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Three-Dimensional Numerical Simulation of Interstellar Cloud-Cloud Collisions and Triggered Star Formation. I

--- Head-On Collisions ---

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We simulate the head-on collisions between isothermal interstellar clouds by using a three-dimensional hydrodynamic code. The cloud-cloud collision is one of the important mechanisms which trigger star formation. We obtain a new dynamical criterion for the gravitational instability instead of the static criterion for the Bonnor-Ebert sphere. In the collision between identical clouds with relative velocity greater than the sound velocity, the shock compressed layer is formed and the central density increases by the square of the impact Mach number. As a result, the enhanced self-gravity induces the instability and the central part of the cloud begins to collapse even if the total mass is smaller than the Bonnor-Ebert critical mass \( M_{\text{BE}} \). The efficiency of this triggering mechanism, however, does not increase by far for the high Mach number collision because of the saturation of the gravity in disk geometry. In the head-on collision between hot clouds with small mass such as \( M < 0.4 M_{\text{BE}} \), they only merge and make a quadrupole oscillation around an equilibrium configuration.

§ 1. Introduction

Interstellar clouds have so large supersonic velocity dispersion that the cloud-cloud collisions are expected to occur with fairly high probability. The radius of a typical molecular cloud with mass \( 10^5 M_\odot \) is 25 pc, the mean random velocity \( v \) is 15 km/s, and a mean mass density \( \langle \rho \rangle \) is \( 10^{-23} \) g cm\(^{-3}\) for interstellar medium. Then the collision rate for this typical cloud is

\[
t_{\text{col}} = \frac{n \sigma v}{10^{-8} \gamma^{-1} \left( \frac{10^5 M_\odot}{M} \right) \left( \frac{\langle \rho \rangle}{10^{-23} \text{g cm}^{-3}} \right) \left( \frac{\sigma}{625 \text{pc}^2} \right) \left( \frac{v}{15 \text{km/s}} \right)},
\]

which is shorter than the evolution time of the galaxy.\(^1\) We consider that the cloud collision makes an important contribution to star formation activity and galaxy evolution.

When the molecular cloud is compressed and the density increases by a cloud-cloud collision, the critical mass of the Jeans instability is considered to be reduced. Since the cooling time of molecular gas is about \( 10^6 \) years, much shorter than the collision time scale, the cloud temperature does not increase. While the cloud-cloud collision is regarded as one of the important mechanisms to trigger star formation, this process is difficult to analyse because it is a very inelastic process. Many problems remain to be solved: What is the outcome of such inelastic collisions? In what probability are stars born? What fraction of cloud mass is converted into stars?

We consider that coalescence and subsequent gravitational instability must be associated with the cloud collisions. At the same time, there are some possibilities that the collision could induce disruption of a cloud into small pieces. These process-
es may have close relation to the stability of a cloud. Up to now, there is only a
stability criterion by analysis of the Bonnor-Ebert sphere that the static gas cloud
whose total mass is less than $M_{BE}$ is stable. Is this conclusion true in the collisional
compression? It is still an open question whether the two clouds merge or be separat­
ed when they make a head-on collision. This different conclusion causes different
cloud-mass-spectra.\textsuperscript{2,3} For modeling stochastically the global structure of galaxies,\textsuperscript{4}
it is necessary to know these results of cloud-cloud collisions, the cross sections and,
if disruption occurs, the branching ratio of mass into cloudlets.

As shown in many radio observations, star forming region reveals fascinating
morphology such as disks or filaments and very complex velocity structure.\textsuperscript{5-7} We
attempt also to answer what kind of dynamical interactions can reproduce such cloud
morphology. We perform three-dimensional (3-D) numerical simulations for some
gas clouds in more general configuration. Above all, the greatest advantage of our
3-D simulations is what we can calculate for grazing or off-center collisions of two
clouds. If this simulation is successful, we would get some insight into the origin of
the cloud rotation, by determining the initial distribution of angular momentum which
is the key point in fragmentation process of a rotating cloud.\textsuperscript{8}

Several works have been made so far, to discuss the gravitational instability
induced by the two-dimensional collisions, such as plane-symmetry\textsuperscript{9,10} or cylindrical
cloud-cloud collisions.\textsuperscript{11} We believe it necessary to search the trigger criterion of
star formation without imposing any geometrical symmetry on cloud configuration.
Unfortunately, the previous 3-D calculations have not discussed in detail the change
of gravitational stability criterion.\textsuperscript{12,13}

For the present, as the first paper in the series of our study on 3-D collisions of
self-gravitating systems, the head-on collisions between interstellar clouds are de­
described. In § 2, we define the cloud-cloud collision process as a physical problem.
The formalism for our 3-D hydrodynamics is presented in § 3. Numerical results of
our simulation are given in § 4. Dynamical criterion for the gravitational instability
is understood in terms of the tensor virial analysis in § 5. Section 6 is devoted to the
condition for the fragmentation and the effect of the equation of state.

§ 2. Physical parameters

In the simulation of two clouds collision, there are several important physical
parameters: total cloud mass (gravitational energy), mass ratio of two initial clouds,
temperature (thermal energy), impact velocity (kinetic energy), impact parameter and
the spin angular momentum of each cloud, etc.

Throughout the series of our papers, we assume that the equation of state (EOS)
is polytropic,

$$P = K \rho^{1+1/n},$$

where $n$ is the polytropic index and $K$ is a constant which is determined by the
entropy of gas.

In the case $n > 5$, the cloud is surrounded by an ambient gas of constant pressure,
Then we define a nondimensional mass \( M_n \) using the invariant constants in equation of motion, \( P_e \) and \( K \), as

\[
M_n = M \left( \frac{nG}{n+1} \right)^{3/2} K^{-2n/(n+1)} P_e^{(n-3)/(2(n+1))},
\]

(2.2)

where \( M \) is the dimensional cloud mass and \( G \) is the gravitational constant. In this paper, we investigate mostly the isothermal case, i.e., \( n = \infty \), and introduce the parameter \( A \) defined by

\[
A = (36\pi/125)M_m^2 = \frac{36\pi}{125} G^3 M^2 P_e/C_s^8,
\]

(2.3)

where \( C_s \) is the constant sound velocity. This parameter \( A \) measures the ratio of both the gravitational energy and volume energy compared to thermal energy. Remember that the parameter \( A \) is proportional to the square of mass, then it will be found as a convenient value when we compare our results with those in the tensor virial analysis.

In 3-D collisions, there are other physical parameters, e.g., the mass ratio of the initial two clouds 1 and 2, \( r = M_1/M_2 (0 < r \leq 1) \). In this paper, however, we take \( r = 1 \), because we would like to concentrate on how the gravitational instability depends on the nondimensional mass \( A \) and the impact Mach number \( V/C_s \) for the head-on collisions.

The criterion for the stability of the isothermal Emden solution is the Bonnor-Ebert criterion, which we call static criterion. It is represented as a maximum mass

\[
M_{BE} = 1.18 \left( C_s^8 / G^3 P_e \right)^{1/2},
\]

(2.4)

beyond which the cloud becomes gravitationally unstable. To make our results clearer, we will stress the difference between the critical mass in dynamical process and the above critical mass \( M_{BE} \). The corresponding parameter, \( A \), is written as

\[
A_{BE} = \frac{36\pi G^3 M_{BE}^2 P_e}{125 C_s^8} = 1.26.
\]

(2.5)

\section{Numerical method}

\subsection{Hydrodynamics by Smoothed Particle Method}

We employ a hydrodynamic code called Smoothed Particle Method. The gaseous system is treated as an ensemble of \( N \) fluid elements, which we call "particles". Each particle has the same mass \( m_0 \) and the similar density distribution, for which we choose the Gaussian type smoothing kernel. The motion of each particle is described in the Lagrangian coordinates. The physical quantities in the Euler coordinates can be calculated as the expectation value from the distribution of \( N \) particles. The advantage of this method is considerable when we treat the 3-D motion of gas without assuming any symmetric structure.

The local densities of fluid at \( x = x_i \) are given by the superposition of all particles.
where $h_j$ is the smoothing length of $j$-th particle. This length is determined locally in accordance with the spatial density,

$$h_j = \zeta \left( \frac{m_0}{\rho(x_i)} \right)^{1/3},$$

(3.2)

where $\zeta$ is the coefficient of order unity.

Each particle obeys the Newtonian equation of motion, which converges to the ordinary Euler equation in the limit $N \to \infty$. The equation of motion for the $i$-th particle is written as

$$\frac{dv_i}{dt} = -\frac{1}{\rho(x_i)} \nabla P(x_i) - \nabla \phi(x_i) - \varepsilon \sum_j Q_{ij},$$

(3.3)

where $\phi$ is the gravitational potential which can be calculated directly by integrating over the mass distribution. The terms, $Q_{ij}$, are the anti-symmetric ($Q_{ij} = -Q_{ji}$) artificial viscosity which are necessary for simulating the plane shock phenomena. The type of $Q_{ij}$ is the same as used in a paper of Miyama, Hayashi and Narita. Using this anti-symmetric viscosity, we succeed in simulating the supersonic collision of fluid and avoiding particle penetration.

In our simulations, we checked the conservation law of mass and angular momentum. The center of total mass $x_{cm} = \sum_i x_i / N$ and the total angular momentum $J = m_0 \sum_i (x_i - x_{cm}) \times (v_i - v_{cm})$ for 4000 particles are conserved within an error of 0.01% and 0.1%, respectively.

In this paper, we normalize all variables by the initial mean density $\rho_0$ and the initial radius $R_0$ of the cloud. The time unit is the corresponding free-fall time, $t_0 = (3\pi/32G\rho_0)^{1/2}$.

### 3.2. Boundary conditions

A self-gravitating isothermal gas cloud is stabilized by pressure due to an intercloud gas, mimicking a molecular cloud in HI medium. In our computation, the clouds are assumed as always being surrounded by the intercloud medium with a constant external pressure $P_e$. The gravity of the intercloud medium is neglected. To simulate this boundary condition, we approximate the effect of external pressure $P_e$ by taking the following $i$-th particle pressure,
This expression is empirical but it is found very satisfactory since it can reproduce the isothermal Emden solution emersed in uniform gas (see Fig. 1).

3.3. Initial conditions

We investigate the collisional process of two spherical clouds which are in hydrostatic equilibrium initially. Although many authors consider the uniform density spheres as the initial condition, it makes the problem more complicated because a uniform density sphere oscillates around the equilibrium configuration until the collision occurs.

We use the Cartesian coordinates \((x, y, z)\) in which the \(x\)-axis is chosen as the collisional one. The \(z\)-axis will be taken as the direction of the total angular momentum for the off-center collisions, but it is degenerate to the \(y\)-axis in the head-on case.

In the cases of supersonic collisions, two clouds are set to have the initial separation, \(D(=4R_o)\). The equilibrium spheres are found to be almost unchanged before the collision as shown in the evolution of maximum and minimum density in Figs. 2~4. The impact velocity is of course increased by the mutual gravitational acceleration in the approaching phase, but the internal velocity change is negligible compared with the initial supersonic velocity because the gravitational energy is smaller than the kinetic energy. If we define the velocity at infinity as

\[
V_\infty^2 = V^2 - \frac{2GM}{D},
\]

there is little difference between \(V\) and \(V_\infty\) in the case of supersonic collisions.

Although the observed interstellar clouds have supersonic motions, we investigate also the subsonic collisions from theoretical interest. We aim at studying the applicability of the Bonnor-Ebert criterion for the slow collisions when the Emden solutions suffer the nonlinear perturbations with the large amplitude. For this purpose, we simulate the slow collision in the cases \(V/C_s=0\) and 0.5 with initial separation \(D=2R_o\), that is, two clouds are in contact with each other initially. For these subsonic cases with \(D=2R_o\), we cannot define the velocity at infinity, because Eq. (3·5) is negative.

3.4. Numerical accuracy of Smoothed Particle Method

Before simulating the head-on collision, we test resolution of our code for presenting the shock structure in our numerical methods. In high supersonic head-on collisions, one-dimensional compression is a good approximation. According to the isothermal shock condition, the density ratio after and before the shock front will rise up to the square of Mach number; \(\rho/\rho_o=(V/C_s)^2\). The cloud is compressed in the direction of collision and forms the plane shock without changing the cloud size in parallel with the shock front. Then the flatness of the compressed layer scales \(b/a=(C_s/V)^2\), where \(a\) is the radius and \(b\) is the thickness of the compressed disk.

The resolution limit of smoothed particle method is almost equal to the particle size, \(h(=\xi(m_0/\rho)^{1/3}=\xi(M/\rho N)^{1/3}\) according to Eq. (3·2)). In the one-dimensional
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In order to resolve such thickness, the ratio \( b/h \) must be greater than 1 as

\[
b/h = \frac{1}{\kappa} (3N/4\pi)^{1/3} (V/C_s)^{-4/3} > 1.
\]

(3.6)

Therefore we have to increase the number of particles for high supersonic collisions. Equation (3.6) indicates that we need at least about \( N = 10000 \) particles to simulate the head-on collision of \( V/C_s = 10 \).

Most of our simulations were carried out using \( N = 4000 \) particles. With a supercomputer FACOM VP200 in Kyoto University, it takes one hour of computational time to calculate 1000 time-steps. For the highly supersonic \( (V > 6C_s) \) models, we took \( N = 8000 \) particles. In the 3-D simulation of a self-gravitating system, the particle number cannot be increased to ten thousand even with our supercomputer for financial reason. Therefore we do not calculate the extremely-supersonic collisions of \( V > 10C_s \).

The results in other works, which employ the particle scheme, have been severely influenced by this limitation. Hausman obtained, with \( N = 200 \) particles,\(^{12}\) the density compression of only a factor 6 to 7 for the isothermal head-on collision of \( V/C_s = 10 \), and he cannot avoid the penetration of particles through the shock layer. Lattanzio et al. calculated, with \( N = 5172 \) particles, the isothermal head-on collision of \( V/C_s = 24.6 \),\(^{13}\) although \( N \sim 360000 \) particles are required from Eq. (3.6).

§ 4. Numerical results

4.1. Survey of the head-on collisions

Results of head-on collisions are separated into two groups depending on the dynamical stability: One is the stable merging which does not collapse and the other is the induced gravitational "collapse" which would correspond to the trigger of star formation. Furthermore, the stable collisions can be characterized by oscillation and strong expansion of merged clouds, which we call "stable" and "expand", respectively. The triggered gravitational instability has two types also: One is the collapse after the shocked disk formation by supersonic collisions \((M \lesssim 0.5M_*\)) and the other is the spherical collapse without disk formation in subsonic collisions \((M \gtrsim 0.5M_*\)). In Figs. 2-4, we show the time evolutions of the maximum density (solid line), the density at the center of mass (dotted line) and the minimum density (dashed line) for each typical collision.

We plot the results of our 40 calculated models in Fig. 5 and tabulate in Table I. In Fig. 5, the results of numerical simulations are classified with three kinds of marks; the "stable" (circle) which we calculated long enough to confirm the merging and oscillations as in Fig. 2, the "expand" (square) which we terminated the calculation in the expanding phase as in Fig. 3 and the "collapse" (cross) whose central density increases exponentially with time as in Fig. 4. In Table I, they are marked with S, E and C, respectively.

Before collision, all the clouds have the mass less than \( M_\infty \), so that they are initially stable. From these clouds, we obtain many "collapse" models. We should
Fig. 2. The time evolutions of the maximum density (solid line), the density at the center of mass (dotted line) and the minimum density (dashed line) for the “stable” model $A=0.2$ and $V/C_s=5$. The shock compression increases the maximum and the central density by the square of impact Mach number and excites the quadrupole oscillation. The minimum density corresponds to that of the particle ejected from the surface by the shock rarefaction wave. This density jump indicates the arrival of such a shock wave.

Fig. 3. The same as Fig. 2 for the “expand” model $A=0.2$ and $V/C_s=6$. The shock compression increases the maximum and the central density, then the strong expansion starts and the central density decreases rapidly.

Fig. 4. The same as Fig. 2 for the “collapse” model $A=0.3$ and $V/C_s=7$. After the shock compression, the maximum and the central density increase exponentially with time by the gravitational instability.

Fig. 5. The results of head-on collision: “collapse” (cross), “stable” (circle) and “expand” (square). The abscissa is the impact Mach number $V/C_s$ and the ordinate is the square of nondimensional mass $A$ defined as $(36\pi G^3 P_r/125C_s^4)M^2$. The critical line for the stability of isothermal Emden solution is shown by dashed line. Two solid curves correspond to the approximate criteria for dynamical stability by tensor virial analysis. The lower line represents the stability condition against the density enhancement induced by the isothermal shock. The upper line is the same condition but including the effect of geometry, that is, the flatness of the compressed layer.
call these collapsing models the triggered star formation by a head-on collision. The dashed line in Fig. 5 represents the half of Bonnor-Ebert critical mass, \( M = M_{\text{BE}}/2 \), \( A = A_{\text{BE}}/4 \). This will correspond to the naive critical mass for the merging clouds. Our results indicate that the head-on collision always induces the star formation if the total mass exceeds \( M_{\text{BE}} \). The two solid lines in Fig. 5 are based on the analysis on the stability of Maclaurin spheroid and the details will be discussed in § 5. The actual head-on collision decreases the critical mass and it is effective for the supersonic case of \( V/C_s = 2 \sim 3 \). This agrees quantitatively with the linear analysis by Gilden.\(^{10}\)

We list the details of each model in Table 1. The \( \rho_{\text{shock}} \) is the central density in units of the initial mean density \( \rho_0 \) at the shock compression phase, which can be read as the first plateau in Figs. 2 \sim 4. The \( \rho_{\text{max}} \) is the maximum density throughout the calculation. In Period column, the time interval from the first oblate phase to the second oblate phase is listed for "stable" models. As for the "collapse" models, the

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<th>( V/C_s )</th>
<th>( \rho_{\text{shock}} ) ( (\rho_0) )</th>
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(continued)
time scale of the exponential density increase is written in the same column. The collapse mass $M_c$ is counted from the number of dense particles such as $\rho > 14\rho_0$. The factor, 14, corresponds to the maximum central density of the stable Bonnor-Ebert sphere. The escape mass $M_{esc}$ is the summation mass of the particle whose energy is positive at the end of the calculation.

4.2. Coalescence in stable head-on collisions

i) Case of merging with oscillations

As a representative, we show the supersonic collision of $A=0.2$ and $V=5C_s$ in Fig. 6, where the equidensity surface and density on $z=0$ plane are plotted. At first, the shock compression occurs and the disk appears with almost uniform density in the $y$-direction. The typical shock structure is clearly seen in Fig. 7. Then the self-gravity increases the central density and makes the steep density gradient at the same time. In the case that the pressure gradient overcomes the self-gravity, the first bounce (hereafter we call this $x$-bounce) begins ($t=1.03$). The shock-compressed disk expands and the shape of cloud becomes prolate ($t=1.43$). It comes from the fact that the expansion is supersonic along the collision axis, while in the transverse directions the disturbance propagates only with the sound velocity. In this way, the stable collisions excite the quadrupole oscillations. The ejected gas are pulled back as the time goes on. The two isothermal clouds merge into one and there is no eruption or fragmentation even for the faster supersonic ($4C_s \leq V \leq 10C_s$) collisions.
Fig. 6. (continued)
This oscillation will dump by some dissipative processes to get a new spherical equilibrium.

In order to consider the physics of this oscillation, we compare the period with the non-radial oscillations of the homogeneous compressible sphere, which was investigated in the linear analysis by Pekeris.\textsuperscript{19) } The stable $p$-mode of quadrupole oscillation has the period $\tau \sim 5\tau_0$, where $\tau_0$ is the free-fall time, $\tau_0 = (3\pi/32G\rho_0)^{1/2}$. In this period, the cloud starts from an oblate shape and it comes back again through a prolate stage. The central density gets the maximum twice during the whole period of this quadrupole oscillation. We find that the agreement between the listed values in the Period column of Table I and $\tau$ are quite satisfactory. The slight variation of period
Fig. 7. The position of particles projected on the $x$-$y$ plane (which belong to the single cloud initially) and the $x$-$y$ components of velocity vectors on $z=0$ plane at the shock formation stage of the supersonic collision $A=0.2$ and $V/C_s=5$. Our artificial viscosity avoids the particle penetration and satisfies the shock condition fairly well. The small fraction of particles is ejected to radial direction in the $y$-$z$ plane. They are pulled back in the later stage.

depending on the impact Mach number can be understood as the effect of unharmonicity of the oscillation with a finite amplitude on the free energy surface (see Fig. 13).

ii) Case of strong expansion

For the highly supersonic collisions ($V/C_s \geq 6$), the first compression along the collision axis proceeds much faster than the ejection to the transverse direction. The larger pressure gradient in the $x$-direction makes the stronger expansion in a prolate shape. Although the expanding velocity saturates around $\sim 3C_s$ even for the faster supersonic collisions, the portion of mass which moves away becomes larger. Due to this strong expansion, the central density much decreases and the void formation occurs near the center of collision (see the dotted line in Fig. 3).

For every model, we calculate the time evolution until we can judge whether the expansion or collapse starts. For the “expand” collisions, however, we do not follow the later evolution for financial reason, as well as most of previous works, since these highly supersonic collisions require many particles for high spatial resolutions as discussed in § 3.4.

Considering these coalescence models, the evaporation is not expected for two reasons. First, we use the isothermal equation of state so that the shock compression proceeds in very inelastic way. The ratio of particles which keep the positive energy after the collision is limited within a several percent as shown in the escape column of Table I. Most of the kinetic energy is dissipated away according to the isothermal shock condition. Second, thinking from the oscillatory merging of moderately supersonic collisions, the effect of external pressure will push back the expanding shell at last.

In Gilden’s 2-D simulations, his five models are classified as the collapsing core or the cloud expansion and these expansion phases are regarded as the evaporation of the cloud into the ambient medium. Since he remarks that the effect of equation of state is small in his simulations, the neglect of external pressure during the time
Fig. 8. The equidensity surface ($\rho=0.6\rho_0$) and the density in the $x$-$y$ plane of the "collapse" head-on collision $A=0.3$ and $V/C_s=7$. First, the shock compressed disk forms, then the gas falls to the collision axis because the density and pressure gradient in the $y$-$z$ plane is small. Along the $x$-axis, the pressure gradient made by the one-dimensional compression can reverse the gas motion. But near the center of collisional interface, the gravity increases and the collapse proceeds.
evolution stage may be responsible for the fact that he did not obtain the stably merging cloud.

We have a particular result which is apparently different from Gilden's calculations. For the collision with the same Mach number and the same mass as his unstable Model II \( A=0.12, V/C_s=2.7 \), we obtain the stable cloud as the result of coalescence. We think this difference is caused by the choice of different initial conditions.

4.3. Triggered collapse by cloud-cloud collisions

i) Supersonic collisions of \( M < 0.5M_{\text{KE}} \)

The "\textit{collapse}" after the collision with \( M < 0.5M_{\text{KE}} \) may be regarded as the triggered star formation in a narrow sense, because the total mass is still less than \( M_{\text{KE}} \) and there is a stable hydrostatic equilibrium if there is no dynamical interaction.

In the case of supersonic collisions, the density during first shock compression, \( \rho_{\text{shock}} \), is almost independent of \( A \) as in Table I. We show the typical evolution of structures in Fig. 8. For this triggered case after shock compression, the ejected gas in the \( y-z \) plane is pulled back \( (t \approx 0.52-0.96) \) and the pressure gradient cannot stop the contraction, but the gravity is dominant only in the central spherical region and the outer parts near the collision axis continue \( x \)-bounce \( (t \approx 0.96-1.25) \). In other words, the collapse proceeds in the prolately elongating cloud. As the result, a supersonically expanding cigar-like cloud and a orthogonal contracting disk are formed. From this simulation, there is a possibility that a new-born star in such a elongated cloud can exist even without rotation or magnetic field.

After the collision the system is separated into the collapsing region and the expanding region whose velocity does not exceed \( |V_x| \leq 3C_s \) irrespective of the Mach
Fig. 11. The equidensity surface ($\rho = 0.6 \rho_0$) and the density in the $x$-$y$ plane of the "collapse" head-on collision $A = 0.4$ and $V/C_s = 3$. Before the disk formation is completed by shock compression, the collapse begins and proceeds. Finally a central star with a surrounding non-rotating disk is expected to be formed.
number as shown in Fig. 9. The presence of the maximum expanding velocity indicates the very inelastic feature of the isothermal head-on collision.

The collapsing density profile does not depend on the initial profile and always shows $\rho \propto r^{-2}$ structure as in Fig. 10, which is the characteristic of the isothermal similarity solution.$^{20,21}$

ii) Massive supersonic collisions of $0.5M_{BE} < M < M_{BE}$$

These collapses also should be called the triggered star formations because, without collision, each cloud is entirely stable. For the case $M > M_{BE}/2$, the combined mass is greater than the Bonnor-Ebert mass so that the possible outcomes of the head-on collision are the gravitational collapse or the cloud disruptions. All our numerical simulations result in the former one.

At the shock formation stage, the dynamical time scale in the central compressed region $t_{dc} = 1/\sqrt{G\rho_c}$ is smaller than the time scale near the cloud surface $t_{ds} = 1/\sqrt{G\rho_s}$ because, using the relation $\rho_c/\rho_s = (V/C_s)^2$, we have $t_{dc} = t_{ds}C_s/V$. Moreover, once the gravitational instability sets in, the collapse proceeds exponentially with time so that the difference of the time scale between the central and the surface region also grows exponentially. As the result, as in Fig. 11, the collapse proceeds almost spherically in a part of the compressed region, but the change around the surface region is little and the fragmentation cannot proceed.

The process of the collapse and the structure of the cloud are very different from the cases $M < M_{BE}/2$. The characteristic of these collapses is the fast onset of the collapse during the disk formation by shock compression. The gravitational instability begins before the shock compression propagates in the whole cloud, because the central free-fall time $t_{dc}$ is shorter than the sound crossing time of the cloud $R/C_s$. Therefore by the time when the central collapse proceeds sufficiently, the x-bounce does not finish and the entire cloud remains in disk-shape. This indicates the star formation with a non-rotating disk cloud.

iii) Massive subsonic collisions of $0.5M_{BE} < M < M_{BE}$

We study whether the Bonnor-Ebert criterion is applicable or not to the dynamical process far from the spherical symmetry. From the numerical simulations, it is confirmed that the cloud with larger mass $M > M_{BE}/2$ always collapses as expected by the static analysis.

The characteristic of these quiet collapses is no formation of the compressed disk. The gravitational instability begins before the compression occurs in the whole region of the cloud. The collision of $A$
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= 0.7, \( V = 0 \) is shown in Fig. 12. In the central part of the compressed regions, the collapse proceeds almost spherically. The radial density profile also fits the isothermal similarity solution of \( \rho \propto r^{-2} \).

For most of the collapsing models presented in i) \(~\) iii), the collapsing mass \( M_c \) shown in Table I increases with the parameter \( A \). This is because, with the larger \( A \), the density contrast of the initial cloud is not small and there are much gas in the central part of the compressed disk. But it does not depend on the impact Mach number so sensitively as long as the impact velocity is supersonic. In Table I, Run 5 seems exceptional for this trend, but it does not mean the collapsing mass is small for such large \( A \), but that the small part of the cloud collapses much faster than the large envelope because we count the collapsing mass at the time when the central density reaches \( 10^4 \rho_0 \). Hence there is a possibility that the remaining cloud will collapse in the later stage.

§ 5. Dynamical stability condition in a head-on collision

5.1. Maclaurin spheroid approximation

In order to analyze our numerical results, especially for the triggered collapse of \( M < M_{\text{BE}}/2 \), we approximate the isothermal cloud by a simple model. In the case of supersonic head-on collision, the shocked layer has relatively uniform density because the dynamical ram pressure dominates the gravity and the density contrast of the initial cloud is small, for example, \( \rho_c/\rho_0 = 2.2 \) for \( M = 0.5 M_{\text{BE}} \). Therefore, the Maclaurin spheroid seems to be a proper approximation for the cloud at the compressed phase. Since the gravitational energy of the uniform density spheroid with the axisymmetric radii \( X = b \) and \( Y = Z = a \) has the analytic expression, the free energy of the isothermal Maclaurin spheroid is given by

\[
F(\xi, \eta; A) = MC_s^2 \left( \frac{\sin^{-1} e}{e} \frac{1}{\xi} \ln(A \xi^2 \eta) + A \xi^2 \eta \right),
\]

where

\[
\xi = (5C_s^2/3GM)a, \quad (5.2)
\]
\[
\eta = b/a, \quad (5.3)
\]
\[
e = (1 - \eta^2)^{1/2}. \quad (5.4)
\]

In Eq. (5.1), the first term is the gravitational energy, the second term is the thermal energy of the isothermal gas and the last is the volume energy due to the external pressure. The parameter \( A \) is the square of nondimensional mass normalized by the external pressure as given in Eq. (2.3).

For a given \( A \), the evolution of the cloud is approximated by the motion of a test particle on this free energy surface. In Fig. 13, we show the free energy surface \( F \) on the \( \xi \cdot \eta \) plane in the case \( A = 1.5 \). The stable energy-minimum point corresponds to the equilibrium configuration for the Maclaurin spheroid which is always spherical (\( \eta = 1 \)) without rotation or magnetic field. The oblate-prolate (quadrupole) oscillation...
discussed in § 4.2 is understood as the motion in the \( \eta \)-direction of a test particle around this minimum point. The saddle point represents the marginally stable spheroid. The test particle will be pulled back by the potential wall in large \( \xi \) region, which means the confinement effect by the external pressure. The lower unlimited energy surface for small \( \xi \) represents the onset of gravitational instability. Therefore, if \( \xi \) becomes smaller by the strong compression which makes the density beyond the equilibrium value, the gravitational collapse can be triggered.

Besides such a variation in the \( \xi - \eta \) plane for a fixed \( A \), there is a drastic change of this energy surface at the critical value of \( A \). For \( A > A_M = 2.847 \), there is no stable spheroid configuration since the free energy \( F \) has no minimum point with respect to the variation in the \( \xi - \eta \) plane. This means that the cloud with the total mass \( A > A_M \) cannot get to the equilibrium even without the dynamical interaction. The critical value \( A_M \) is slightly different from the Bonnor-Ebert critical mass, \( A_{BE} = 1.26 \), because the actual Emden solution has the density contrast.

5.2. Dynamical compression

Using this free energy, we test analytically the stability of the shock-compressed layer. In head-on collisions, this compressed region can be approximated in two ways:

(i) One-dimensional compression: In the case of highly supersonic collisions, one-dimensional compression is a good approximation. Therefore we consider that the layer has the flatness \( \eta_1 = (C_s/V)^2 \) and the radius \( a \) is the initial \( \xi_0 \) of the equilibrium with the half mass.

(ii) Spherical density enhancement: In the case of slow collisions, one-dimensional shock condition cannot be used. We include only the effect of density enhancement by changing the variable \( \xi \) as \( \xi_1 = (C_s/V)^{2/3} \xi_0 \). While the shape effect of colliding region is too complex for this simple analysis, we assume that the cloud is nearly spherical, i.e., \( \eta_1 = 1 \).

Besides the above assumptions, we assume that the kinetic energy is negligibly small when the compressed region forms. Then we investigate the stability of these two configurations on the free energy surface of Eq. (5·1) by the motion of a test particle from the initial position as
\[
\begin{align*}
\text{(i) } & (\xi_0, \eta_0) = (\xi, (C_s/V)^2), \\
\text{(ii) } & (\xi_0, \eta_0) = (\xi (C_s/V)^{2/3}, 1).
\end{align*}
\]

For a given \( A \), the initial cloud size \( \xi_0 \) is determined by the equilibrium. We must investigate the test particle motion on the energy surface of \( F(\xi, \eta; 4A) \) because the total mass is \( 2M \). If the initial position is on the unstable region (small \( \xi \) part), the colliding cloud is expected to start collapsing. On the other hand, near the equilibrium point (the energy minimum region) of \( F(\xi, \eta; 4A) \), it will oscillate around this point. For a given impact Mach number, \( V/C_s \), there is a critical \( A \) at which \( (\xi_1, \eta_1) \) is on the metastable point of energy surface such that

\[
\frac{\partial F(\xi, \eta; 4A)}{\partial \xi} \bigg|_{\xi=\xi_1, \eta=\eta_1} = 0,
\]

\[
\frac{\partial^2 F(\xi, \eta; 4A)}{\partial \xi^2} \bigg|_{\xi=\xi_1, \eta=\eta_1} > 0.
\]

We draw this metastable condition as the critical line on the \( V/C_s - A \) plane in Fig. 5. The upper solid line is case (i) of fast collision and the lower one corresponds to case (ii) of slow collision. These lines are scaled by the factor \( A_{\text{cm}}/A_M \) in order to apply the analysis of the Maclaurin spheroid approximation to that of the Emden solution which we used as the initial condition.

When only the density enhancement is considered, the self-gravity becomes much strong and the critical line goes down as the Mach number increases (case ii). While in one-dimensional compression (case i), the gravity does not become so strong but the thermal energy increases. Therefore, the critical value of \( A \) is almost unchanged for highly supersonic collisions. The free energy surface in Fig. 13 explains the above difference. The energy barrier between the stable minimum point and the gravitationally unstable region in the direction of \( \xi \)-axis is small (\( \eta \gtrsim 0.5 \)), while the thermal energy increase toward the small \( \eta \)-direction makes the barrier larger (\( \eta \ll 1 \)).

The compression by the head-on collisions makes the critical mass of the isothermal cloud smaller than \( A_{\text{cm}}/4 \). The consequent criterion from our numerical surveys runs the intermediate region of the above two lines. Especially, for low Mach number collisions, the lower line (case ii) is good enough and the upper line (case i) agrees with the criterion for high Mach number collisions. Since the upper line includes the effect of shape change by the shock and the lower line stresses on the density enhancement, the fact that the numerical criterion lies between these two lines indicates that they are the limiting cases.

\section{Coalescence and fragmentation}

In our simulation, the disruption is not obtained even by the strong isothermal shock. We conclude that the isothermal cloud-cloud collisions are very inelastic and they result in the coalescence without disruption in the cases \( V \lesssim 10C_s \). The gravitational instability proceeds at the center of the merged system and we expect the single star formation.
As for the theory on cloud evolution, our results support the idea that small clouds coalesce to form larger clouds. The gravitational instability leads to a star formation when the combined mass exceeds the critical mass. That is, the head-on collision of isothermal clouds always leads to the onset of star formation after increasing cloud mass by doing some coalescences.

Many of the previous works on the one-dimensional collisions expected the fragmentation at the interface layer. They are based on the stability analysis of the uniform layer with infinite extent, although it is not good in the actual 3-D collisions. In the collision of two spherical clouds, the initial density distribution of each cloud and the configuration that the spheres overlap without impact parameter make the column density largest on the collision axis in the compressed layer. There are inward directed components of gravity and the gravitational instability is induced in the central region faster than in the outer region. The time for fragmentation in the outer region is much longer than the central collapsing time. And also the former time is longer than the supersonic expansion time scale. Therefore the gravitational instability in the outer region is less likely if $V \lesssim 10C_s$.

We used the isothermal equation of state to investigate the pure dynamical effect of cloud-cloud collision. However, there are some discussions on the isothermality of the cloud. Taking account of the thermal property of the interstellar molecular gas, the efficient cooling by CO molecules is able to make the shock compressed layer so cold and dense that the critical mass can become small enough to fragment. Energy equations have been solved in 1-D collisions by Smith, Sabano and Tosa. They suggest the temperature of the compressed region becomes about a third of the initial temperature. Hence we try to use the more realistic equation of state than the isothermal one to investigate the effect of equation of state and the condition for fragmentation. Including the effect of such a shock cooling, we run the model in which the gas pressure scales mainly

$$\rho = \rho_0 (\frac{\rho}{\rho_0})^{0.73}.$$  \hspace{1cm} (6.1)

Larson gives this expression by fitting the result of hydrostatic molecular cloud model and by the radio observation of interstellar clouds.

The modified EOS we use is shown in Fig. 14. The shock compression emphasizes the molecular cooling and decreases the cloud temperature. In dense region, because the cloud becomes opaque, the equation of state changes to adiabatic one and increases the gradient in the $\rho$-$T$ plane. We assume the first change occurs at the shock compressed density and the second change starts when the temperature decreases to a third of initial value.
Table II. Non-isothermal head-on collisions.

<table>
<thead>
<tr>
<th>Run</th>
<th>A</th>
<th>V/Cs</th>
<th>ρ_{shock} (ρ₀)</th>
<th>ρ_{max} (ρ₀)</th>
<th>Period (t₀)</th>
<th>M_c/M_{tot} collapse</th>
<th>M_{escl}/M_{tot} escape</th>
<th>type</th>
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<tr>
<td>41↑</td>
<td>0.15</td>
<td>3.0</td>
<td>10</td>
<td>63</td>
<td>1.4</td>
<td>0.0</td>
<td>0.0</td>
<td>S</td>
</tr>
<tr>
<td>42↑</td>
<td>0.20</td>
<td>3.0</td>
<td>11</td>
<td>&gt;10^4</td>
<td>0.11</td>
<td>0.37</td>
<td>0.0</td>
<td>C</td>
</tr>
<tr>
<td>43↑</td>
<td>0.30</td>
<td>3.0</td>
<td>11</td>
<td>&gt;10^4</td>
<td>0.09</td>
<td>0.46</td>
<td>0.0</td>
<td>C</td>
</tr>
<tr>
<td>44↑</td>
<td>0.40</td>
<td>3.0</td>
<td>14</td>
<td>&gt;10^4</td>
<td>0.08</td>
<td>0.50</td>
<td>0.001</td>
<td>C</td>
</tr>
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<td>4.0</td>
<td>15</td>
<td>&gt;10^4</td>
<td>0.13</td>
<td>0.26</td>
<td>0.014</td>
<td>C</td>
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<tr>
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<td>4.0</td>
<td>18</td>
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<td>0.002</td>
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<tr>
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<td>0.50</td>
<td>4.0</td>
<td>22</td>
<td>&gt;10^4</td>
<td>0.08</td>
<td>0.50</td>
<td>0.001</td>
<td>C</td>
</tr>
</tbody>
</table>

↑ Non-isothermal equation of state (Fig. 14) which includes the approximate cooling effects is used so that the adiabatic core forms when the ρ_{max} approaches 10^4. The time interval presented in the period column is measured from ρ_{max}=10^2 to ρ_{max}=10^3 for these non-isothermal “collapse” models.

All the results for the cooling models are summarized in Table II. Due to the decrease of thermal pressure, the first density increase by the shock compression is enhanced compared with the isothermal shock condition. Consequently, the triggering effect is enhanced and its critical line shifts lower in the V/Cs-A plane. The collapsing mass shows a similar tendency to the case of isothermal collision; it increases with the nondimensional mass A, but does not depend strongly on the impact Mach number.

The great difference from the isothermal collisions is the structure of halos outside the collapsing region. In the isothermal collisions, only the merged cloud is left behind. While, for our modified EOS, many small clumps appear after the collision. Figure 15 shows the results of such a collision where the shock cooling is effective and the temperature decreases as the density increases. Then due to the thermally-enhanced gravitational instability, the disk fragments into the small pieces with the size comparable to the disk thickness. In this case, each small clump has the maximum density about ρ_{max}=2-4ρ_{surf}. The clump mass is too small to collapse...
even for the decreased temperature except the central collapsing region. Such a fragmentation or disruption is manifest for the case of small $A$, which has a small density contrast. While in the case of large $A$, the central collapse proceeds faster (see Period column of Table II) and dominates the spherical gravitational field before the fragmentation occurs in the outer region. Therefore we conclude that the shock cooling and the small density concentration in the transverse direction of collision axis are the necessary conditions for the disruption by the relatively low Mach number head-on collision, $V \lesssim 4C_s$.

§ 7. Summary

1) Head-on cloud-cloud collisions can really trigger star formation. The critical mass obtained from the static analysis can be reduced by $\sim 20\%$ due to the dynamical shock compression in a head-on collision of $V/C_s = 2-3$. For example, two stable clouds of $M > 0.4 M_{BE}$ become unstable when they experience the supersonic head-on collision. The collapsing mass in triggered star formation varies from 30% to 60% of the total mass, as the total mass increases from $0.4 M_{BE}$ to $M_{BE}$.

2) However, the efficiency for such a trigger does not increase with the impact Mach number. Because in the high Mach number collision the compression occurs almost in one-dimensional way and a very thin disk is formed, the gravitational force does not increase as much as expected by the simple scaling for the spherical density enhancement.

3) Disruptions or fragmentation by the head-on collision is less likely as long as the gas remains isothermal in the velocity region, $V \lesssim 10C_s$. The merged cloud does not diverge into the ambient medium on behalf of the boundary condition that clouds are always exerted by the constant external pressure. The typical evolution of the stable collision is the quadrupole oscillation around an equilibrium state. The effective cooling which decreases the temperature at the shocked layer is found to be necessary for the fragmentation to occur. Although the star formation occurs only at the collision center, many cloudlets whose masses are smaller than the Jeans mass appear.

4) From the simple initial condition without rotation or magnetic fields, a head-on collision makes the fascinating structures, for example, the new-born star with the expanding prolate cigar and the contracting disk perpendicular to it.

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