Kernel methods for pier scour modeling using field data

Mahesh Pal, N. K. Singh and N. K. Tiwari

ABSTRACT

Three kernel-based modeling approaches are proposed to predict the local scour around bridge piers using field data. Modeling approaches include Gaussian processes regression (GPR), relevance vector machines (RVM) and a kernlised extreme learning machine (KELM). A dataset consisting of 232 upstream pier scour measurements derived from the Bridge Scour Data Management System (BSDMS) was used. The radial basis kernel function was used with all three kernel-based approaches and results were compared with support vector regression and four empirical relations. Coefficient of determination value of 0.922, 0.922 and 0.900 (root mean square error, RMSE = 0.297, 0.310 and 0.343 m) was achieved by GPR, RVM and KELM algorithm respectively. Comparisons of results with support vector regression and Froehlich equation, Froehlich design, HEC-18 and HEC-18/Mueller predictive equations suggest an improved performance by the proposed approaches. Results with dimensionless data using all three algorithms suggest a better performance by dimensional data.

Key words | field scour data, Gaussian process regression, kernlised extreme learning machine, pier scour, relevance vector machine, support vector machines

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INTRODUCTION

Partial blocking of flow of a river stream by a bridge pier changes flow pattern around it significantly. This blocking of flow allows a buildup of adverse pressure gradient just upstream of the bridge pier, forcing the boundary layer to go under a three-dimension separation (Laursen & Toch 1956; Breusers *et al*. 1977; Chiew 1984), thus allowing the formation of a horseshoe vortex around a bridge pier. The formation of a horse shoe vortex causes a drastic change in the shear stress distribution around the pier, leading to an excessive removal of sediments near the structure (Melville & Coleman 2000). Removal of excessive sediments and subsequent reduction of bed elevation near the piers exposes the foundations of a bridge. The amount of reduction in bed elevation below an assumed natural level is termed as scour depth. Excessive reduction of bed elevation near the piers may expose the foundation of a bridge, which may result in structural collapse as well as loss of life and property. The temporal variation and the maximum depth of scour at a river bridge pier depends on catchment and river characteristics, flood flow transport, river-bed material and the geometry of the bridge pier (Melville & Coleman 2000). Thus, accurate estimation of equilibrium depths of local scour around bridge piers is a vital issue in the hydraulic design of bridges. Equilibrium scour depth is achieved when transport of bed material into and out of a scour hole becomes equal. Several methods and formulas have been proposed for estimation of the equilibrium depth of local scour near bridge piers (Deng & Cai 2010). The main problem with different empirical formulas is that most of these are based on dimensional analysis using small-scale laboratory experiments with non-cohesive and cohesive uniform bed material under steady-flow conditions (Kandasamy & Melville 1998; Ansari *et al*. 2002). The equations developed using laboratory research have not been adequately verified by using field data and the scour prediction methods developed based on laboratory data did not always produce good results for field conditions (Melville 1975; Jones 1984; Dargahi 1990). Due to scale effect, laboratory settings oversimplify or ignore the complexities of natural rivers, thus the scour-depth equations based on

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laboratory flume data were found to overestimate scour depth measured at bridge piers (Mueller & Wagner 2005).

Machine learning techniques, such as artificial neural network, support vector machine, model tree and genetic programming, have extensively been used in predicting pier and abutment scour using laboratory and field datasets (Trent et al. 1999; Kambekar & Deo 2003; Choi & Cheong 2006; Azamathulla et al. 2006, 2009, 2010; Bateni et al. 2007a, 2007b; Guven & Gunal 2008; Guven et al. 2009; Ayoubloo et al. 2010; Kaya 2010; Muzzammil & Ayyub 2010; Muzzammil 2010; Shin & Park 2010; Ghazanfari-Hashemi et al. 2011; Goyal & Ojha 2011; Pal et al. 2011, 2012; Toth & Brandimarte 2011; Azamathulla 2012; Hong et al. 2012; Khan et al. 2012; Ghaemi et al. 2013; Najafzadeh et al. 2013). Most of these studies indicate improved performance by machine learning approaches in comparison to the empirical relations. Among various machine learning approaches used to predict pier scour, support vector machines (SVM), a kernel-based approach, was found to provide improved predictive performance in comparison to a back-propagation neural network and model tree approach (Pal et al. 2011, 2012; Hong et al. 2012). There has been an increase in the use of other kernel-based algorithms, such as Gaussian process regression (GPR), relevance vector machines (RVM) and (ELM), as modeling tools in civil engineering (Khalil et al. 2005; Samui 2007, 2012; Flake et al. 2010; Pal & Deswal 2010). To the best of our knowledge, no study has reported their use in pier scour modeling. One of the major advantages of kernel-based machine learning algorithms is the requirement of fewer user-defined parameters in comparison to a back-propagation neural network. A back-propagation neural network algorithm requires the setting up of several parameters such as learning rate, momentum factor, number of hidden layers and nodes in each hidden layer. Being an iterative process, the user also has to define the number of iterations. A large number of training iterations may force a neural network to over-train, thus affecting the predictive capabilities of the algorithm. The presence of local minima due to the use of non-convex unconstrained minimization is another problem with the use of a back propagation neural network.

Keeping in mind the improved performance of kernel-based approaches in various civil engineering applications, the present study is designed to evaluate the potential of GPR, RVM and a kernlised extreme learning machine (KELM) in modeling the pier scour using a field dataset. For comparison of results, SVM, a widely used kernel-based algorithm, back-propagation neural network and four empirical relations were used. A number of empirical relations are proposed to predict pier scour using laboratory studies and a study by Mueller & Wagner (2005) suggests that out of 26 empirical equations to predict the pier scour using laboratory dataset, no single equation is conclusively better than the rest. In this study, four empirical relations were used: HEC-18, HEC-18/Mueller equation, Froehlich (1988), and Froehlich Design (1988) and were found to perform well for pier scour modeling (Mueller & Wagner 2005).

MODELING TECHNIQUES

Three kernel-based machine learning approaches, GPR, RVM and ELM, were used to predict pier scour in the present study. A brief description of these approaches is provided below.

Gaussian process regression

Gaussian processes (GP) are a natural generalization of the Gaussian distribution, with mean and covariance as a vector and matrix, respectively (Neal 1998). GPR is a useful nonparametric regression approach due to its theoretical simplicity and good generalization ability, as well as providing probabilistic output (Rasmussen & Williams 2006). The use of kernel functions relates to GPR well with SVM (Vapnik 1995) and RVM (Candela 2004).

The main assumption of GPR is that y is defined by $y \sim f(x) + \xi$, where $\xi \sim N(0, \sigma^2)$. The symbol ~ in statistics means *sampling for*. In GPR, for every input x there is an associated random variable $f(x)$, which is the value of the stochastic function f at that location. In this study, it is assumed that the observational error ξ is normal independent and identically distributed, with a mean value of zero ($\mu(x) = 0$), a variance of σ^2 and $f(x)$ drawn from the Gaussian process on χ specified by k. That is:

$$Y = (y_1, \ldots\ldots\ldots y_n) \sim N(0, K + \sigma^2 \boldsymbol{I})$$

where $K_{ij} = K(x_i, x_j)$, and $\boldsymbol{I}$ is the identity matrix.

For a given vector of the test data $\boldsymbol{X}_*$, the predictive distribution of the corresponding output $\boldsymbol{Y}_*/(\boldsymbol{X}, \boldsymbol{Y}), \boldsymbol{X}_* \sim N(\mu, \Sigma)$ is Gaussian, where:

$$\mu = K(\boldsymbol{X}_*, \boldsymbol{X})\left(K(\boldsymbol{X}, \boldsymbol{X}) + \sigma^2 \boldsymbol{I}\right)^{-1} \boldsymbol{Y} \quad (1)$$

$$\Sigma = K(\boldsymbol{X}_*, \boldsymbol{X}_*) - \sigma^2 I - K(\boldsymbol{X}_*, \boldsymbol{X}) \times \left(K(\boldsymbol{X}, \boldsymbol{X}) + \sigma^2 I\right)^{-1} K(\boldsymbol{X}, \boldsymbol{X}_*) \quad (2)$$

If there are n training data and n_* test data, then $K(\boldsymbol{X}, \boldsymbol{X}_*)$ represents the $n \times n_*$ matrix of covariances evaluated at all pairs of training and test datasets, and this is similarly true for the other values of $K(\boldsymbol{X}, \boldsymbol{X})$, $K(\boldsymbol{X}_*, \boldsymbol{X})$ and $K(\boldsymbol{X}_*, \boldsymbol{X}_*)$; here $\boldsymbol{X}$ and $\boldsymbol{Y}$ are the vector of the training data and training data labels y_i.

A specified covariance function is required to generate a positive semi-definite covariance matrix K, where $K_{ij} = K(x_i, x_j)$. The term kernel function used in SVM is equivalent to the covariance function used in GPR. With the known kernel function and degree of noise σ^2, Equations (1) and (2) would be enough for inference.

During the training process of GPR models, the user needs to choose a suitable covariance function, its parameters and the degree of noise. In the case of GPR with a fixed value of Gaussian noise, a GP model can be trained by applying Bayesian inference, i.e. by maximizing the marginal likelihood. This leads to the minimization of the negative log-posterior:

$$p(\sigma^2, k) = \frac{1}{2} Y^T \left(K + \sigma^2 I\right)^{-1} Y + \frac{1}{2} \log\left|K + \sigma^2 I\right| - \log p(\sigma^2) - \log p(k) \quad (3)$$

To find the hyperparameters, the partial derivative of Equation (3) can be obtained with respect to σ^2 and k, and minimization can be achieved by gradient descent. Readers are referred to Kuss (2006) for more details about GPR and different covariance functions.

Relevance vector machine

RVM is another recent development in kernel-based machine learning approaches and can be used as an alternative to SVM for the prediction of pier scour. The RVM is based on a Bayesian formulation of a linear model with an appropriate prior (where the prior of a parameter is the probability distribution that represents the uncertainty about the parameter before the training data are examined) that results in a sparser solution than that achieved by SVM. RVM is based on a hierarchical prior, where an independent Gaussian prior is defined on the weight parameters in the first level, and an independent Gamma hyper prior is used for the variance parameters in the second level (Tipping 2001). This results in an overall student-t prior on the weight parameters, which leads to model sparseness (Tipping 2001). Key advantages of the RVM over the SVM include a reduced sensitivity to the hyperparameter settings, an ability to use non-Mercer kernels, provision of a probabilistic output for a given dataset, producing more sparse solution then SVM with no need to define the soft margin parameter C as used with SVM.

For the given dataset (x_i, y_i) with n number of samples, RVM tries to predict $\tilde{y}$ for a query x according to $\tilde{y} = f(\boldsymbol{X}; \mathbf{g}) + \varepsilon_n$ where $\varepsilon_n \sim N(0, \sigma^2)$ is the independent zero-mean Gaussian distributed random noise having variance σ^2 and $\mathbf{g} = (g_0, g_1, \ldots\ldots g_n)^T$ is a weight vector of the basis function $\boldsymbol{\Phi}(x)$. The function $f(x)$ for RVM can be written as:

$$f(\boldsymbol{X}, \mathbf{g}) = \sum_{i=1}^{n} g_i K(x, x_i) + g_0 = \sum_{i=1}^{n} g_i \boldsymbol{\Phi}(x) \quad (4)$$

Based on the assumption of independence of $\tilde{y}$, the likelihood of the dataset can be represented by:

$$p(\boldsymbol{y}/\mathbf{g}, \sigma^2) = \frac{1}{2\pi\sigma^2} \exp\left\{-\frac{1}{2\sigma^2} \|\boldsymbol{y} - \boldsymbol{\Phi}(x)\mathbf{g}\|\right\} \quad (5)$$

where $\boldsymbol{\Phi}$ is a $n \times (n+1)$ matrix. RVM uses a Bayesian perspective and constrains parameter g and σ^2 by defining a prior probability distribution over weights (Tipping 2001):

$$p(\mathbf{g}/\boldsymbol{\alpha}) = \prod_{i=1}^{n} n\left(g_i/0, \alpha_i^{-1}\right)$$

where $\boldsymbol{\alpha}$ is a vector of $(n+1)$ hyperparameter.

After defining the prior probability of the parameters, the posterior over the weights can be obtained by the

Bayesian rule:

$$p(\boldsymbol{g}/\boldsymbol{y}, \boldsymbol{\alpha}, \sigma^2) = \frac{1}{(2\pi)^{(n+1)/2}} \exp\left\{-\frac{1}{2}(\boldsymbol{g}-\boldsymbol{\mu})^T \boldsymbol{\Sigma}^{-1}(\boldsymbol{g}-\boldsymbol{\mu})\right\} \quad (6)$$

where posterior covariance and mean are defined as:

$$\boldsymbol{\Sigma} = \left(\sigma^{-2}2\boldsymbol{\Phi}^T 2\boldsymbol{\Phi} + \boldsymbol{D}\right)^{-1}, \quad \boldsymbol{\mu} = \sigma^{-2}\boldsymbol{\Sigma}\boldsymbol{\Phi}^T \boldsymbol{y} \text{ and } \boldsymbol{D} = \text{diag}(\alpha_1, \alpha_2, \ldots, \alpha_n) \quad (7)$$

To obtain the marginal likelihood for the hyperparameters, Tipping (2001) suggested using:

$$p(\boldsymbol{y}/\boldsymbol{\alpha}, \sigma^2) = (2\pi)^{-n/2}|\text{Cov}|^{-1/2}\exp\left\{\frac{1}{2}\boldsymbol{y}^T \text{Cov}^{-1}\boldsymbol{y}\right\} \quad (8)$$

where covariance is defined by $cov = \sigma^2 \boldsymbol{I} + \boldsymbol{\Phi}\,\boldsymbol{D}^{-1}\boldsymbol{\Phi}^T$ and optimizing Equation (8) with respect to $\boldsymbol{\alpha}$ and σ^2. Tipping (2001) also suggested using an iterative re-estimation method to obtain the values of $\boldsymbol{\alpha}$ and σ^2 using the following equation:

$$\alpha_i^{\text{new}} = \frac{1-\boldsymbol{\alpha}_i \boldsymbol{\Sigma}_{ii}}{\boldsymbol{\mu}_i^2} \text{ and } (\sigma^2)^{\text{new}} = \frac{\|\boldsymbol{y}-\boldsymbol{\Phi}_{\boldsymbol{\mu}}\|^2}{n - \sum_{i=1}^{n}(1-\alpha_i \boldsymbol{\Sigma}_{ii})} \quad (9)$$

RVM works by repeating Equation (9), while updating the posterior statistics in Equation (7), until all $\boldsymbol{\alpha}_i$ becomes smaller than the pre-defined value such that the corresponding model parameter g_i reduces to zero. The assignment of an individual hyperparameter to each weight is the reason for the sparse property of RVM. Further details about RVM is available in Tipping (2001).

Kernlised extreme learning machine

The ELM is a single hidden layer neural network (Huang et al. 2006). It uses randomly assigned input weights and bias, not requiring adjustment of input weights like a back-propagation method. For the given dataset, an ELM having H hidden neurons and activation function $f(x)$ can be represented as:

$$\sum_{i=1}^{H} \boldsymbol{\alpha}_i f(\boldsymbol{x}_j) = \sum_{i=1}^{H} \boldsymbol{\alpha}_i f(\boldsymbol{w}_i.\, \boldsymbol{x}_j + c_i) = \boldsymbol{e}_j \quad \text{where } j = 1, \ldots\ldots, n, \quad (10)$$

where $\boldsymbol{w}_i$ and $\boldsymbol{\alpha}_i$ are the weight vectors connecting inputs and the ith hidden neuron and the ith hidden neuron and output neurons respectively; c_i is the complex bias of the ith hidden neuron. Huang et al. (2006) proposed that Equation (10) can be written in a compact form and this is represented by the following equation:

$$\boldsymbol{A\alpha} = \boldsymbol{Y} \quad (11)$$

where $\boldsymbol{A}$ is called the hidden layer output matrix of the neural network (Huang et al. 2006).

Most of the research works reporting the use of neural network have used back-propagation learning algorithms to adjust the set of weights $(\boldsymbol{w}_i, \boldsymbol{\alpha}_i)$ and biases. The back-propagation learning algorithm requires specifying the value of learning rate, momentum, and does not guarantee that the absolute minimum of the error function will be found. Thus, during the training phase, the learning algorithm can have local minima and may over-train. To overcome these problems, Huang et al. (2006) propose to use the smallest norm least squares solution of $\boldsymbol{A\alpha} = \boldsymbol{Y}$. In most cases of the use of ELM, the number of hidden neurons is much less than the number of training samples, thus making $\boldsymbol{A}$ a non-square matrix and there may not exist $\boldsymbol{\alpha}$ such that $\boldsymbol{A\alpha} = \boldsymbol{Y}$, instead one may need to find $\boldsymbol{\alpha}'$ (Huang et al. 2006). Thus, the solution of Equation (11) becomes:

$$\boldsymbol{\alpha}' = \boldsymbol{A}^{\Psi}\boldsymbol{Y} \quad (12)$$

where $\boldsymbol{A}^{\Psi}$ is the Moore–Penrose generalized inverse of matrix $\boldsymbol{A}$ (Serre 2002).

Recently, Huang et al. (2012) proposed using orthogonal projection and kernel methods in the design of ELM. According to the orthogonal projection method; $\boldsymbol{A}^{\Psi} = (\boldsymbol{A}^T\boldsymbol{A})^{-1}\boldsymbol{A}^T$ if $\boldsymbol{A}^T\boldsymbol{A}$ is non-singular or $\boldsymbol{A}^{\Psi} = \boldsymbol{A}^T(\boldsymbol{A}\boldsymbol{A}^T)^{-1}$ if $\boldsymbol{A}\boldsymbol{A}^T$ is non-singular. Huang et al. (2012) suggested adding a positive value $1/\rho$ (where ρ is a user-defined parameter) to the diagonal of $\boldsymbol{A}\boldsymbol{A}^T$ or $\boldsymbol{A}^T\boldsymbol{A}$ in the calculation of the output weights $\boldsymbol{\alpha}$ which provide a more stable solution of ELM with better generalization capabilities in comparison to least square solution. Thus, to have a stable ELM algorithm, one can have:

$$\boldsymbol{\alpha} = \boldsymbol{A}^T\left(\frac{\boldsymbol{I}}{\rho} + \boldsymbol{A}\boldsymbol{A}^T\right)^{-1}\boldsymbol{Y} \quad (13)$$

with a corresponding output function of ELM defined by:

$$\boldsymbol{h}(\boldsymbol{x})\boldsymbol{\alpha} = \boldsymbol{h}(\boldsymbol{x})\boldsymbol{A}^T\left(\frac{\boldsymbol{I}}{\rho} + \boldsymbol{A}\boldsymbol{A}^T\right)^{-1}\boldsymbol{Y} \tag{14}$$

Huang *et al.* (2012) also proposed using a kernel function if the hidden layer feature mapping $\boldsymbol{h}(\boldsymbol{x})$ is unknown. A kernel matrix for ELM can be represented as follows:

$$\chi_{\text{ELM}} = \boldsymbol{A}\boldsymbol{A}^T : \chi_{\text{ELM}i,j} = \boldsymbol{h}(\boldsymbol{x}_i) \cdot \boldsymbol{h}(\boldsymbol{x}_j) = K(\boldsymbol{x}_i, \boldsymbol{x}_j) \tag{15}$$

where $K(\boldsymbol{x}_i, \boldsymbol{x}_j)$ is a kernel function. Now the output function (Equation (11)) can be written as:

$$\begin{bmatrix} K(\boldsymbol{x}, \boldsymbol{x}_1) \\ \vdots \\ K(\boldsymbol{x}, \boldsymbol{x}_n) \end{bmatrix}^T \left(\frac{I}{\rho} + \chi_{\text{ELM}}\right)^{-1}\boldsymbol{Y} \tag{16}$$

In kernel function based implementation of ELM, knowledge of hidden layer feature mapping and the number of hidden nodes is not required, instead a kernel function corresponding to $\boldsymbol{h}(\boldsymbol{x})$ can be used. Different kernel functions such as linear, polynomial and radial basis function, as used with SVM, can be used with kernel-based ELM.

DATASET

Of the total 493 pier scour measurements available in the Bridge Scour Data Management System (Mueller & Wagner 2005), 232 dataset for upstream scour were used in this study to predict pier scour. The observations with scour in cohesive material (a total of five data out of 493 measurements) were removed because the time required for scour to reach its maximum depth in cohesive material is considerably longer than in non-cohesive material from this analysis. Data with pier type 'group', 'unknown bed material', 'missing value of any input variable' and having 'zero scour' (a total of six data for zero scour with non-cohesive material) were removed from the total dataset. Furthermore, data with scour measurements at the downstream side of piers were removed. The effect of debris on the scour depth was also provided in the dataset and grouped in four categories (i.e. unknown, insignificant, moderate and substantial). Debris accumulation near a pier often makes measurement of maximum scour impossible and may increase the scour near the pier due to a larger obstruction to flow (Song *et al.* 1989). Thus, all scour measurements with 'substantial' and 'moderate' effect of debris (a total of 40 data, having 14 data with scour measurement at the downstream side and three data with pier type group) were removed from the dataset. Finally, the remaining dataset consisting of both live-bed and clear water scour with 232 upstream scour measurements was used in the present study. The dataset was divided randomly so as to use 154 data for training different kernel-based algorithms and the remaining 78 data for testing the created models.

Studies by Lee *et al.* (2007) and Azamathulla *et al.* (2010) have used only non-dimensional datasets in modeling the pier scour with field dataset, so the present work compares the performance of different algorithms with both dimensional and non-dimensional datasets in order to study the influence, if any, of non-dimensional analysis with field dataset. Altogether seven input parameters, namely pier shape factor (*Ps*), pier width (*Pw*), skew of the pier to approach flow (*skew*), critical velocity of the flow (V_c), depth of flow (h), D_{50} (i.e. the grain size of bed material in mm for which 50% is finer) and gradation of bed material σ, were used to predict the scour depth with the dimensional dataset. Critical velocity was calculated by using the equation suggested by Neill (1973) and provided in Mueller & Wagner (2005). In order to predict the normalized scour depth (scour/P_w), seven input parameters were reduced to six non-dimensional parameters (i.e. P_s, skew angle, $V/\sqrt{gh}$, h/P_w, D_{50}/P_w and σ). Minimum, maximum, mean and standard deviation values of all input and output parameters used in this study are provided in Table 1.

DETAILS OF ALGORITHMS

The use of GPR, SVR and KELM requires the use of a kernel function. A number of kernels are discussed in the literature, but studies suggest the effectiveness of radial basis kernel function in the case of SVM in the majority of civil engineering applications (Pal & Mather 2003; Gill *et al.* 2006; Pal &

Table 1 | Characteristics of the train and test data used in this study

Input parameter	Train data Min	Max	Mean	St. dev.	Test data Min	Max	Mean	St. dev.
Dimensioned data								
P_s	0.7	1.3	0.973	0.210	0.7	1.3	0.988	0.202
P_w	0.3	5.5	1.558	1.156	0.3	5.5	1.397	1.151
Skew	0	85	9.260	18.629	0	65	9.897	18.373
V_c	0.43	4.05	1.58	1.066	0.38	3.44	1.572	1.054
h	0.3	22.5	4.552	4.019	0	22.4	3.796	3.579
D_{50}	0.12	95	18.978	26.758	0.15	95	19.473	25.097
σ	1.2	20.3	3.650	3.294	1.2	21.8	3.605	2.901
Scour	0.1	7.1	1.121	1.272	0.1	6.2	0.938	1.059
Non-dimensioned data								
V/gh	0.046	0.784	0.288	0.144	0	0.607	0.257	0.133
h/P_w	0.333	10.444	2.998	1.826	0.4	11	2.937	1.774
D_{50}/P_w	0.07	120	18.428	29.056	0.07	120	21.717	31.799
Scour/P_w	0.091	2	0.738	0.455	0.108	2.333	0.74	0.491

The unit of measurements for P_w, h and scour depth is in meter, velocity of flow is in meter/second, D_{50} is in mm and *skew* is measured in degrees.
$P_s = 1.3$ for square nosed-piers, 1.0 for round-nosed piers and 0.7 for sharp-nosed piers.

Goel 2006; Goel & Pal 2009). For a fair comparison of results, a radial basis kernel (RBF) $e^{\left(-\gamma|\boldsymbol{x}-\boldsymbol{y}|^2\right)}$, where γ (kernel width) is a kernel specific, was used with all three machine learning algorithms used in this study.

In addition to the choice of the kernel function GPR, RVM and KELM requires the setting up of kernel specific parameters. GPR and KELM also require optimum values of the Gaussian noise (added to the diagonal of the covariance matrix) and the parameter ρ respectively. Several approaches, such as a manual method and a grid search method, are proposed for the selection of user-defined parameters with different algorithms. In the present study, a manual method, which involves carrying out a large number of trials by using different combinations of user-defined parameters with different algorithms, was used to select the optimal value of user-defined parameters. Optimal values of various user-defined parameters are chosen in a way so as to minimize the root mean square error values with test dataset. Table 2 provides the optimal values of user-defined parameters used with different algorithms in the present study.

Coefficient of determination, root mean square error (RMSE), index of agreement (Willmott & Wicks 1980) and mean absolute error (MAE) was used to compare the performance of different kernel-based algorithms to model the bridge pier scour values. The use of MAE was encouraged by the study by Willmott (1982), which suggests that this measure is less sensitive to extreme values than RMSE. The index of agreement value varies between zero and one; a value closer to 1 indicates better agreement between actual and predicted values. The cumulative probability approach and Levene's test was also considered to judge the suitability of different algorithms in predicting pier scour. The cumulative probability approach requires plotting the ratio of the predicted to measured scour values (i.e. discrepancy ratio) against the calculated cumulative probability (Abu-Farsakh 2004). The cumulative probability factor is calculated by first arranging the ratio of predicted to the measured scour values in ascending order and indexing them with 1 to s numbers, where s is

Table 2 | Optimal value of user-defined parameters used in this study

Algorithm	RBF kernel
GPR	Noise = 0.4, $\gamma = 5$
RVM	$\gamma = 0.44$
KELM	$\rho = 12$, $\gamma = 0.34$

the total number of data points in the testing set. Then, for each of the relative amounts, the cumulative probability factor for a sample (i) can be calculated as follows:

$$P(\%) = \frac{i}{s+1} * 100 \quad (17)$$

The ratio of predicted to actual pier scour value at the cumulative probability of 50% (i.e. P50) indicates the tendency of overestimating or underestimating the pier scour values by the modeling approach. If the computed value of 50% cumulative probability is closer to a ratio of unity, it suggests a better agreement between actual and predicted values. A value less than unity suggests underprediction whereas values greater than unity indicates overprediction.

Levene's test is a non-parametric test and tests the null hypothesis that variances of the predicted values have equal variances as that of actual dataset (Khan *et al.* 2006). Equal variances across predicted values by different algorithms are called homogeneity of variances. The test is less sensitive to departures from normality. The hypothesis for Levene's test can be performed by choosing a level of significance and then comparing the value of calculated critical value to that of critical value (0.05 used in the present study) obtained from the F distribution table. If the calculated critical value is smaller than 0.05, the hypothesis about equality of the sample population variances that there is a difference between the variances in the predicted values by different algorithms can be accepted.

RESULTS

Table 3 provides the coefficient of determination and RMSE value obtained by using GPR, RVM and KELM algorithms with the test dataset. The coefficient of determination value of 0.922, 0.922 and 0.900 (RMSE = 0.297, 0.310 and 0.343 m) was obtained by using the GPR, RVM and KELM algorithms, respectively. Results suggest an almost similar performance by all three kernel-based algorithms in predicting pier scour with field dataset. In comparison to the coefficient of determination value of 0.897 and 0.880 (RMSE = 0.356 and 0.388 m) provided by RBF kernel-based SVM and back-propagation neural network (Pal *et al.* 2011), the results from Table 3 suggest that the proposed kernel-based algorithms achieve better predictive accuracy with the used dataset. The results in terms of RMSE suggest an improved performance by GPR in comparison to other two kernel-based algorithms.

Table 3 | Performance indicators with different algorithms using dimensional data

	RBF kernel			
Algorithm	R^2	**RMSE (m)**	**MAE (m)**	**Index of agreement**
GPR	0.922	0.297	0.224	0.97
RVM	0.922	0.310	0.245	0.98
KELM	0.900	0.343	0.247	0.98

Figure 1 provides the graph plotted between actual and predicted value of scour depth obtained by using GPR, RVM and KELM algorithms with the test dataset. Comparison of Figures 1(a)–1(c) suggests that RVM is able to better predict higher scour values in comparison to GPR and KELM approaches.

In order to compare the performance of proposed kernel-based algorithms in predicting the normalized pier scour (scour/P_w), seven input parameters were reduced to six non-dimensional parameters. Table 1 indicates no significant difference in the statistics of the non-dimensional training and test dataset. For a fair comparison of results with dimensional dataset, the same user-defined parameters as used with dimensional data were used to train different algorithms. For consistency in predicting scour values, predicted normalized scour depth was converted to scour depth and appropriate statistical measures were re-calculated. The coefficient of determination and RMSE values obtained by using GPR, RVM and KELM algorithms (Table 4) suggest a better performance by KELM with non-dimensional data. A comparison of statistical measures from Tables 3 and 4 indicates better predictive performance by different algorithms with dimensional dataset. Figure 2 provides a plot between actual and predicted scour values obtained by using non-dimensional inputs. A comparison of Figures 1 and 2 suggests that the majority of the predicted values lie away from the line of perfect agreement with non-dimensional data. This confirms the findings of earlier studies by Azamathulla *et al.* (2006), Zounemat-Kermani *et al.* (2009) and Pal *et al.* (2011, 2012), suggesting a better

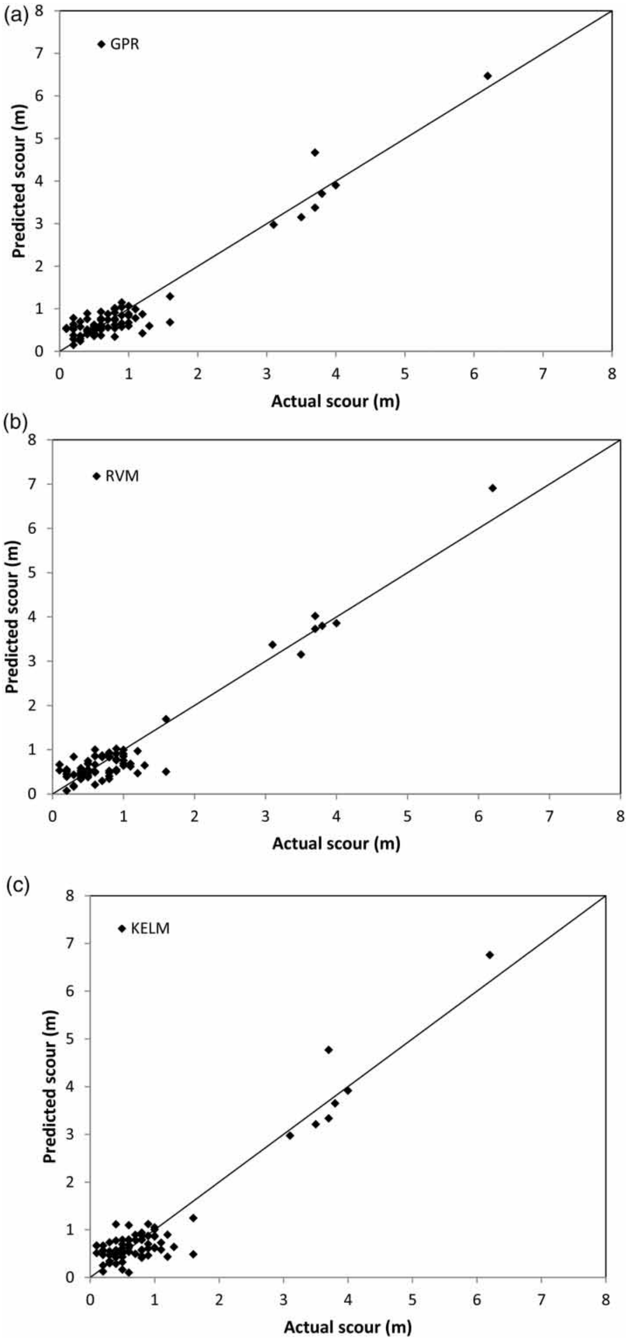

Figure 1 | Actual vs. predicted scour: (a) GPR; (b) RVM; and (c) KELM algorithm using dimensional dataset.

Table 4 | Coefficient of determination and RMSE values using non-dimensional data

	RBF kernel			
Algorithm	**R^2**	**RMSE (m)**	**MAE (m)**	**Index of agreement**
GPR	0.790	0.511	0.299	0.945
RVM	0.760	0.535	0.365	0.936
KELM	0.816	0.479	0.307	0.951

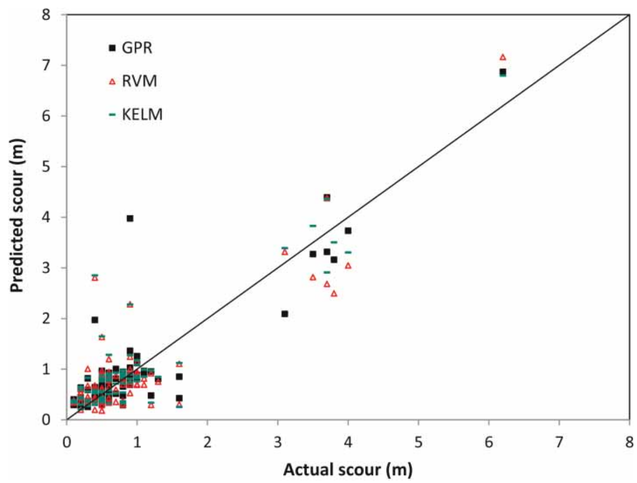

Figure 2 | Actual vs. predicted scour using non-dimensional data.

performance with dimensional dataset in predicting pier scour by different machine learning algorithms.

The same dataset as used with GPR, RVM and KELM was used to predict scour depth using four empirical relations. A comparison of coefficient of determination and RMSE values (Tables 3 and 5) indicates improved predictions of scour depth by the proposed modeling approaches in comparison to empirical relations. Figure 3 provides a graph between actual and predicted values of scour depth using the Froehlich Equation (1988), Froehlich Design (1988), HEC-18 and HEC-18/Mueller equations. Of the four empirical relations used in this study, P50 values provided in Table 5 indicate that Froehlich (1988) underpredicts most of the values, whereas the remaining three equations overpredict the scour depth with the used dataset. The poor performance of empirical relation may be attributed to the reason that these equations are deliberately designed to provide conservative estimate of scour prediction.

To compare the performance of GPR, RVM and KELM in predicting pier scour using dimensional dataset, the cumulative probability analysis curve for all three algorithms is also plotted (Figure 4). Plot in Figure 4 (P50 values with

Table 5 | Coefficient of determination and RMSE with empirical relations

Name	Equation	R^2	RMSE (m)	MAE	Index of agreement	P50 value
Froehlich Equation (1988)	$0.32\,P_s\,g^{-0.1}\,V^{0.2}\,h^{0.36}\,P_w^{0.62}\,D_{50}^{-0.08}$	0.384	0.957	0.590	0.554	0.484
Froehlich Design (1988)	$0.32\,P_s\,g^{-0.1}\,V^{0.2}\,h^{0.36}\,P_w^{0.62}\,D_{50}^{-0.08}+P_w$	0.433	1.540	1.00	0.832	2.157
HEC-18	$2.0\,K_1\,K_2\,K_3\,g^{-0.215}\,V^{0.43}\,h^{0.135}\,P_w^{0.65}$	0.319	1.670	1.31	0.800	2.786
HEC-18/Mueller Equation (1996)	$2.0\,K_1\,K_2\,K_3\,K_4\,g^{-0.215}\,V^{0.43}\,h^{0.135}\,P_w^{0.65}$ where $K_4=0.4\left(V-V_c'/V_c'-V_{c95}'\right)^{0.15}$	0.394	1.360	0.902	0.836	1.682

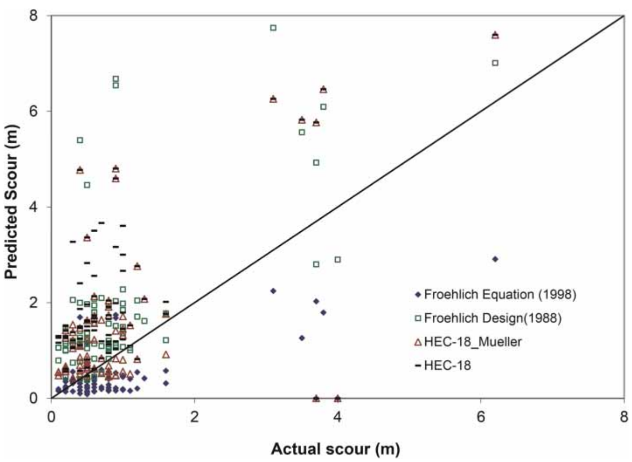

Figure 3 | Actual vs. predicted scour using four empirical relations.

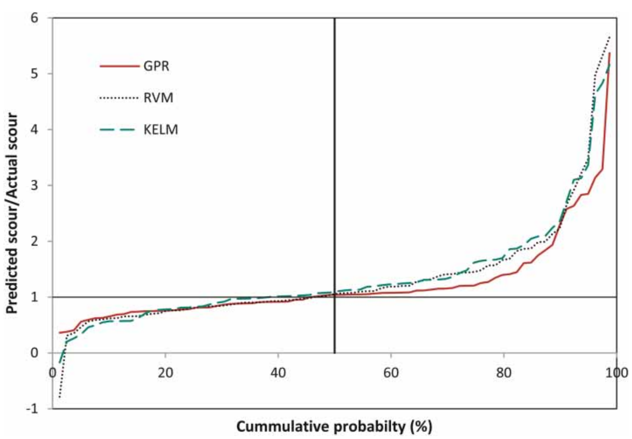

Figure 4 | Cumulative probability diagram for GPR, RVM and KELM algorithm with test dataset.

GPR = 1.03, RVM = 1.03 and KELM = 1.01) suggests that all three KELM, GPR and RVM algorithms perform equally well in predicting the pier scour, as P50 value is very close to 1 with all three algorithms.

Table 6 | Results of the Levene's test using testing dataset

Algorithm	Levene's statistic	*p*-value
GPR	0.238	0.626
RVM	0.156	0.693
KELM	0.223	0.634

The Levene's test is also used to test whether the predicted scour values using GPR, RVM and KELM algorithms have statistics and dependence properties similar to those of the actual scour values of the test dataset (Table 6). The results from Table 6 indicate that there is no statistically significant difference between the sample population mean and variance of the actual test dataset and the predicted values by different machine learning algorithms (all *p*-values are above 0.05). This suggests that all kernel-based algorithms used in this study can effectively be used to predict pier scour using the field dataset.

SENSITIVITY ANALYSIS

The results in the previous section suggest that all three kernel-based modeling approaches are capable of generalization well in predicting pier scour within the range of the input parameters used in this study. This section discusses the influence of the four parameters (pier width, flow velocity, depth of flow and pier shape), which has a major influence on the depth of pier scour. This is achieved by testing the model created by using the training data in the previous section with a hypothetical test dataset. The hypothetical test dataset is created by varying one input parameter while keeping all other input parameters constant. In this section, the effect of four parameters (pier width, velocity of flow, pier shape and depth of flow) of pier scour is studied using all three modeling approaches.

To study the effect of the pier width, depth of flow and flow velocity on the bridge pier scour, all three parameters were varied by increasing their values by 0.2, while other parameters were kept constant. This way, 10 datasets having different pier width, depth of flow and velocity of flow were created and used for sensitivity analysis while scour value was kept constant. The results plotted in Figures 5–7 suggest that GPR was able to create conditions similar to physical modeling in justifying that pier scour depth increases with increasing pier width, flow velocity and depth of flow (Richardson *et al.* 1993). The results of RVM were similar to GPR except in the case of the effect of flow velocity on scour depth. In spite of improved predictive capabilities on test data, Figures 5–7 suggest that the KELM algorithm was not able to justify its suitability in creating conditions similar to physical models in modeling the pier scour.

A plot of another sensitivity analysis involving the use of piers of different geometry, while keeping other parameters constant and using all three algorithms, are provided in Figure 8. Results from this plot suggest that

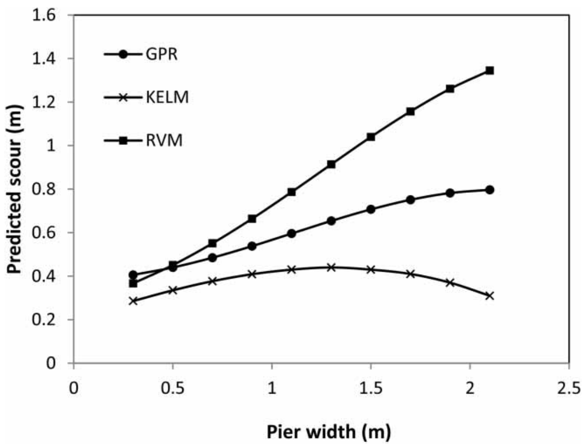

Figure 5 | Variation in scour depth with increasing pier width using all three kernel-based algorithms.

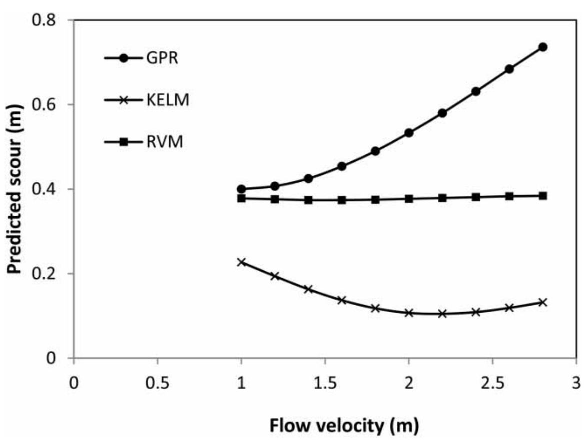

Figure 6 | Variation in scour depth with increasing flow velocity using all three kernel-based algorithms.

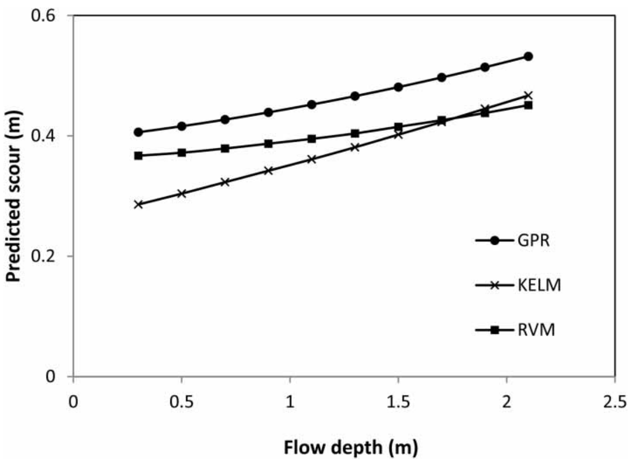

Figure 7 | Variation in scour depth with increasing depth of flow using all three kernel-based algorithms.

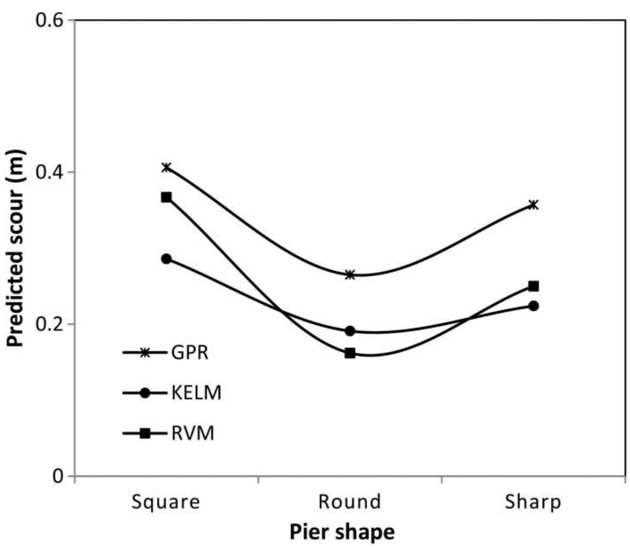

Figure 8 | Variation in scour depth with different type of pier shape using all three kernel-based algorithms.

the use of a circular pier causes minimum scour with all three algorithms, which confirms the results provided by physical models in modeling the pier scour (Richardson et al. 1993).

DISCUSSION

Both GPR and RVM are probabilistic linear models, thus producing probabilistic output instead of point predictions as provided by KELM. RVM can be considered a particular case of GPR models. Being non-sparse and non-parametric, GPR can provide better performance than RVM (Candela 2004), which is justified by the results of this study. Compared to SVM, RVM produces a much sparser solution, requiring only 20 relevance vectors in comparison to 151 support vectors by SVM out of a total of 154 training data to create the model, but may produce local minima because of the use of an expectation maximization based learning approach (Tipping 2001). The choice of kernel function may be an important issue in the design of a kernel-based algorithm when used in water resource engineering. In order to limit the scope of this study, only the RBF kernel function was used. Further studies need to be carried out to judge the suitability of various kernel functions for different datasets.

CONCLUSIONS

This paper investigates the potential of three kernel-based algorithms in predicting the local scour using field dataset. Results suggest encouraging performance by different algorithms. The results can be summarized as follows:

1. The study showed that all three kernel-based methods perform well in predicting pier scour with field dataset and provide improved performance in comparison to RBF based SVR and back-propagation neural network.
2. A comparison of results using dimensional and non-dimensional input parameters suggests a better performance by dimensional input parameters with this dataset.
3. The results of statistical testing indicate comparable performance by all three approaches in predicting pier scour values.
4. Sensitivity analysis suggests that GPR and to some extent RVM were successful in modeling the physical process of scour. The results of this study suggest that increasing the value of flow velocity and pier size causes an increase in scour around the bride piers. Pier scour was also found to increase with increasing flow depth but the variation was not much with increasing value of depth of flow. Results also indicate the unsuitability of KELM in modeling the physical process of scour with the used dataset, thus suggesting that GPR and RVM can effectively be used as a new modeling tool for various applications in water resource engineering in comparison to well established SVM.

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