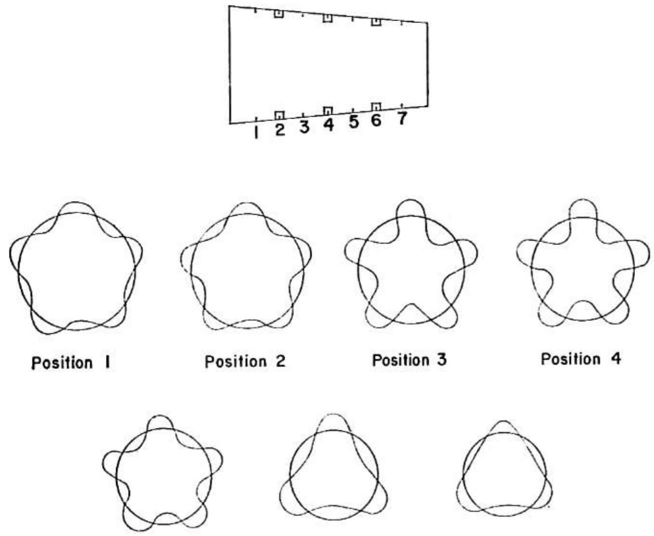

Fig. 7 Mode shape of stiffened cone (f = 511 cps)

4 Stiffened Cone. The cone shown in Fig. 5(c) has symmetrical ring stiffeners. The size of the problem remains the same as the three previous problems. However, the results are quite different. Starting with the fourth natural frequency, the lobar number (n) at each end of the cylinder is different. Fig. 7 illustrates one of these "mixed modes."

Mixed modes of this type have also been determined experimentally [11] and the authors state that "No adequate theory... has taken into account the difference in the number of circumferential waves found at the major and minor diameters of the shell." Discrete mass techniques present a useful and adequate theory to calculate such modes. The reason discrete mass techniques account for mixed and irregular modes is that the basic lobar mode is not assumed *a priori*. The mixed and irregular modes are simply the result of the general eigenvalue solution. Thus any mode can be determined if the size of the discrete mass model is adequate.

Conclusions

Discrete mass techniques for vibration analysis of shells present an extremely valuable analytical tool, particularly for shells not amenable to classical solutions. The latter includes unsymmetrical shells as well as symmetrical shells having irregular or mixed mode shapes. There are countless numbers of shell structures that have frequencies and modes of vibrations that are irregular. The authors propose that the use of discrete mass techniques for these problems is an expedient approach.

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DISCUSSION

A. L. Ross[4]

The strength of numerical tools such as were used by the authors is dramatically illustrated by this paper—especially for the applications to problems that would be extremely difficult to handle with other than discrete element methods.

An interesting difference between the methods of solutions for the symmetric and unsymmetrically stiffened cylinder that appears to have been overlooked by the authors deserves, I believe, some mention at this point.

In the closed form solution the referenced authors, Arnold and Warburton [6], were concerned only, for example, with circumferential variations of radial deflection of the form $\cos n\theta$ which, for their rotationally symmetric cylinder, was of course quite adequate.

In the numerical solutions the present authors likewise considered only symmetric (about a vertical centerline) modes, which again is quite adequate for the rotationally symmetric shells, but entirely inadequate for finding *all* the natural frequencies of the rotationally unsymmetric shell. This is because antisymmetric (about a vertical centerline) as well as symmetric modes of vibration will exist. Of course for rotationally symmetric shells the actual orientation ("phase") of the vibration modes will be determined by the type (and "direction") of the forcing function. However, for shells with a single plane of symmetry the "orientation" of the individual modes is not a function of the forcing function, except as to the magnitude of participation of the mode. As a matter of practical importance in shock or vibration analysis, the exclusion of either can give absurd results for some forcing functions.

This latter point can be better emphasized by examining the equations that would be applicable for determining the response of the cylinder (for each mode) due to an impulsive translational shock motion applied laterally (in any direction) through the supports, or the response due to a lateral vibratory displacement at the supports.

For shock and vibration response these expressions are (respectively):

$$\bar{d}_{in} = \frac{V_0}{\omega_n} P_n d_{in}$$

and

$$\bar{d}_{in} = \frac{P_n d_0 d_{in}}{\left[\left(\frac{\omega_n{}^2}{\omega^2} - 1\right)^2 + \frac{1}{Q^2}\frac{\omega_n{}^2}{\omega^2}\right]^{1/2}}$$

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where

$$P_n = \frac{\sum_i \beta_i m_i d_{in}}{\sum_i m_i d_{in}^2}$$

and β_i is the direction cosine of the angle between the degree of freedom (displacements) d_i, and the direction of the translational shock or vibration stimulus, and Q is the magnification (damping) factor (the assumption of small damping of a particular type has been made in the equation). V_0 is the impulsive velocity change and d_0 is the magnitude of the vibratory forcing motion. The other symbols follow the nomenclature of the authors.

The important point to be observed from these equations is that symmetric modes (about the vertical centerline) will not be excited by a translational horizontal excitation since for every positive deflection in the numerator of the participation factor, P_n, there is an equal negative deflection. Similarly antisymmetric modes will not be excited for vertical motion.

One may then postulate that for the unsymmetrically stiffened shells the authors' solution would be adequate for evaluating the response to vertical motion of the supports but not for lateral translational motion in any other direction.

The authors were therefore quite justified in considering only one half of the shell. However two problems (with different boundary conditions) should have been solved—one to determine symmetric (about the vertical geometric symmetry line) modes and the other to determine antisymmetric modes of vibration.

It appears that the authors used the "symmetry boundary conditions" of zero slope, shear, and circumferential displacement of the vertical centerline. In order to determine the "antisymmetric" modes the conditions of zero radial deflection, traction, and moment should be used at the centerline.

Authors' Closure

In his discussion, Mr. Ross is correct in pointing out that for a horizontal forcing function, the antisymmetrical modes of vibration would be used, rather than the symmetrical modes. The symmetrical modes would be used to calculate the response to a forcing function that is symmetric with respect to the vertical axis. Since the solution is linear, the simultaneous use of both symmetric and antisymmetric modes is unnecessary. If a forcing function, such as a shock loading, had a vertical and horizontal component, the complete solution would involve the linear combination (considering phase) of the two solutions. The antisymmetric modes would be calculated using the same mathematical model (taking advantage of vertical symmetry), with boundary conditions of zero radial deflection, horizontal deflection, and moment at the centerline. This calculation would only require a doubling of computer time, and it would be more economical than simultaneously calculating the full model for both symmetric and antisymmetric modes.