(d))
Fig. 1. Many body pion absorption process: (a) Two nucleon process. (b) Two delta process (4 nucleon process). (c) Sequential three nucleon process. (d) Successive three nucleon process. The dashed line denotes pion, while the solid and double solid lines are for nucleons and delta isobars. The interaction is denoted by the wavy line, which is assumed as due to $\pi + \rho$ exchange with the short-range correlation.

was just estimated roughly with the experience of calculating the other three body process (Fig. 1(c)). Hence, in this paper, we would like to calculate the successive pion absorption process (Fig. 1(d)) explicitly following their method. In addition, we would like to calculate the $2\Delta$ mechanism with the same spirit of calculating the three body mechanisms.

This paper is arranged as follows. In §2, the successive three body absorption process will be formulated. The $2\Delta$ process is discussed then by modifying some part of the three body process. The numerical results will be provided in §3. We will discuss then the difference between the microscopic estimate of the $2\Delta$ process worked out here with a rough estimate of the $2\Delta$ process performed by Brown et al. The summary of this work is provided in §4.
§ 2. Many body pion absorption processes

We shall study here in detail pion absorption by three nucleons through the successive mechanism (Fig. 1(d)). A rough estimate of absorption cross section of this mechanism was performed by Oset et al. The pion absorption rate per unit volume is given by

$$\Gamma^{(3)}(q) = -\frac{1}{q} \text{Im} \Pi^{(3)}(q),$$

where $q$ is the incident pion momentum, and $\Pi^{(3)}(q)$ is the pion self-energy which corresponds to the optical potential ($V_{\text{opt}} = 1/2q^0 \Pi^{(3)}(q)$) of successive three body absorption (Fig. 2). We get the following formula for the pion self-energy $\Pi^{(3)}(q)$ by applying the Feynman rules (Fig. 2):

$$-i\Pi^{(3)}(q) = -\left(\frac{f^*}{\mu}\right)^4 \left(\frac{f^*}{\mu}\right)^4 \left(\frac{f^*}{\mu}\right)^2 \sum_{\text{isospin}} \int \frac{d^4k}{(2\pi)^4} \frac{d^4p}{(2\pi)^4} \frac{d^4p'}{(2\pi)^4} iG_n(k)$$

$$\times \left[ i^2G_\Delta(q+k)^2G_\Delta(q+k-p)^2iG_n(q+k-p-p') \right]$$

$$\times \left[ i^2D_\pi^2(p) i^2D_\pi^2(p') (-i)p^2U_n(p) (-i)p^2U_n(p') \right]$$

$$\times \langle m_s | S(-q) | M_s | S_d | p | M_s' | S^\dagger p | m_s' \rangle$$

$$\times \langle m_s' | S(-p') | M_s'' | S_d | p' | M_s''' | S^\dagger q | m_s \rangle$$

$$\times \langle m_t | T_A | M_t' | T_d | M_t | T_{\omega} | m_t' \rangle$$

$$\times \langle m_t' | T_{\omega} | M_t'' | T_d | M_t''' | T_{\omega} | m_t \rangle$$

with the nuclear density $\rho$, the pion mass $\mu$, the nucleon mass $M$ and the vertex coupling constants $f$ for $\pi-N-N$, $f^*$ for $\pi-\Delta-N$, and $f_d$ for $\pi-\Delta-\Delta$. $G_n(k)$, $G_\Delta(q)$ and $D_\pi(p)$ are nucleon, delta and pion propagators:

$$G_n(k) = \frac{1-n(k)}{k^0 - \mathbf{k}^2/2M + i\varepsilon} + \frac{n(k)}{k^0 - \mathbf{k}^2/2M - i\varepsilon},$$

$$G_\Delta(q) = \frac{1}{q^0 - \omega_\Delta - q^2/2M + i\varepsilon/2},$$

$$D_\pi(p) = \frac{1}{p^0 - \mathbf{p}^2 + i\varepsilon},$$

where $M_\Delta$ is the $\Delta$ mass and $\omega_\Delta$ is the mass difference between delta and nucleon; $\omega_\Delta = M_\Delta - M$. $n(k)$ is the occupation probability. $U_N(p)$ is the Lindhard function for $p-h$ excitation (Fig. 3(a)).

By making use of the low density approximation, $^6$

$$\text{Im} U_N(p) = -\pi \rho \delta(p^0 - p^2/2M) P_F(p),$$

where $P_F(p)$ is the Pauli blocking factor; $P_F(p) = 1 - \theta(2 - p/k_F)(1 - (3/4)(p/k_F) + (1/16) \times (p/k_F)^3)$ with $k_F$ being the Fermi momentum of nucleons, which is related with density.
Fig. 2. The pion self-energy graph for the successive three nucleon process. The absorption rate is calculated by taking the imaginary part of this self-energy.

\[
\text{Im}\Pi^{(3)} = -\frac{\pi}{2} \rho^3 \int \frac{d^3 p}{(2\pi)^3} \left\{ \frac{D_\pi^2(p) D_\pi^2(p')}{D_\pi^2(p) D_\pi^2(p')} \right\}^2 \times \left[ \frac{24}{36} q^2 (p \cdot p')^2 - \frac{24}{36} p^2 (p' \cdot q)^2 - \frac{9}{36} p^2 (p' \cdot q)^2 \right] \times \delta(q^0 - p^2/2M - p'^2/2M - (q - p - p')^2/2M),
\]

where the nucleon kinetic energy \( k^2/2M \) has been neglected with respect to \( q^0 \). In addition, we have used the consideration developed by Oset et al. and replaced \( G_d \) by |\( G_d |^2 \) using Eq.(2.11) of Ref. 6). Oset et al.\(^6\) considered in their rough estimate of this process only one pion exchange process for \( \Delta-N \) interactions. However, we need to consider not only the one-pion exchange but also the \( \rho \)-exchange, modified by the presence of the short-range nuclear correlations. Following Refs. 6) and 10), we modify the one-pion exchange interaction

\[
V_\pi^{ij}(p) = \left( \frac{f}{\mu} \right)^2 D_\pi(p) p^2 \vec{p}_i \vec{p}_j \quad (3a)
\]

as follows:

\[
\tilde{V}_\pi^{ij}(p) = A(p) (\delta_{ij} - \vec{p}_i \vec{p}_j) + B(p) \vec{p}_i \vec{p}_j, \quad (3b)
\]
where $A(p)$ is the interaction strength of the transverse mode and $B(p)$ is that of the longitudinal mode. $\vec{p}$ is the unit vector of the momentum $p$. The functional forms for each mode are given by

$$A(p) = \left(\frac{f}{\mu}\right)^2 \left[ F_{\pi}^2(p)D_\pi(p)p^2C_\pi - \frac{1}{3} q_c^2 \tilde{F}_\pi^2(p) \tilde{D}_\pi(p) \right]$$

$$+ \tilde{F}_\pi^2(p) \tilde{D}_\pi(p) \left( p^2 + \frac{2}{3} q_c^2 \right),$$

$$B(p) = \left(\frac{f}{\mu}\right)^2 \left[ F_{\sigma}^2(p)D_\sigma(p)p^2 - \tilde{F}_\sigma^2(p) \tilde{D}_\sigma(p) \right]$$

$$\times \left( p^2 + \frac{1}{3} q_c^2 \right) - \frac{2}{3} q_c^2 \tilde{F}_\sigma^2(p) \tilde{D}_\sigma(p)C_\sigma,$$

where $F_\pi$ and $F_\sigma$ are $\pi$ and $\rho$ vertex form factors, chosen as a monopole type, $q_c$ is a correlation parameter, chosen as the $\omega$-meson mass ($q_c \sim m_\omega$) and $\tilde{D}_\pi, \tilde{D}_\sigma, \tilde{F}_\pi, \tilde{F}_\sigma$ are the corresponding propagators and form factors obtained by substituting $p^2$ by $p^2 + q_c^2$ for each term. $C_\pi$ is the ratio $(f_\pi/m_\pi)^2/(f_\mu)^2$. Consequently, Eq.(2) is modified as follows:

$$\text{Im } I^{(a)}(q) = -\frac{\pi}{2} \delta^3 \int \frac{d^3 \rho}{(2\pi)^3} \frac{d^3 \rho'}{(2\pi)^3} |G_\delta(q)|^2 |G_\delta(q - p)|^2 P_\pi(q - p - p') P_\pi(p)$$

$$\times P_\pi(p') \left( \frac{f_\pi}{\mu} \right)^2 \left( \frac{f_\pi}{f} \right)^4 \frac{5}{2} q^2 [A^3(p)A^2(p') - \frac{2}{3} (1 + \cos^2 \phi)$$

$$- \frac{4}{3} (1 - \cos^2 \theta) - \frac{1}{2} (1 - \cos^2 \theta') + (1 - \cos^2 \theta' - \cos^2 \theta + \cos \phi \cos \theta \cos \theta') + \frac{25}{9}]$$

$$+ A^3(p)B^2(p') \left[ - \frac{2}{3} (1 - \cos^2 \phi) - \frac{2}{3} (1 - \cos^2 \theta) + \frac{1}{2} \cos^2 \theta'$$

$$- \cos \phi \cos \theta \cos \theta' + \frac{25}{18} \right] + B^2(p)A^2(p') \left[ - \frac{2}{3} (1 - \cos^2 \phi) - \frac{1}{3} \cos^2 \theta$$

$$- \frac{1}{4} (1 - \cos^2 \theta') - \cos \phi \cos \theta \cos \theta' + \frac{25}{18} \right] + B^2(p)B^2(p')$$

$$\times \delta(q^2 - p^2/2M - p'^2/2M - (q - p - p')^2/2M)$$

with $\cos \theta = \vec{p} \cdot \vec{q}$, $\cos \theta' = \vec{p}' \cdot \vec{q}$ and $\cos \phi = \vec{p} \cdot \vec{p}'$.

The terms $B(p)$ and $B(p')$ still contain the pion pole at $p^0 = (p^2 + \mu^2)^{1/2}$. However, the phase space integrand peaks around $p^0 \sim p^2/2M$, $p^0 \sim p'^2/2M$ which is far from the pion pole. Thus the $p$ and $p'$ dependence of the effective interactions is rather smooth, and it allows us to make an approximation to take averages of these interactions. If we also average $\cos^2 \theta$ to $1/3$, $\cos^2 \theta'$ to $1/3$, $\cos \phi \cos \theta \cos \theta'$ to $1/9$ and average out the Pauli blocking factors, then we obtain the following expression:
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\[
\text{Im} \mathcal{I}^{(3)}(q) = -\frac{\pi}{2} \rho^3 \left( \frac{f_\Delta}{\mu} \right)^2 \left( \frac{f^* \mathcal{J}}{f} \right)^4 q |G_d(q)|^2 P_F^3(\bar{p}) \\
\times \frac{5}{2} \left[ \frac{10}{9} A^4(\mathbf{p}) + \frac{10}{9} A^2(\mathbf{p}) B^2(\mathbf{p}) + \frac{5}{18} B^4(\mathbf{p}) \right]_{\text{av}} \int \frac{d^3 \mathbf{b}}{(2\pi)^3} \frac{d^3 \mathbf{b}'}{(2\pi)^3} |G_d(q - \mathbf{p})|^2 \\
\times \delta(q - \mathbf{p}^2/2M - \mathbf{p}'^2/2M - (q - \mathbf{p} - \mathbf{p}')^2/2M)
\]

with an average momentum \( p_{\text{av}} = p'_{\text{av}} = \bar{p} = (2Mq^0/3)^{1/2} \) for effective interaction and for the Pauli blocking factors. Since we expect that the energy of each final nucleon peaks around \( q^0/3 \), we substitute \( q^0/3 \) to \((q - \mathbf{p} - \mathbf{p}')^2/2M\) in the energy conservation \( \delta \)-function. After performing the phase space integrations, we obtain

\[
\text{Im} \mathcal{I}^{(3)}(q) = -\frac{\pi}{2} \rho^3 \left( \frac{f_\Delta}{\mu} \right)^2 \left( \frac{f^* \mathcal{J}}{f} \right)^4 q |G_d(q)|^2 P_F^3(\bar{p}) \times \frac{5}{2} \left[ \frac{10}{9} A^4(\mathbf{p}) + \frac{10}{9} A^2(\mathbf{p}) B^2(\mathbf{p}) + \frac{5}{18} B^4(\mathbf{p}) \right]_{\text{av}} \\
\times \int_{-1}^{1} d \cos \theta |G_d(q - \mathbf{p})|^2
\]

with \( p_{\text{max}} = (2M \cdot 2q^0/3)^{1/2} \). Together with Eq.(1), we obtain the absorption rate of the successive mechanism as follows:

\[
\Gamma^{(3)}(q) = \frac{\pi}{(2\pi)^4} \left( \frac{f_\Delta}{\mu} \right)^2 \left( \frac{f^* \mathcal{J}}{f} \right)^4 q |G_d(q)|^2 P_F^3(\bar{p}) \\
\times \frac{5}{2} \left[ \frac{10}{9} A^4(\mathbf{p}) + \frac{10}{9} A^2(\mathbf{p}) B^2(\mathbf{p}) + \frac{5}{18} B^4(\mathbf{p}) \right]_{\text{av}} \\
\times \int_{0}^{p_{\text{max}}} dp M p^2 \sqrt{(2q^0/3 - p^2/2M)2M} \\
\times \int_{-1}^{1} d \cos \theta |G_d(q - \mathbf{p})|^2.
\]

We shall perform double fold integrals numerically and discuss the results in the next section.

Next we would like to study pion absorption by four nucleons through the double delta mechanism of Fig.1(b). We also study it through the corresponding piece of the optical potential shown in Fig. 5. Comparison of Figs. 2 and 4 tells us that we get the diagram of Fig. 4 by replacing the outside \( p-h \) excitation (Fig. 3(a)) in Fig. 2 with the diagram of Fig. 3(b) which corresponds to two body absorption. Hence, we can easily evaluate the contribution from this
mechanism by replacing \((f/\mu)^2 p^2 \text{Im} U_N(p)\) in our non-sequential three body mechanism; Eq.(5) by \(\text{Im} \Pi^{(2)}(p)\). Using\(^6\)

\[
\text{Im} U_N(p) = -\pi \rho \delta(q^0 - p^2/2M - (q - p - p')^2/2M - p^2/2M)P_r(p),
\]

\[
\text{Im} \Pi^{(2)}(p) = -\frac{1}{27\pi} \rho^2 |G_d(p)|^2 \left(\frac{f}{\mu}\right)^2 p^2 M (M p^0 - p^2/4)^{1/2} \times P_r^2(p)[2A^2(p') + B^2(p')],
\]
and averaging \(\cos^2 \theta\) to 1/3, \(\cos^2 \theta'\) to 1/3, \(\cos \theta \cos \theta'\) to 1/9, we obtain

\[
\text{Im} \Pi^{(2d)}(q) = -\frac{1}{54\pi} \rho^4 \left(\frac{f}{\mu}\right)^2 |G_d(q)|^2 q^2 [2A^2(p') + B^2(p')] \times P_r^2(p)
\]

\[
\times \frac{d^3 p}{(2\pi)^3} \frac{d^3 p'}{(2\pi)^3} |G_d(q - p)|^2 |G_d(p)|^2 M \sqrt{M p^0 - p^2/4}
\]

\[
\times P_r(q - p - p') P_r(p'),
\]

\[
\times \frac{5}{2} \left[ \frac{10}{9} A^2(p) A^2(p') + \frac{5}{9} (A^2(p) B^2(p') + B^2(p) A^2(p')) \right] + \frac{5}{18} B^2(p) B^2(p')
\]

\[
\text{Av}[\text{Integrand}],
\]

We also average out the Pauli blocking factors, and assume that the outgoing two nucleons in both sides of the interaction specified by four momenta \(p\) share the incoming energy \(q^0; p^0 = q^0/2\).

For \(A(p)\) and \(B(p)\) of interactions 1 and 2 in Fig. 1(b), we can make the approximation as before. However, we cannot perform the same approximation to interaction 3, because there is no restriction between energy \(p^0\) and momentum \(p\) which are carried by this interaction and therefore the pion propagator here can be on mass-shell in the region that we study. Concerning the momentum \(p\), we shall make a detailed numerical investigation of the integrand, which will be discussed in the following section. From this we find that the integrand of \(\mathcal{P}\) integration peaks sharply around \(p_{\text{av}} = 3.6\mu\) and we shall also average out the amplitude of \(p\) by using the average momentum \(p_{\text{av}} = 3.6\mu\). Consequently, the absorption rate of the double-\(\Delta\) mechanism is given by

\[
\Gamma^{(2d)}(q) = \frac{1}{27\pi (2\pi)^3} \rho^4 \left(\frac{f}{\mu}\right)^2 |G_d(q)|^2 q^2 [2A^2(p') + B^2(p')],
\]

\[
\times P_r^2(p) \frac{5}{2} \left[ \frac{10}{9} A^2(p) A^2(p') + \frac{5}{9} (A^2(p) B^2(p') + B^2(p) A^2(p')) \right]
\]

\[
\times P_r^2(p) \frac{p_{\text{max}}^2}{3} \int_0^{p_{\text{max}}} dp \sqrt{M p^0 - p^2/4} |G_d(p)|^2
\]

\[
\times \int d \cos \theta |G_d(q - p)|^2 p_{\text{av}} = q^0/2,
\]

where \(p_{\text{max}} = \sqrt{4M p^0} = \sqrt{2M q^0}\) and \(p_{\text{max}} = \sqrt{2M (q^0 - p^0)} = \sqrt{M q^0}\). Average momenta \(p_{\text{av}}\)
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for interaction 1 and \( p_{\omega}^2 \) for interaction 2 are chosen \( p_{\omega}^2 = (2M \cdot q^2 / 4)^{1/2} \), which are also used for the Pauli blocking factors.

We shall discuss here the medium effects on the delta propagator and the interactions. As for the delta propagators we shall add the self-energy term \( \Sigma_d(q) \) as

\[
G_d(q) = \frac{1}{q^0 - \omega_d - q^2 / 2M + i\Gamma / 2 - \Sigma_d(q)}.
\]

For the self-energy \( \Sigma_d \), we will take a phenomenological approach and use the \( \Delta \)-spreading potential.\(^6\) In addition we have to add the Pauli correction explicitly. Following Ref. 6), we shall take

\[
\text{Im} \, \Sigma_d = \text{Im} \, \Sigma_d^{sp} + \text{Im} \, \Sigma_d^{Pauli} = -42\rho/\rho_0(\text{MeV}) + \text{Im} \, \Sigma_d^{Pauli},
\]

where the Pauli correction term \( \text{Im} \, \Sigma_d^{Pauli} \) rather complicated and it is given in the Appendix of Ref. 6). The real part of the self-energy and \( \Delta \)'s single particle kinetic energy will cancel out approximately in the resonance region, thus we omit them in the calculations.\(^6\) For the \( \Delta \) width we take the expression of Ref. 10) (correcting a misprint, \( M_\Delta / \sqrt{s} \rightarrow M_\Delta / \sqrt{s} \) in A. 4 and A. 8).

We must also consider the effect of the renormalization of the virtual mesons involved in the process. However, it is not our purpose to study it in detail. We will just show how to modify them. All one has to do is to substitute\(^6\)

\[
A^2(p) \rightarrow \left| \frac{\bar{A}(p)}{1 - \bar{A}(p)U(p)} \right|^2,
\]

\[
B^2(p) \rightarrow \left| \frac{\bar{B}(p)}{1 - \bar{B}(p)U(p)} \right|^2
\]

in the appropriate places. Lindhard function \( U(p) \) for \( \rho - h \) and \( \Delta - h \) in Eqs. (12·a) and (12·b) is approximated as\(^6\)

\[
U_{\rho h} = -\frac{M}{\bar{p}^2} \rho - i\frac{M}{2\pi\bar{p}} \frac{3\pi^2\bar{p}}{2}^{2/3},
\]

\[
U_{\Delta h} = \frac{4}{9} \frac{(f^*)^2}{f} \rho \left[ \frac{1}{\bar{p}^2 - \omega_\Delta + i\Gamma / 2} - \frac{1}{\bar{p}^2 + \omega_\Delta} \right],
\]

and \( U(p) \) is given by \( U_{\rho h}(p) + U_{\Delta h}(p) \).

§ 3. Numerical calculation

We shall discuss here first how to determine the average momentum \( p_{av} = 3.6\mu \), which has been discussed in the previous section for \( 2\Delta \) process. The term \( B(p) \) contains the pion pole, which happens at \( p = \sqrt{p_0^2 + \mu^2} = \sqrt{(q^0 / 2)^2 + \mu^2} \). We define a function \( f(q, p) \) as

\[
f(q, p) = p_{\rho\omega}^2 \int_{-1}^{1} d\cos \theta \bar{p}^2 \sqrt{Mf^0 - \bar{p}^2 / 4} |G_d(p)|^2 |G_d(q - p)|^2,
\]
Fig. 5. The phase space integrand $J(q, p)$ for the two delta process for various pion energies. The peak is around $3-4 \mu$. The pion pole positions are indicated by arrows at various energies. Note that the pion pole does not exist with $p^0 = q^0/2$ at $T_n = 100$ MeV.

which appears in Eq. (10) in conjunction with the pion propagator. If $J(q, p)$ is extremely small at the pion pole, we can expect that the contribution of this singular point to the integration is small and we may average out this interaction as before. We calculate $J(q, p)$ numerically, and show the results in Fig. 5 for various pion energies. We find that $J(q, p)$ peaks around $3.0 \sim 4.0 \mu$ and therefore it is small enough around the pion pole, the position of which is indicated by an arrow. Hence, we may perform an approximation that the interaction is averaged out and represented by the energy $q^0/2$ and the momentum $p_{av} = 3.6 \mu$.

The absorption rates for the successive three body mechanism $\Gamma^{(3)}_\pi$ in Eq. (7) and for double-$\Delta$ mechanism $\Gamma^{(2\Delta)}_\pi$ in Eq. (10) are calculated at the normal density $\rho_0$. The numerical results are shown in

Fig. 6. (a) The pion absorption rates of various processes in which all the medium effects are considered. The calculations are performed at the normal density $\rho = \rho_0$; solid line—2 body process; dash-dotted line—sequential 3 body process; dash-double-dotted line—successive 3 body process; broken line—$2\Delta$ process. The inserted figure at the upper corner is obtained by expanding the scale by factor 10. (b) The pion absorption rates of various processes, which are calculated without the medium corrections on the delta propagators and the interactions.
We find the ratio \( \frac{\Gamma^{(2)}}{\Gamma^{(3)}} \) is \( 30 \sim 60(\%) \) in the energy region from 50 MeV to 400 MeV. It is close to the guessed value (\( \sim 50\% \)) found in the previous work by Oset et al.\(^5\) On the other hand, the ratio \( \frac{\Gamma^{(2A)}}{\Gamma^{(2N)}} \) is very small (\( \sim 0.3\% \) at 150 MeV). The ratio can go up to 1 only above \( T_{\pi} \sim 400\) MeV. Even if we neglect medium correction effect on the \( \Delta \)-propagators and interaction, this ratio goes to only \( \sim 3\% \) as shown in Fig. 6(b), which is extremely small compared with the value estimated by Brown et al.\(^5\) Hence, we would like to discuss where this large difference is originated.

Brown and his collaborators estimated that the ratio \( \frac{\Gamma^{(2A)}}{\Gamma^{(2N)}} \) would be on the order of 1. They provided this result in the following way.\(^5\) The expression of the ratio \( \frac{\Gamma^{(2A)}}{\Gamma^{(2N)}} \) is given by assuming initial nucleon pairs are at rest,

\[
\frac{\Gamma^{(2A)}}{\Gamma^{(2N)}} = \frac{\text{Im} \int \frac{d^3p_1}{2\pi} \frac{d^3p_2}{2\pi} \delta(p_1 + p_2 - q) G_{2A} \langle \pi NN | \delta H | \Delta N \rangle \langle \Delta N | V_{2A} | \Delta \Delta \rangle^2}{\text{Im} \int \frac{d^3p_1}{2\pi} \frac{d^3p_2}{2\pi} \delta(p_1 + p_2 - q) G_{2N} \langle \pi NN | \delta H | \Delta N \rangle \langle \Delta N | V_{2N} | NN \rangle^2},
\]

(15)

where \( G_{2A} \) and \( G_{2N} \) are product of energy denominators for \( 2\Delta \) and for \( 2N \), respectively:

\[
G_{2A} = \left[ \omega - \omega - i\frac{1}{2} \right]^2 (\omega - 2\omega_\Delta - p_\Delta^2/2M_\Delta - p_\pi^2/2M_\pi + i\gamma_1/2 + i\gamma_2/2),
\]

(16a)

\[
G_{2N} = \left[ \omega - \omega - i\frac{1}{2} \right]^2 (\omega - p_\pi^2/2M - p_\Delta^2/2M + i\varepsilon).
\]

(16b)

\( \gamma_1 \) and \( \gamma_2 \) are the decay width of each one of delta forming \( 2p \cdot 1h \) state. Performing the phase space integrations, and spin isospin summations, they found

\[
\frac{\Gamma^{(2A)}}{\Gamma^{(2N)}} \sim \frac{1}{2} \left( \frac{M_\Delta}{M} \right)^{3/2} \left( \frac{\gamma}{\omega_\Delta} \right) R \sim 0.1R \quad \text{at} \quad \omega \sim \omega_\Delta
\]

with \( R \) being the matrix elements, which are taken out from the phase space integrations.

They substitute \( \Delta \)-spreading potential (s.p. potential) to \( \gamma \) as

\[
\frac{\gamma}{2} \sim 40\text{ (MeV)} \rho/\rho_0.
\]

However, the many body delta decay width \( \gamma \) contains not only the decay width due to two body absorption but also the decay width of the higher multinucleon absorp-
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tions and of the quasi-elastic scattering. The two body decay width is provided as
30% of $\gamma$ around the resonance energy by Oset et al.\textsuperscript{9) Hence we must multiply 0.3 to
the previous value which leads to $\Gamma^{(2A)}/\Gamma^{(2N)} \sim 3 \times 10^{-2} R$.

In Ref. 5), the ratio of the $V_{AA}$ and $V_{NN}$ matrix elements in Eq.(15) is estimated
under the long-range assumption for the $2A$ process. They find the ratio being $R \sim
10$ and hence the $2A$ and $2N$ processes would be on the same order. The long-range
nature of the $2A$ process, in addition, demands more initial nucleon pairs to be
involved for pion absorption than the $2N$ process with the short-range character. As
a consequence, the $2A$ process would be larger than the $2N$ process.

We claim here, however, that the $2A$ process is not of long-ranged. The average
momentum transfer for $2A$ came out to be $p_{av} \sim 3.6 \mu$. We note the $\Gamma^{(dd)}$ in Eq.(15) has
an expression similar to that of $J(q, \rho)$ in Eq.(14) except for the factor $\sqrt{M\rho^2 - p^2 / 4}$, which arises from the availability of energy for decay of delta. If we include this
factor for $\Gamma^{(dd)}$, the integrand for $p$-integration peaks around a similar value. On the
other hand, that for $2N$ is $p_{av}^{(2N)} \sim q^0 M \sim 4 \mu$ at the resonance energy. Hence, we would
have to estimate the ratio $R$ under the short-range assumption for both the $2A$ and $2N$
processes. We find the ratio about 1. In addition there is no further enhancement
expected due to the initial nuclear pairs. As a net, $\Gamma^{(2A)}/\Gamma^{(2N)}$ would be $\sim 3\%$. This
crude estimate agrees with the previous result without the medium effects on the delta
and the interaction.

\section{4. Conclusion}

We have studied the successive three body and the $2A$ processes for pion absorp-
tion to complete the discussions originated by Oset et al. in Ref. 6), where the impor-
tance of the three body pion absorption process was demonstrated. We have found
that the successive three body process is about $30\sim 60\%$ of the sequential three body
process in the energy region between $T_x \sim 50\sim 400\text{MeV}$. This result supports the
estimate of Oset et al.\textsuperscript{6) We have found very small contributions from the $2A$ process at the resonance
energy in contrast to the crude estimate of Brown et al.\textsuperscript{5) We trace back this
difference as caused mainly by the long-range assumption for the $2A$ process taken in
Ref. 5). Our phase space calculation indicates a large momentum transfer $p_{av} \sim 3.6 \mu$.

We shall, therefore, conclude that pion absorption at the resonance energy is
shared about equal amount by the two nucleon process and the three nucleon process.
As the energy is increased, the 2 nucleon process becomes dominant, while as the
energy is increased to $\sim 400$ MeV, the 2 delta (4 nucleon) process begins to dominate
the absorption process. As the energy is increased more, the 4 nucleon process starts
to be important.

\begin{thebibliography}{9}
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