Asymmetries in the Single Smuon Production $e^+ e^- \rightarrow \mu^+ \tilde{\mu}^-$

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We investigate the longitudinal asymmetries, the forward-backward asymmetries and the polarized forward-backward asymmetries in the single production of supersymmetric partner of muon: $e^+ e^- \rightarrow \mu^+ \tilde{\mu}^-$. As the most salient feature, these asymmetries depend on the mass spectrum of the supersymmetric partners not only primarily through the threshold effects but also secondarily through the decay width of zino and the Feynman propagators of exchange type. As the consequence of this feature, it is shown that the asymmetries are of crucial importance in discriminating two typical $N=1$ supergravity GUT models, i.e., the canonical model and the no-scale model and in clarifying the possible breaking mechanism of supersymmetry.

§ 1. Introduction

Supergravity (SUGRA) and superstrings have arrested much attention of high energy theorists as promising candidates for unified theories of all the particle interactions including gravity.\(^1\) Supersymmetry (SUSY) plays an essential role in the modeling at the symmetry limit, i.e., the high energy limit. In the scenario of supersymmetric unification, $N=1$ SUGRA GUT is reduced to the supersymmetric Weinberg-Salam model with global SUSY breaking terms at the low energy mass scale $m_w \approx 10^2$ GeV.\(^2,3\) Supersymmetric Weinberg-Salam models commonly predict several supersymmetric particles, i.e., SUSY partners, with masses of $O(m_w)$. It is therefore of physical urgency to provide phenomenologically suitable method for examining the possible existence of the SUSY partners and affording an active confirmation to SUSY.\(^2,4\) The mass spectrum of these SUSY partners depends on the global SUSY breaking terms. Consequently, it is of importance to set up a reliable machinery in order to determine the mass spectrum of the SUSY partners and to understand the symmetry breaking mechanism of SUSY below the Planck mass scale $M_P \approx 10^{19}$ GeV.

Chiappetta et al. have shown that the asymmetries of the SUSY partner pair production in $e^+ e^-$ collisions are of significance in the supersymmetric phenomenology.\(^5,6\) The $e^+ e^-$ colliding data with longitudinally polarized electron beam are, in fact, soon available at SLC.\(^7\) Since SUSY partners are different in spin, chiral coupling and mass from ordinary correspondents, new kinds of asymmetries are expected in the production of the SUSY partners. In particular, the polarization asymmetries of the scalar muon (smuon) pair production $e^+ e^- \rightarrow \mu^+ \mu^-$ materialize the non-renormalization theorem,\(^5\) which is one of the essential properties of SUSY.\(^8\) The asymmetries for the SUSY partner pair production, however, show rather restricted dependences on the mass spectrum of the SUSY partners only through the Feynman propagators of exchange type and are not very efficient for the purpose of determining the mass spectrum of the SUSY partners.
The single production of SUSY partner in $e^+e^-$ collisions has, on the other hand, arrested active attention of high energy phenomenologists$^{(9)-13)}$ because the process can occur even if the masses of singly produced SUSY partners are larger than the beam energy $\sqrt{s}/2$. In previous papers$^{14)}$ in particular, we have studied the polarization asymmetries as well as the cross sections of the single scalar electron production $e^+e^-\rightarrow e^+\bar{\tau}\bar{e}^-$, and have shown that the asymmetries crucially depend on the mass spectrum of the SUSY partners. In addition$^{14)}$ it has been noted that these asymmetries may play the role of an efficient machinery in distinguishing the $N=1$ SUGRA GUT models.

In this paper the detailed examination of the single smuon production $e^+e^-\rightarrow \mu^+\bar{\tau}\bar{\mu}^-$ is performed. We are primarily concerned with calculating the polarization asymmetries and the forward-backward asymmetries. Since the generation change between initial and final states yields restricted types of the Feynman diagrams, the analysis of $e^+e^-\rightarrow \mu^+\bar{\tau}\bar{\mu}^-$ turns out to be much more efficient than that of $e^+e^-\rightarrow e^+\bar{\tau}\bar{e}^-$ in investigating the sensitive dependence of the asymmetries on the mass spectrum of SUSY partners. It has been argued that the cross sections of $e^+e^-\rightarrow \mu^+\bar{\tau}\bar{\mu}^-$ depend on the masses of particles into which the SUSY partner (zino) of $Z^0$ can decay$^{13)}$. It is then natural to expect that the asymmetries of $e^+e^-\rightarrow \mu^+\bar{\tau}\bar{\mu}^-$ depend not only on these masses but also on the smuon mass $M_\mu$ and the scalar electron mass $M_e$. In order to clarify SUSY breaking mechanism below $M_F$, therefore, it is of importance to investigate the possible dependence of the asymmetries on the mass spectrum of the SUSY partners in $e^+e^-\rightarrow \mu^+\bar{\tau}\bar{\mu}^-$. This is accomplished as follows.

In $\S$ 2 the calculational machinery is set up for the cross sections and the asymmetries of $e^+e^-\rightarrow \mu^+\bar{\tau}\bar{\mu}^-$ under the standard ansatz of the minimal SUSY Weinberg-Salam model. The mass parameters of the SUSY partners are introduced on the basis of two typical $N=1$ SUGRA GUT models, i.e., the canonical model and the no-scale model. In $\S$ 3 numerical results are shown. It is then clarified that the dependence on the mass spectrum of the asymmetries is controlled by the threshold effects, the decay width of zino and the Feynman propagators of exchange type. Finally in $\S$ 4 principal conclusions on the asymmetries of the single smuon production are summarized. In particular, the significance of the polarized forward-backward asymmetries is emphasized. Further discussion is devoted to estimating the background contribution.

**§ 2. Calculational machinery**

In the low energy limit, in general, $N=1$ SUGRA GUTs are reduced to supersymmetric Weinberg-Salam models with soft global SUSY breaking terms. We take the minimal SUSY Weinberg-Salam model$^4)$ hereafter, which is characterized by the superfields listed in Table I and minimal superpotential. As is well known, the soft breaking scalar masses $m_0$, the soft breaking gaugino masses $m_{1/2}$ and the trilinear coupling constants in the soft global SUSY breaking terms depend on the mechanisms of the local SUSY breaking, or the models taken at $M_F$.$^{15)-17)}$
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Table I. Superfields.

<table>
<thead>
<tr>
<th>superfield</th>
<th>boson</th>
<th>fermion</th>
<th>$SU(2)_L$</th>
<th>$U(1)_Y$</th>
</tr>
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<tr>
<td>vector</td>
<td>$V^a$</td>
<td>$\lambda^a$</td>
<td>adj.</td>
<td></td>
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<tr>
<td></td>
<td>$V'$</td>
<td>$\lambda'$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>chiral</td>
<td>$L_a^i$</td>
<td>$\bar{l}_a^i$</td>
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<td></td>
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<tr>
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<td>$\bar{d}_a^i$</td>
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</tr>
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<td></td>
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<td>$d_a$</td>
<td>-3</td>
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<td></td>
<td></td>
<td>$\bar{d}_a^i$</td>
<td>$d_a$</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\bar{h}_a^i$</td>
<td>$h_a^i$</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\bar{h}_a^i$</td>
<td>$h_a^i$</td>
<td>-1</td>
</tr>
</tbody>
</table>

The interaction Lagrangians relevant to our calculations of the amplitudes for $e^+ e^- \rightarrow \mu^+ \bar{\nu} \bar{\mu}^-$ yield

$$\mathcal{L}_{\mu \nu \gamma} = e \gamma_{\mu} l A_{\nu} + \frac{e}{4 \cos \theta_W \sin \theta_W} \bar{\mu} \gamma_{\mu} (1 - \gamma_5 - 4 \sin^2 \theta_W) l Z^\mu ,$$  \hspace{1cm} (1)

$$\mathcal{L}_{\mu \nu \gamma} = ie A_{\mu} (\bar{\mu} \gamma_{\mu} \bar{l}_L + \bar{l}_R \gamma_{\mu} \bar{\mu} \bar{l}_R)$$

$$- \frac{ie}{\cos \theta_W \sin \theta_W} Z_{\mu} \left[ \left( -\frac{1}{2} + \sin^2 \theta_W \right) \bar{\mu} \gamma_{\mu} \bar{l}_L + \sin^2 \theta_W \bar{l}_R \gamma_{\mu} \bar{\mu} \bar{l}_R \right],$$  \hspace{1cm} (2)

and

$$\mathcal{L}_{\mu \nu \gamma} = -\sqrt{2} e \left[ \bar{l} \frac{1}{2} (1 - \gamma_5) \bar{\mu} \bar{l} - \bar{l} \frac{1}{2} (1 + \gamma_5) \bar{\mu} \bar{l} \right]$$

$$- \frac{\sqrt{2} e}{\cos \theta_W \sin \theta_W} \left[ \bar{l} \frac{1}{2} (1 + \gamma_5) \bar{\mu} \bar{l} \left( - \frac{1}{2} + \sin^2 \theta_W \right) \cos \phi \right]$$

$$+ \bar{l} \frac{1}{2} (1 - \gamma_5) \bar{\mu} \bar{l} \left( - \frac{1}{2} + \sin^2 \theta_W \right) ( - i ) \sin \phi$$

$$+ \bar{l} \frac{1}{2} (1 - \gamma_5) \bar{\mu} \bar{l} \left( - \frac{1}{2} + \sin^2 \theta_W \right) ( i ) \sin \phi \right] + \text{h.c.},$$  \hspace{1cm} (3)

where $l$ and $\bar{l}$ read the charged leptons and their SUSY partners, respectively, and $\bar{z}_1[\bar{z}_2]$ denotes the heavy [light] zino, i.e., the mixed state of the SUSY partner of $Z_0$ and the neutral higgsinos $\tilde{h}_1^{0i}$. These mass eigenstates have been analytically obtained as the natural consequence of the assumptions that the SUSY Higgs mass parameter is neglected and that the same value is set to be the two vacuum expectation values of Higgs particles.\textsuperscript{41,20} In what follows, the masses of leptons $l$ as well as quarks $q$ are legitimately neglected in comparison with the masses of the SUSY partners. Feynman diagrams for $e^+ e^- \rightarrow \mu^+ \bar{\nu} \bar{\mu}^-$ are shown in Fig. 1.

We next define by $\epsilon_\lambda^{(2)}$; $\lambda = \pm$ the electron [positron] with helicity $\lambda$. It must be
mentioned that the helicity amplitudes \( \mathcal{M}(e^+ e^- \rightarrow \mu^+ \bar{\mu} L; R) \) automatically vanish
\[
\mathcal{M}(e_+ e^- \rightarrow \mu^+ \bar{\mu} L; R) = \mathcal{M}(e^- e^- \rightarrow \mu^+ \bar{\mu} L; R) = 0,
\]
under the ansatz of \( m_{e^\pm} = 0 \), if and only if incident electrons and positrons have the same helicity. Applying the formula for three-body differential cross sections, we obtain the polarized cross sections of \( e^+ e^- \rightarrow \mu^+ \bar{\mu} \):
\[
\frac{d^4\sigma}{dE_1 dE_2 d\cos\theta d\cos\theta_{\text{F}}^1} = \beta E_1 E_2 \left[ \sum_{\text{spin}} \left| \mathcal{M}(e^+ e^- \rightarrow \mu^+ \bar{\mu} L; R) \right|^2 \right] \left( 4 \eta^2 \sin^2 \theta \sin^2 \theta_{\text{F}}^1 - \eta^2 \right)^{1/2};
\]
\[
\eta = \sqrt{1 - \beta E_1 E_2 + M_{\mu_1; R}^2 - M_{\mu^2}^2 + 2E_1 E_2 (1 - \beta \cos \theta \cos \theta_{\text{F}}^1),}
\]
where \( E_1, E_2 \) and \( \theta_{\text{F}}^1, \theta_{\text{F}}^2 \) mean the energy and the scattering angle of \( \mu^+, \bar{\mu} \), respectively. In the explicit evaluation of the helicity amplitudes, then, the decay widths of \( Z_1, \bar{Z}_1, \bar{Z}_2 \) are naturally introduced. For the detailed forms of the decay widths, we refer to the Appendix.

The asymmetries are defined by integrating the differential cross sections (5) as follows:
\[
i) \text{the longitudinal asymmetries:}
\]
\[
A_{\text{L}} = \frac{\sigma(+) - \sigma(-)}{\sigma(+) + \sigma(-)}; \quad \sigma(\pm) \equiv \sigma(e^+ e^- \rightarrow \mu^+ \bar{\mu} \pm);
\]
\[
ii) \text{the differential longitudinal asymmetries:}
\]
\[
A_{\text{DL}} = \left[ \frac{d\sigma(+) - d\sigma(-)}{d\cos\theta_{\mu} - d\cos\theta_{\mu}} \right] \left[ \frac{d\sigma(+) + d\sigma(-)}{d\cos\theta_{\mu} + d\cos\theta_{\mu}} \right],
\]
\[
iii) \text{the polarized forward-backward asymmetries:}
\]
\[
A_{\text{FB}^\pm} = \frac{\sigma_\theta(\pm) - \sigma_\theta(\pm)}{\sigma_\theta(\pm) + \sigma_\theta(\pm)}; \quad \sigma_\theta(\pm) = \int \int_0^\pi d\sigma^\pm d\cos\theta_{\mu},
\]
and
\[
iv) \text{the forward-backward asymmetries:}
\]}
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\[ A_{FB} = \frac{\sigma_{e^+} - \sigma_{e^-}}{\sigma_{e^+} + \sigma_{e^-}}; \quad \sigma_{e^+} = \frac{1}{2} [\sigma_{e^+; \beta(+)} + \sigma_{e^+; \beta(-)}]. \quad (9) \]

As can evidently be seen from Eq. (4), these asymmetries are fully described in terms of $\sigma(\pm)$ just for the case of the longitudinal polarized electron beam and no additional information on asymmetries is available, in fact, even if both $e^+$ and $e^-$ beams are simultaneously polarized.

Practical estimation of the asymmetries defined above inevitably needs fixing the mass parameters of the SUSY partners. In order to examine whether or not the asymmetries play the role of an efficient machinery in distinguishing $N=1$ SUGRA GUT models, hereafter, two typical models are adopted as follows and the mass parameters are determined by the use of the mass relations attainable in the low energy limit of these models.

One model is the canonical model (CN model) which has the canonical kinetic terms for the scalar fields.\(^{18,22}\) Therefore this model will be most natural so far as the structure of the kinetic terms is concerned. The other is the no-scale model (NS model) which has the no-scale structure in the sense that the weak mass scale $m_w$ is dynamically induced from the Planck mass scale $M_p$.\(^{3}\) As has recently been shown by Witten,\(^{23}\) this model is the effective theory in $d=4$ as a consequence of compactification of the extra dimensions in the $d=10$ $N=1$ SUGRA or the superstring.

The mass relations of SUSY partners at the weak mass scale can be determined by the use of the renormalization group (RG) equations, where $m_0$ and $m_{1/2}$ play the role of the initial parameters at the GUT scale. For the CN model, we employ the RG equations developed by Ibáñez et al., in which $G = SU(5)$.\(^{18}\) For the NS model, on the other hand, we apply the RG equations pioneered by Ellis et al., in which $G = E_6$.\(^{24}\) In order to accommodate with the experimental information, in addition, we can legitimately set up $\alpha_{em}(m_w) = 1/128$, $\sin^2 \theta_w(m_w) = 0.22$ and $m_z = 93 \text{ GeV}$. The mass spectra of SUSY partners which are concerned directly with $e^+ e^- \rightarrow \mu^+ \bar{\nu} \bar{\mu}^-$ then read as follows:

1) the CN model:

\[ M_{\tilde{\tau}} \approx 0.47 m_{1/2}, \quad (10\cdot a) \]
\[ M_{\tilde{t}_u} \approx [m_0^2 + 0.16 m_{1/2}^2]^{1/2}, \quad (10\cdot b) \]
\[ M_{\tilde{t}_d} \approx [m_0^2 + 0.58 m_{1/2}^2]^{1/2} \quad (10\cdot c) \]

and

\[ M_{\tilde{\chi}^+_{1 \pm}} \approx [m_{\chi_0^2} + 0.12 m_{1/2}^2]^{1/2} \pm 0.35 m_{1/2}, \quad (10\cdot d) \]

2) the NS model:

\[ M_{\tilde{\tau}} \approx 0.26 m_{1/2}, \quad (11\cdot a) \]
\[ M_{\tilde{t}_u} \approx 0.94 m_{1/2}, \quad (11\cdot b) \]
\[ M_{\tilde{t}_d} \approx 1.40 m_{1/2} \quad (11\cdot c) \]
and

\[
M_{\tilde{\chi}_1^\pm} \simeq [m_\theta^2 + 0.04 \, m_{1/2}^2]^{1/2} + 0.19 \, m_{1/2} \ .
\]  
(11·d)

Here, the extra \(Z^0\) boson, its SUSY partner, other extra matter fields and their SUSY partners predicted in the \(E_6\) NS model have not been considered\(^{24),33}\), the principal motivation of which will briefly be discussed in § 4. As can evidently be seen from Eqs. (10) and (11), the parameter \(m_0\) is present in the mass relations of the CN model, while is absent in those of the NS model. This guarantees the most salient feature of the NS model that the global SUSY breaking is eventually controlled only through gaugino mass \(m_{1/2}\).\(^{25)-28}\)

There is evidently one and only one arbitrary parameter in the NS model, while there are two in the CN model. Since we are principally concerned with clarifying the difference between the two typical \(N=1\) SUGRA GUT models, contaminative complications in numerical works are to be avoided. Not only from a phenomenological point of view but also from an aesthetic point of view, hereafter, \(M_f\) is considered as a common parameter for the two models. Then \(M_{\tilde{\chi}_1^\pm}\) turn out to be the same in both models, and vice versa. Furthermore, we choose the mass ratio:

\[
\hat{\zeta} = m_{1/2}/m_0 \simeq 0.6 ,
\]  
(12)

as the other input parameter in the CN model. The same value of \(M_{\tilde{\chi}_1}\) in both models is surely guaranteed under the condition (12). As a result, the model difference is reduced to the mass difference of left handed scalar leptons \(M_{\tilde{\chi}_1}\) in the two models. In addition, recent experiments seem to give a bound \(M_{\tilde{\chi}} \gtrsim 20\) GeV.\(^{34}\) Considering Eqs. (11·a) and (11·b) as well as this bound, we put tentatively \(M_f=10\) GeV [mass sets I; III] or \(20\) GeV [mass sets II; IV; V; VI]. All the mass sets which we have taken are shown in Table II. It is free for us to assume that \(M_{\tilde{\ell}_{1,2,3}} = M_{\tilde{\chi}_{1,2}}\) and \(M_{\tilde{\ell}} = 60\) GeV in the mass sets I~IV, which might be at least qualitatively not inconsistent with the presently available mono-jet events at CERN SPS collider experiments.\(^{29}\) We next examine the mass sets V and VI where the constraints on \(M_{\tilde{\ell}}\) and \(M_{\tilde{\chi}}\) are removed. It has, in fact, been suggested that the family symmetry breaking can be formulated in superstring theories\(^{30}\) or in the no-scale SUGRA models,\(^{21}\) and that the \(\tilde{q}\)'s are considerably heavy in the superstring phenomenology.\(^{24),32}\)

### Table II. Mass sets (in GeV).

<table>
<thead>
<tr>
<th>Mass sets</th>
<th>The CN model</th>
<th>The NS model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>I</td>
<td>II</td>
</tr>
<tr>
<td>(M_f)</td>
<td>10</td>
<td>20</td>
</tr>
<tr>
<td>(M_{\tilde{\chi}_1})</td>
<td>36</td>
<td>72</td>
</tr>
<tr>
<td>(M_{\tilde{\chi}_2})</td>
<td>38</td>
<td>77</td>
</tr>
<tr>
<td>(M_{\tilde{\chi}_3})</td>
<td>86</td>
<td>79</td>
</tr>
<tr>
<td>(M_{\tilde{\ell}})</td>
<td>101</td>
<td>109</td>
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<tr>
<td>(M_{\tilde{\ell}_{1,2,3}})</td>
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<td>60</td>
</tr>
<tr>
<td>(M_{\tilde{\ell}})</td>
<td>36</td>
<td>72</td>
</tr>
<tr>
<td>(M_{\tilde{\ell}})</td>
<td>38</td>
<td>77</td>
</tr>
</tbody>
</table>
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between the masses of the $\tilde{l}$'s and $\tilde{q}$'s of different generations have not yet been known, however. In the mass set V we tentatively take $M_{\tilde{\ell}_L} = 0.5 M_{\tilde{\ell}_R}$, and in the mass set VI we take $M_{\tilde{q}} = 120$ GeV, while other masses are left the same as in the set IV. As has already been touched upon in § 1, the asymmetries may depend on the masses of the particles into which $\tilde{z}_{1,2}$ can decay as well as $M_{\tilde{\ell}}$ and $M_{\tilde{\ell}}$. If only it is energetically allowed, the decay of $\tilde{z}_{1,2}$ into $q\bar{q}$ channels is assured to yield dominant contributions to decay widths $I_{\tilde{z}_{1,2}}$ owing to the color factor. For the explicit forms of decay widths, we refer to the Appendix. The dependence on $M_{\tilde{q}}$ of the asymmetries are then studied in comparison with the results for the sets IV and VI.

§ 3. Results

The cross sections and various asymmetries of $e^+e^-\rightarrow \mu^+\bar{\mu}^-$ are systematically depicted in Fig. 2 and Figs. 3~5, respectively. Enhancement of the cross sections and rapid change in the asymmetries are striking at the thresholds which are peculiar to the single smuon production. First of all, the integrated cross sections are not efficient in discriminating the aforementioned $N=1$ SUGRA GUT models, as a consequence of a single resultant mass parameter $M_{\tilde{\ell}}$. Secondly, the asymmetries clarify the difference between the CN model and the NS model, because mass spectra of the SUSY partners are different in the two models.

In Fig. 2, we show the integrated cross sections for the mass sets I~IV. Besides

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![Fig. 2](https://academic.oup.com/ptp/article-abstract/79/1/159/2277477)

Fig. 2. Integrated cross section $\sigma$ versus total energy $\sqrt{s}$ for $e^+e^-\rightarrow \mu^+\bar{\mu}^-$ in the CN model (a) and the NS model (b). Dashed curve and solid curve correspond to the mass sets with $M_{\tilde{\ell}}=10$ GeV and those with $M_{\tilde{\ell}}=20$ GeV, respectively.
Table III. Threshold masses.

<table>
<thead>
<tr>
<th>Threshold masses$^a$</th>
<th>Feynman diagrams$^b$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_f + M_{\bar{\nu}_e\nu_e}$</td>
<td>(A), (B), (C), (D)</td>
</tr>
<tr>
<td>$2M_{\bar{\nu}_e\nu_e}$</td>
<td>(B)</td>
</tr>
<tr>
<td>$M_f + M_{\bar{\nu}_e\nu_e}$</td>
<td>(C), (D)</td>
</tr>
</tbody>
</table>

$^a$ We neglect muon mass $m_\mu$.

$^b$ Each set of diagrams contribute to the threshold effects at the correspondent threshold, $\sqrt{s} =$ threshold mass.

Numerical results of asymmetries are summarized in Figs. 3 and 4 for the mass sets I; III and for II; IV, respectively. The longitudinal asymmetries $A_{LR}$; the forward-backward asymmetries $A_{FB}$; the polarized forward-backward asymmetries $A_{FB}^{(p)}$ and $A_{FB}^{(q)}$ are shown in Figs. 2(a); (b); (c) and (d), respectively. The difference between asymmetries for the mass sets with $M_f=10$ GeV and those with $M_f=20$ GeV is apparent. We are now in a position to discuss the difference between asymmetries for the two models, i.e., the CN model and the NS model. Factorizability yields the remarkable similarity of Fig. 1(a) with the usual muon pair production in the present analysis. As a consequence of its low threshold, Fig. 1(a) naturally plays the role of an overall contribution in comparison with Figs. 1(b)~(c).

Figure 3 shows the asymmetries for the mass sets I and III in which $M_f=10$ GeV. When $\sqrt{s} \geq 2M_{\bar{\nu}_e\nu_e}$, all the asymmetries of $e^+e^- \rightarrow \mu^+\bar{\nu}\mu\bar{\nu}$ reasonably resemble to those of the prominent $Z^0$ peak, sharp rises appear at appropriate $\sqrt{s}$ in all sets, which correspond to thresholds that are related directly or indirectly to $e^+e^- \rightarrow \mu^+\bar{\nu}\mu\bar{\nu}$. These thresholds are systematized in Table III. In the integrated cross sections, however, threshold effects which correspond to $M_f + M_{\bar{\nu}_e\nu_e}$ and $2M_{\bar{\nu}_e\nu_e}$ are not apparent. It is noted that $M_{\bar{\nu}_e\nu_e} < M_{\bar{\nu}_e\nu_e}$.

Fig. 3. Asymmetries $A_{LR}, A_{FB}, A_{FB}^{(p)}$ and $A_{FB}^{(q)}$ as a function of $\sqrt{s}$ in (a), (b), (c) and (d), respectively. Dashed curve and solid curve correspond to the mass sets I and III, respectively.
Asymmetries in the Single Smuon Production \( e^+ e^- \rightarrow \mu^+ \bar{\mu}^- \). As will be envisaged from Ref. 5), this is due to the fact that dominant contributions come from the \( \mu^+ \bar{\mu}^- \) pair production figure 1(b). In fact, \( A_{LR} \approx -0.24 \) at \( \sqrt{s} = m_z \) which is just the value obtained for the longitudinal asymmetry for \( e^+ e^- \rightarrow \mu^+ \bar{\mu}^- \)

\[
A_{LR} = \frac{2a_\mu b_\mu}{(a_\mu^2 + b_\mu^2)^{1/2}} \approx -0.24 ,
\]

where

\[
a_\mu = \frac{1 + 4 \sin^2 \theta_w}{4 \cos \theta_w \sin \theta_w} \tag{14}
\]

and

\[
b_\mu = \frac{-1}{4 \cos \theta_w \sin \theta_w} \tag{15}
\]

For both \( \sqrt{s} \geq 2M_{\bar{\mu}} \) and \( \sqrt{s} = m_z \), the difference in \( A_{LR} \) between the two models is successfully explained in terms of the fact that \( 2M_{\bar{\mu}} < m_z \) in the CN model and \( 2M_{\bar{\mu}} > m_z \) in the NS model. Both \( A_{FB} \) and \( A_{B_\mu} \) are nearly zero in this \( \sqrt{s} \) region, showing the features of the scalar-particle pair production. In lower energies \( \sqrt{s} < 2M_{\bar{\mu}} \), all the asymmetries fully substantiate the difference between the two models. In particular \( A_{FB} \) has opposite signs for the CN model and the NS model over the wide \( \sqrt{s} \) region. These differences are obtained as a typical consequence of the fact that \( M_{\bar{\mu}} + M_{\bar{\mu}} \) in the NS model is larger than that in the CN model.

In Fig. 4 we show the asymmetries for the CN model and the NS model, in which \( M_{\bar{\mu}} = 20 \text{ GeV} \). When \( \sqrt{s} \geq 2M_{\bar{\mu}} \), all the asymmetries naturally behave in resemblance to those of \( e^+ e^- \rightarrow \mu^+ \bar{\mu}^- \). \( A_{LR} \) for the mass set IV simulates that of \( e^+ e^- \rightarrow \mu^+ \bar{\mu}^- \) in the region in which \( \sqrt{s} > m_z \). For the mass set II, on the other hand, \( A_{LR} \) is very similar to that for \( e^+ e^- \rightarrow \mu^+ \bar{\mu}^- = \mu_R^+ \mu_R^- + \mu_L^+ \mu_L^- \) in the case \( \sqrt{s} \geq 2M_{\bar{\mu}} \approx 154 \text{ GeV} \). As can be seen from these arguments, the difference in \( A_{LR} \) between the two models is correctly described in terms of the fact that \( 2M_{\bar{\mu}} > 2M_{\bar{\mu}} \). \( A_{FB} \) and \( A_{B_\mu} \) are nearly zero in the \( \sqrt{s} \) region, in which \( \sqrt{s} \geq 2M_{\bar{\mu}} \) because of the symmetric properties of the scalar-particle pair production. When \( \sqrt{s} < 2M_{\bar{\mu}} \), first of all, the changes of all the asymmetries at threshold \( \sqrt{s} = M_{\bar{\mu}} + M_{\bar{\mu}} \approx 99 \text{ GeV} \) are quite remarkable for both models. This change is caused as a consequence of the fact that contributions from the \( \bar{\tau} \bar{\tau}_2 \) pair production figures 1(c) and (d) to the cross sections become important above this threshold. For the energy region \( M_{\bar{\mu}} + M_{\bar{\mu}} < \sqrt{s} < 2M_{\bar{\mu}} \), the asymmetries depend upon whether or not there exists sufficient contribution from the \( \mu_L \) production and materialize characteristically the difference between the two models. In Fig. 4(a), \( A_{LR} \) for the mass set IV makes a turnover near the threshold \( \sqrt{s} = M_{\bar{\mu}} + M_{\bar{\mu}} \approx 129 \text{ GeV} \), while that for the set II increases monotonically. This is due to the fact that for the set II, the contributions from the \( \mu_L \) production cancel those from the \( \mu_R \) production. In contrast to \( A_{LR} \), in Fig. 4(b), the contributions from the \( \mu_L \) production amplify the turnover effect near the threshold \( \sqrt{s} = M_{\bar{\mu}} + M_{\bar{\mu}} \) in \( A_{FB} \). As regards \( A_{B_\mu} \), in Figs. 4(c) and (d), we find that \( A_{B_\mu} \) is positive and \( A_{B_\mu} \) is negative in the energy region \( M_{\bar{\mu}} + M_{\bar{\mu}} < \sqrt{s} < 2M_{\bar{\mu}} \) for the set IV. This is understood as a consequence of the fact that
only the $\bar{\mu}_R$ production contributes in this energy region and cross sections satisfy the inequalities $\sigma_\phi(-) > \sigma_\phi(+) > \sigma_R(-) > \sigma_R(+)$. For the mass set II, there also exist the contributions from the $\bar{\mu}_L$ production and the cross sections satisfy the inequalities $\sigma_\phi(+) > \sigma_\phi(-) > \sigma_R(-) > \sigma_R(-)$. These contributions are small, however, compared with those from the $\bar{\mu}_R$ production. Taking the totality of the $\bar{\mu}_{R,1}$ productions into account, cross sections satisfy the inequalities $\sigma_\phi(-) > \sigma_\phi(+) > \sigma_R(+) > \sigma_R(+)$, and both $A_{LR}^{(+)}$ and $A_{LR}^{(-)}$ are negative for the set II. Consequently $A_{FB}^{(+)}$ typically exemplify the difference between the CN model and the NS model. It is really remarkable that $A_{FB}^{(+)}$ has different signs in the two models.

We have so far examined the asymmetries for the mass sets I~IV which are characterized by different threshold masses. Let us now turn our attention to investigating the possible dependence of the asymmetries on $M_\phi$ or $M_\tilde{q}$ in the NS model. This dependence is not related to the threshold effects as follows: $M_\phi$ dependence comes through the propagators of exchange type in the lowest order diagrams and $M_\tilde{q}$ dependence comes through the decay widths of $\bar{\tilde{z}}_{1,2}$. Figure 5 shows $A_{LR}^{(+)}$ for the mass set V[VI] in which $M_\phi[M_\tilde{q}]$ is replaced by a different number in the set IV. For the purpose of comparison $A_{LR}^{(+)}$ for the set IV are also shown in Fig. 5. Effects on $A_{LR}^{(+)}$ of taking a smaller $M_\phi$ or a larger $M_\tilde{q}$ are seen in Fig. 5. We find that the contributions from Figs. 1(c) and (d) to the cross sections are substantially increased by this change in $M_\phi$ or $M_\tilde{q}$, above the threshold $\sqrt{s} = M_\gamma + M_\tilde{z}_1$. In Fig. 5(a), we show the changes of $A_{LR}^{(+)}$ caused by the increases of the cross sections when $M_\gamma + M_\tilde{z}_1 < \sqrt{s} < 2M_\mu_\gamma$. As a remarkable contrast to the mass set IV, it is found that the contribution

Fig. 4. Asymmetries $A_{LR}, A_{FB}, A_{LR}^{(+)}$ and $A_{LR}^{(-)}$ as a function of $\sqrt{s}$ in (a), (b), (c) and (d), respectively. Dashed curve and solid curve correspond to the mass sets II and IV, respectively.
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§ 4. Discussion

We have investigated in the present paper the polarized asymmetries and the forward-backward asymmetries of the single smuon production $e^+ e^- \rightarrow \mu^+ \bar{\nu} \mu^-$, and examined the typical $N=1$ SUGRA GUT models, i.e., the CN model and the NS model. The models yield the different mass spectra for the SUSY partners, as an inevitable consequence of difference in the global SUSY breaking mechanism. In the present work, we have taken no account of the extra $Z^0$ boson, its SUSY partner as well as other extra matter fields accompanied by their SUSY partners, the existence of which has been predicted in the $E_6$ NS models. This is based on the fact that the extra $Z^0$ mass has not yet been established even in the superstring phenomenology but rather been conjectured to be sufficiently large in the conventional phenomenology.\footnote{In order to make more detailed analysis on asymmetries of the models, however, these additional particles might eventually need to be taken into account.}

First of all, it has been shown that the asymmetries of $e^+ e^- \rightarrow \mu^+ \bar{\nu} \mu^-$ depend sensitively on the mass spectrum of the SUSY partners through the threshold effects and play the role of an efficient machinery in distinguishing the two models. In particular, it has been emphasized that the polarized forward-backward asymmetries $A_{\text{FP}}$ typically substantiate the difference between the two models in the single smuon production region $\sqrt{s} < 2 M_{\tilde{\mu}}$. In addition, it has been pointed out that the difference is naturally materialized not only in the longitudinal asymmetries $A_{\text{LR}}$ but also in the

![Fig. 5. Differential asymmetries $A_{\text{FP}}$ as a function of $\cos \theta_\mu$ at $\sqrt{s} = 130$ GeV (a) and at $\sqrt{s} = 150$ GeV (b). Solid curve, dash-dotted curve and dotted curve correspond to the mass set IV, V and VI, respectively.](https://academic.oup.com/ptp/article-abstract/79/1/159/2277477)
forward-backward asymmetries $A_{FB}$ in a characteristic way, respectively.

Moreover, the dependence of the asymmetries on $M_{\tilde \ell}$ and $M_{\tilde \mu}$ has been investigated in the NS model. Most typical conclusions are as follows. First, the asymmetries substantially depend on $M_{\tilde \ell}$ through the decay widths $\Gamma_{\tilde \ell_1,2}$ of zinos which decay into $q\bar q$ for the region $\sqrt s > M_{\tilde \tau} + M_{\tilde \mu}$ in so far as $M_{\tilde \tau} + M_{\tilde \mu} < 2M_{\tilde \mu}$. Second, the asymmetries significantly depend on $M_{\tilde \mu}$ through the propagators of exchange type if and only if $M_{\tilde \tau} + M_{\tilde \mu} < \sqrt s < 2M_{\tilde \mu}$.

In order to perform the phenomenologically persuasive analysis, the difference between the two models has been postulated to be successfully described in terms of the mass difference of left handed scalar leptons $M_{\tilde \ell_L}$ in the practical investigation of the asymmetries. This postulate has really been materialized by recognizing $M_{\tilde \tau}$ as a common parameter in the two models and choosing the mass ratio $\xi = m_{1/2}/m_0 \approx 0.6$ as the other input parameter in the CN model. If we assume the soft breaking gaugino mass $m_{1/2}$ as a common parameter in the two models, in fact, we then obtain $M_{\tilde \tau,1,2}$ as well as $M_{\tilde \mu}$ in addition to $M_{\tilde \ell_L}$ as the resultant mass parameters. It may be worth mentioning that $M_{\tilde \ell_L}$ can be set to be common under an appropriate choice of $\xi$. At any rate, therefore, the mass spectra of the SUSY partners will inevitably turn out to be complicated in a phenomenologically opaque fashion. This is the reason why we have postulated $M_{\tilde \tau}$ as the common parameter in the present work.

As can be envisaged from the present analysis of $e^+ e^- \rightarrow \mu^+ \tilde \tau \tilde \mu^-$ as well as our previous analyses of $e^+ e^- \rightarrow e^+ \tilde \tau \tilde e^-$, the dependence of the asymmetries on the mass spectrum of the SUSY partners is substantially materialized in the single slepton production in comparison with the slepton pair production. The principal reason is as follows. With regard to the Feynman propagators of the SUSY partners, there exists annihilation type in addition to exchange type in $e^+ e^- \rightarrow l^+ \tilde \tau \tilde l^-$, while the former is completely absent in $e^+ e^- \rightarrow l^+ \tilde l^-$. The threshold effects intrinsic to $e^+ e^- \rightarrow l^+ \tilde l^-$ are then naturally described in terms of the existence of annihilation propagators of the SUSY partners. This has been exemplified in the present paper. We do not go into details but merely mention the most characteristic features of the smuon pair production $e^+ e^- \rightarrow \mu^+ \mu^-$. Both $A_{FB}^{(g)}$ and $A_{FB}$ inevitably vanish as a typical consequence of the symmetric properties of the scalar particle pair production. On the contrary, both $A_{FB}^{(q)}$ and $A_{FB}$ exist and simulate, to some extent, the threshold effects of $e^+ e^- \rightarrow \mu^+ \tilde \tau \mu^-$ at $\sqrt s = 2M_{\tilde \mu}$ as well as $\sqrt s = 2M_{\tilde \mu}$. Therefore, the detailed analysis of the single smuon production provides us with an indispensable prerequisite to the fruitful investigation of the asymmetries.

Frankly speaking, however, it is worth mentioning that the cross sections of the single smuon production below the pair production threshold $2M_{\tilde \mu}$ read at most $10^{-2}$ pb, which is fairly small in magnitude in comparison with the smuon pair production cross sections. With the tentative luminosity of LEP-I, which reads $L \sim 10^{32}$ cm$^{-2}$ s$^{-1}$, we might naively expect $1 \sim 10$ events of $e^+ e^- \rightarrow \mu^+ \tilde \tau \mu^-$ in three-month run. Appreciable progress of experimental technique is hoped.

So far we have confined ourselves to the asymmetries of $e^+ e^- \rightarrow \mu^+ \tilde \tau \mu^-$. It is a matter of course that the process $e^+ e^- \rightarrow \mu^- \tilde \tau \mu^+$ yields the same final state as $e^+ e^- \rightarrow \mu^+ \tilde \tau \mu^-$ in the sense of two accolinear muons $\mu^\pm$ plus large missing energy for the
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stable photino. From a point of view of the final state detection, therefore, both $e^+e^- \to \mu^+ \bar{\nu}_\mu$ and $e^+e^- \to \mu^- \bar{\nu}_\mu$ are to be consistently taken into account. The integrated cross sections then turn out to be twice in magnitude at least in the single smuon production region. As a consequence of the numerical re-analysis, both $A_{FB}$ and $A_{FB}^{(s)}$ are shown to be at most reduced by a factor two. This bound is evidently brought to realization for the isotropic $\mu^+$ distribution of the smuon $\bar{\nu}_\mu$ decay. As a result, $A_{FB}$ is modified to yield the slightly mild angular dependence. On the other hand, no modification of $A_{L_h}$ is surely guaranteed by the same chiral properties of $L_i$ as those of $L_i$. At least from the qualitative point of view, therefore, the possible dependence of the asymmetries on the mass spectrum of the SUSY partners are not substantially modified.

On the basis of these theoretical and phenomenological observations, let us conclude by emphasizing that the detailed analysis of the single smuon production successfully provides us with a perspective machinery in clarifying the global SUSY breaking mechanism through the possible dependence of the asymmetries on the mass spectrum of the SUSY partners.

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Appendix

In this appendix we summarize the relevant decay widths of $\bar{\nu}_L$, $Z^0$ and $\tilde{z}_{1;2}$ which have been evaluated in the lowest order.

1) Scalar muon:

$$\Gamma_{\bar{\nu}_L \rightarrow \bar{\nu}_L} = \frac{\alpha}{2} M_{\bar{\nu}_L} \left( \frac{M_{\tilde{z}}^2}{M_{\bar{\nu}_L}} \right)^2 .$$

(A·1)

The possible contributions of the decay modes $\bar{\nu}_L \rightarrow \tilde{z}_{1;2} \nu_L$ are small in comparison with $\Gamma(\bar{\nu}_L \rightarrow \bar{\nu}_L)$ and have been neglected.

2) $Z^0$ boson:
\[ \Gamma_{3\gamma} = \Gamma(Z^0 \to f\bar{f}) + \Gamma(Z^0 \to \bar{f}f) + \Gamma(Z^0 \to \bar{z}_3\bar{H}^0), \quad (A\cdot 2) \]

\[ \Gamma(Z^0 \to f\bar{f}) = \frac{a}{6} m_z \frac{m_z}{\cos^2 \theta_w \sin^2 \theta_w} \left[ (T_{3\gamma} - e_r \sin^2 \theta_w)^2 + (-e_r \sin^2 \theta_w)^2 \right], \quad (A\cdot 3) \]

\[ \Gamma(Z^0 \to \bar{f}f) = \frac{a}{12} \frac{m_z}{\cos^2 \theta_w \sin^2 \theta_w} \left[ (T_{3\gamma} - e_r \sin^2 \theta_w)^2 \left( 1 - \frac{4M_T^2}{m_z^2} \right)^{3/2} \right. \]

\[ \left. + (e_r \sin^2 \theta_w)^2 \left( 1 - \frac{4M_T^2}{m_z^2} \right)^{3/2} \right] \quad (A\cdot 4) \]

and

\[ \Gamma(Z^0 \to \bar{z}_3\bar{H}^0) = \frac{a}{12} \frac{m_z}{\cos^2 \theta_w \sin^2 \theta_w} \frac{M_{\bar{z}_1}}{M_{\bar{z}_1} + M_{\bar{z}_2}} \left( 1 - \frac{M_{\bar{z}_1}^2}{M_{\bar{z}_2}^2} \right)^2 \left( 1 + \frac{M_{\bar{z}_2}^2}{2M_{\bar{z}_2}^2} \right). \quad (A\cdot 5) \]

3) Massive neutralinos (zinos):

\[ \Gamma_{\bar{z}_1} = \Gamma(\bar{z}_1 \to f\bar{f}) + \Gamma(\bar{z}_1 \to Z^0\bar{H}^0), \quad (A\cdot 6) \]

\[ \Gamma_{\bar{z}_2} = \Gamma(\bar{z}_2 \to \bar{f}f), \quad (A\cdot 7) \]

\[ \Gamma(\bar{z}_1 \to \bar{f}f) = \frac{a}{12} \frac{M_{\bar{z}_1}}{\cos^2 \theta_w \sin^2 \theta_w} \frac{M_{\bar{z}_1}}{M_{\bar{z}_1} + M_{\bar{z}_2}} \]

\[ \times \left[ (T_{3\gamma} - e_r \sin^2 \theta_w)^2 \left( 1 - \frac{4M_T^2}{m_z^2} \right)^2 + (e_r \sin^2 \theta_w)^2 \left( 1 - \frac{4M_T^2}{m_z^2} \right)^2 \right] \quad (A\cdot 8) \]

and

\[ \Gamma(\bar{z}_1 \to Z^0\bar{H}^0) = \frac{a}{12} \frac{M_{\bar{z}_1}}{\cos^2 \theta_w \sin^2 \theta_w} \frac{M_{\bar{z}_1}}{M_{\bar{z}_1} + M_{\bar{z}_2}} \left( 1 - \frac{M_{\bar{z}_1}^2}{M_{\bar{z}_2}^2} \right)^2 \left( 1 + \frac{M_{\bar{z}_2}^2}{2M_{\bar{z}_2}^2} \right). \quad (A\cdot 9) \]

Here, \( T_{3\gamma} \) is the \( T_3 \) quantum number of \( f_\gamma \) and \( e_r \) is the charge of \( f_\gamma \) in units of \( e \). For the details, we refer to Table I. The helicity amplitudes \( \mathcal{M}^{(i)} \) in \( \S \ 2 \) are calculated by the full use of \( (A\cdot 1) \sim (A\cdot 9) \) under the assumptions of three generations, three colors, \( M_{\bar{z}_1} = M_i, M_{\bar{z}_2} = M_{\bar{z}_2} \) and the same value for the masses of scalar quarks in different kinds of flavours.

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