With the accuracy and repeatability of the profiling system established, the next step has been to investigate the variability of coated abrasive samples and to develop practical and acceptable procedures for collecting and analyzing samples. Much of this work is statistical in nature, requiring the processing of a large number of profiles, and is at the present time only partially completed. So far it has been found that the profile direction has no significant influence on the statistical results. Also, averaging the data from several short profiles of a "loci" area leads to more consistent results than those of single line profiles of the same total length, considering for the moment only those statistics related to the vertical distribution of the profile heights. Borrowing terminology from the field of surface finish measurement, it would appear that the combination of shorter profiles reduces the effects of waviness on the profile statistics.

**Interpretation of Results**

The statistical measures which have been developed for profile analysis are being used to determine the range of variability in particular product lines, for comparative analysis of similar products, as an aid in the development of improved products and to study the correlation between product characteristics and performance. These applications involve the comparison of combinations of measures which are related to specific product characteristics.

Rather than attempt an explanation of any one of these applications here, it seems more appropriate to present examples of a more general type of result.

One statistical measure referred to in the program description was $\mu_n$, the number of positive crossings of a horizontal reference line in a unit length of profile. One would expect this number to be related to grit size. In Fig. 7, experimental values of $\mu_n$ have been plotted for standard coated products in nine grit sizes. The $\mu_n$ values were taken at the mean level of the profiles. The plot shows that over this range of grades there is a very nearly linear relationship between crossings per inch and grade. In the lower section of the same figure the ratio of the calculated maximum number of grains per inch to the actual values has been plotted versus grade. The maximum values were calculated using the nominal grit diameters for these screen grade sizes.

All the products included here were "closed-coat" which means that there was an excess of grain available during coating. Under these conditions it appears that the grain density is predictable and that as the grade gets finer the product becomes more "open-coated." Also, the trend toward open-coatedness is linear with grit size.

In Fig. 8, the variation of $\beta$ with height is shown for three grades of product. $\beta$, again, is the ratio of the length of a horizontal reference line falling within the profile to the total line length. If the abrasive particles were all of the same size and located at the same height, the $\beta$ distribution would represent the average cross section of a single grit. In actuality there is a limited vertical distribution of particles and this accounts for the shape of the far left-hand and far right-hand portions of the curves. The $\beta$ curves show the vertical distribution of possible contact area and, since the increase in contact area accompanying wear of the abrasive particles is one of the major factors determining performance, $\beta$ distributions should prove useful in optimizing coatings.

**Conclusion**

Profile analysis of coated abrasive surfaces has been developed to the point that it provides useful measures of product characteristics. These measures have proved to be of value in the comparative analysis of coated abrasive products and in studies of the relationship of product characteristics to performance.
Perhaps, rather than making inferences about the general distribution of the grits, the cause and effect relationship for a given product should be investigated so that corrective action could be taken on the "production line" if there were an irregularity in the expected values of their statistical parameters.

The discussers question whether the six statistical parameters the authors suggest do indeed insure physical product uniformity for the user. Fig. 11 portrays two hypothetical examples of typical profile segments which cannot be differentiated by using the statistical parameters: mean, μ; variance, σ²; and land to void ratio, β. Intuitively, profiles A and B in Fig. 11 do not represent product uniformity.

Stralkowski, et al., 3 recently completed a preliminary investigation in which a two parameter stochastic model was developed to describe the nature of grinding wheel profiles. The physical characteristics of the wheels, including grit size and hardness, were related to the two parameters of the model and the overall variance of the profile. Different grit wheels were shown to have different values for the two parameters. It is possible that this preliminary approach to characterizing a grinding wheel profile by stochastic models would be a more meaningful and a more efficient technique to insure product uniformity.

H. T. McAdams 4

Mr. Story and Mr. Keyes are to be congratulated for bringing to fruition a practical means for statistical characterization of abrasive profiles. With such a rapid and precise method available for profile measurement, abrasive engineers are in a position to conduct significant theoretical studies of the grinding process as well as to maintain more exact quality control of abrasive products.

There are a number of directions in which profile studies of abrasive surfaces can be extended. First, as noted by the authors, there is a need for relating product characteristics, as measured by profiling, to product performance on the job. Second, there is a need to develop a more complete understanding of the manufacturing parameters affecting surface profiles and of how these parameters can be manipulated to produce a desired profile. Finally, even though profile direction has been found to have no significant influence on statistical results, this result is an artifact resulting from the random manner in which grit particles are applied to the backing sheet in normal manufacturing practice. Controlled spatial arrangement of abrasive particles is a design option which, to be effectively exploited, needs to be examined from the standpoint of particle registry in directions both parallel and transverse to the cutting direction. By virtue of the precision of the authors' instrumentation, a complete three-dimensional description of the abrasive surface is possible by combining a number of closely spaced parallel profiles.

Let us consider the relation between abrasive topography and cutting and performance. Certainly many factors other than surface topography influence the performance of coated abrasives, particularly those mechanical and chemical properties of the grit particles which affect particle fracture and attritive wear. Whether the force to which a grit particle is subjected is sufficient to cause fracture is determined in part by the grain depth of cut. This depth, in turn, is influenced by the separation of cutting particles in the direction of abrasive motion and hence by the surface profile. As wear lands develop on the active abrasive particles, cutting rates decrease unless there is a corresponding increase in normal force; therefore, the effective life

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4 Cornell Aeronautical Laboratory, Inc., Buffalo, N. Y.
history of a coated abrasive is determined, in part, by the rate of growth of wear lands.

Forces on the grit can be deduced from consideration of their normal and tangential components. Normal forces, for a given rate of cut, increase with the area of the wear land and thus would be expected to be related to the $\beta$ function, which indicates the manner in which the dimensions of the wear lands change as the grits wear away. Tangential forces increase with grain depth of cut which, as noted, depends upon the spacings between effective grits in the cutting direction. It is in this connection that the $\mu$ function is equally important. Although the relevance of the $\beta$ and $\mu$ functions to performance is evident, much additional work remains to be done to provide an adequate theory of the relation between abrasive topography and the grinding process.

The response of surface profile to changes in manufacturing parameters has been addressed by the authors in their Figs. 7 and 8. They observe that $\mu$ (as measured at mean height), would be expected to increase with grade number: for a closed coating, the number of particles per inch should increase as particle size decreases. They conclude, also, that if one computes the maximum number of grits per inch and considers the ratio between this quantity and $\mu$, the product tends to be more "open-coated" as grade becomes finer. This behavior is perhaps associated to some extent with particle orientation, but it may also reflect differences in particle shape for the various size grades. For the ratio max/actual to be unity, it must be assumed that the particles are in direct contact with each other and are scanned in a direction consistent with the dimension taken as "size." Particle size statistics are expressions of the physico-geometric properties of particulate systems, and distributions of these statistics depend on both particle geometry and the physical principles employed to sense that geometry. In particular, particle size distributions are induced distributions determined by the scanning procedure employed. Therefore, geometrical and statistical constraints, as well as the definition of "closed-coat," must be given due consideration in interpreting the implications of the observed trend. Similarly, care must be taken in interpreting the $\beta$-curve. The authors assert that "if all particles were of the same size and located at the same height, the $\beta$-distribution would represent the average cross-section of a single grit." It must be noted, however, that none of these grits might assume the "average" shape.

Both the $\beta$ function and $\mu$ function are, in reality, quite non-unique: a given $\beta$ or $\mu$ curve can be synthesized from an infinite variety of particle size and shape distributions. For example, consider Fig. 12. The profile is made up of particles with triangular or quadrilateral cross-sections. However, the profile can be well approximated by rectangular sections as shown, and permutation of the rectangular sections along the base axis would not affect the $\beta$ curve (although it would, of course, affect the $\mu$ curve). Furthermore, if specific orderings of the grits are involved, then the $\beta$ and $\mu$ functions may need to be augmented by other quantities so that this ordering may be taken into account.

In Fig. 13, the grits of a given height distribution are ordered in three different ways; however, the $\beta$ and $\mu$ functions for all orderings are identical. Evidently $\beta$ and $\mu$, computed as mean values, do not present an adequate description of the profile. The statistical distribution of spacings between grits as well as of the lengths of grit cross-sections is important. It is to this point that the work on Markov chains (cited as the authors' reference [2]) is addressed.

The authors have considered profiles of a coated abrasive taken in planes perpendicular to the datum plane and in various directions. Because of the fact that chip removal in grinding is a sequential process, however, it is necessary to consider the registry of grits in a direction transverse to the direction of their motion and to view chip-removal in a three-dimensional frame of reference. Such an approach can be taken by developing elevation...

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contour maps of the abrasive surface, rather than elevation profiles.

The instrumentation described by the authors is well suited to such a mapping approach and has been demonstrated in a recent application to the hypsometry of a 36-grit coated abrasive. The authors provided twelve digitized profiles taken in parallel tracks separated laterally 0.005 in. In each track, elevations were recorded at 0.001 in. intervals (Fig. 14). By means of a computer program developed for a wide variety of contouring applications, contour maps of the abrasive were developed. Employing general vector-space theory and generalized regression analysis, the program develops an equation of the surface in terms of locally defined surface "patches" and can provide plan view contours at selected elevation levels.

These contours, shown in Fig. 15, are of interest in analyzing the chip generation process and in predicting abrasive wear loads. How these contours interact in chip removal is shown in Fig. 16. A chip removed by grit No. 1, for example, leaves on the workpiece a new surface which may be encountered not by a single grit but possibly by several, such as Nos. 3, 4, 6, and 7; the forces experienced by these grits will be similarly affected. In the event of controlled registry, an orderly process of chip generation might be observed and a more nearly uniform distribution of loads might result.

This discussion has attempted to present some of the far-reaching consequences of the approach taken by Mr. Story and Mr. Keynes toward improved characterization of abrasive products. It is to be hoped that they will continue work aimed at exploiting the full potential of the method.

Authors' Closure

The observation by Deutsch and DeVries that the stylus method of profiling is not new was hardly necessary, particularly since several references were made in our paper to the similarities between our own application and that of surface finish measurement. Also, it was, and still is, the opinion of the authors that a full discussion of the limitations of the stylus method of profiling was unnecessary in this paper. We are well aware of these limitations and have cited references in which these aspects of profiling are treated in some detail. Certainly no claim is made that coated abrasive profiles are an exact reproduction of the true profile, only that, with all their limitations, they contain much useful information.

In the second paragraph of their discussion it is stated that the diameter of the stylus is an important consideration, and that regardless of included angle, a minimum diameter is preferable, yet in the next paragraph it is noted that the included angle portion of the stylus should extend above the highest peak. The second observation is correct and it follows that the choice of tip radius and included angle are the important criteria and the final diameter is immaterial. The cutting space cross section shown in their Fig. 49 is, incidentally, that of a grinding wheel, not a coated abrasive. The two are very different and a predominant feature of a coated abrasive cross section is the heavy filleting around the grit particles provided by the size coating. Thus there is much less chance of hang-up of the stylus in profiling coated abrasives than there is in profiling grinding wheel surfaces.

The point is made in both discussions that the statistical parameters used by the authors are insensitive to certain variations in particle distribution which have to do primarily with ordering of the grit particles. Although in the example given by Deutsch and DeVries this is not the case (their Fig. 11) since both $\mu$ and the $\mu$ distance values would be sensitive to this ordering, the point is valid. In this instance, however, it is irrelevant. McAdams has touched on the overriding practical consideration which must be recognized when he notes the "random manner" in which the grit particles are applied to the backing. The crushing, grading, and coating processes used in the manufacture of coated abrasives are all bulk processes in which only certain "average" characteristics can be controlled. Though there are mechanisms present in the coating process which can produce patterns on a gross scale, the probability of specific ordering of the types discussed over a range of more than a few grit diameters is nil.

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