Thermodynamics of Open Bosonic Strings in Background Fields

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Thermodynamic properties of open bosonic strings in the presence or absence of external electromagnetic background fields are obtained in terms of state densities. According to open bosonic string theories in background fields we must distinguish two types of string, neutral and charged strings. Because of different dynamical features of these strings, thermodynamical properties are very different from each other and we discuss stability of these strings. We also obtain properties of open strings in the absence of the background fields.

§ 1. Introduction

The superstring theory is expected to be successful in unifying all fundamental interactions and matters. Many works have been done in order to study realistic and phenomenological aspects of strings. Since typical energy region of strings is extremely high, it is difficult to compare predictions of the theory with low energy experiments. Any unified theory provides its own scenario on evolution of the universe. If the string theory succeeds in unifying all interactions and matters, it must give its own scenario of the evolution, which is probably quite different from the predictions of the standard model. It is not straightforward to apply the theory to cosmological problems, but we need to formulate a thermodynamical framework based on the string theory. In the present paper we will investigate thermodynamical properties of open strings.

It is well known that strings have exponentially growing degeneracies of mass spectra. Hagedorn first observed that there exists an ultimate temperature in a model with such a mass spectrum in a low energy region. In a last few years the development of string theories stimulated studies on thermodynamics and statistical mechanics of strings. Most of them adopted the same formalism as Hagedorn's, which is known as a string gas model. Alvarez studied canonical ensemble and microcanonical ensemble by using $\sigma$-model. All of them have shown that strings have the same strange properties, such as ultimate temperature and negative specific heat, as already shown by Hagedorn for the dual model. In these works partition functions of canonical ensembles were calculated. One of the authors (H.O.) presented some features of string gas beyond the ultimate temperature by using microcanonical ensemble. Leblanc presented the partition function by the real time method in a framework of quantum field theory of strings. As shown by Polchinski by using the imaginary time formalism in Polyakov string theory, in the lowest order calculations the partition function is the same result as in the string gas model.

In this paper we calculate the state densities and some thermodynamical quantities of open bosonic strings in the presence or absence of background fields directly...
from a quantum theory by using the ideal gas model.\textsuperscript{7)} The present calculation, which is done in a framework of quantum mechanics, is found to be the same results as the string field theory in the lowest order.\textsuperscript{8i,9)} This suggests that the present calculation has sufficient validity. Theories of a string in background electromagnetic fields\textsuperscript{10)} suggest that there are two types of open strings, neutral and charged. It is essential to distinguish these two types of open strings because they behave differently in background fields. The exact definitions of a charged and neutral string are given in § 3. These strings satisfy different boundary conditions and, therefore, Green's functions of these strings are not the same. We expect also that thermodynamical properties may be different for these two different strings. We consider the following four cases:

(1) strings in the absence of background fields,
(2) neutral strings in the presence of background fields,
(3) charged strings in the presence of background fields and
(4) neutral and charged strings in the presence of background fields.

We will be especially interested in comparison of Case (1) with others. We restrict ourselves to open strings, and we need consider only electromagnetic fields but not gravitational fields in the space of higher dimensions as the background fields.

This paper is organized as follows. Section 2 contains the calculation of state densities and some quantities of open strings in the absence of background. In § 3 calculations of open strings in background fields are given. In § 4 discussion and conclusion are presented. In the Appendix we give some definitions and equations to clarify meaning of approximations in statistical mechanics. We perform all calculations in the light-cone gauge.

\section*{§ 2. Thermodynamics of strings in the absence of background fields}

In this section we calculate the state density of one open bosonic string and discuss some thermodynamical properties of the system of open bosonic strings. Thermodynamical variables are energy density for single string and volume of the system. One of the authors (H.O.) also discussed the state densities of several closed strings. In the following we apply the same formalism to this case. The details of the calculation are shown in Ref. 7).

In our method the state density is obtained from the Green's function by integration over spatial momenta. This relation is written as

\begin{equation}
    d(E, V) = \frac{1}{\alpha'} \lim_{\epsilon \to 0} \text{Im} \ 2E \alpha' G_E,
\end{equation}

where

\begin{equation}
    G_E = \text{Tr} \left( \frac{-1}{\alpha' (E^2 - H_0^2 + i\epsilon)} \right).
\end{equation}

Here $H_0$ is non-perturbative Hamiltonian and given by $|p|^2 + (1/\alpha')m$ where $m$ is $\alpha'$(mass)$^2$. $\text{Tr}$ means the sum of all states. The degeneracies of states for $m$ are given by string mass spectra. $G_E$ is written as
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\[ G_\epsilon = \sum_\pi \frac{V}{(2\pi)^D} \int d\Omega d|P||P|^{D-1} \frac{-1}{\alpha'(E^2 - |P|^2 - \frac{1}{\alpha'} m + i\epsilon)}, \]  

(2.3)

where \( D \) is the dimension of the space. After tedious calculation\(^7\) we obtain the following results for \( D=25 \).

\[ d(E, V) = \sum_{r=0}^\infty K 2 E \left( E^2 - \frac{1}{\alpha'} r \right)^{23/2} a_r \Theta \left( E - \sqrt{\frac{1}{\alpha'} r} \right), \]  

(2.4)

where \( K = V/(7.5\times10^{21}) \) and \( V \) is the volume of the system and \( a_r \) are numbers of degeneracies. In the above equation we discarded the tachyon state. The tachyon states are an unavoidable feature of bosonic string theory. This is probably due to inappropriate choice of vacuum or inconsistency of the theory. We are interested in equilibrium states rather than dynamical instability of vacuum. The tachyon state may not affect thermodynamical properties and, therefore, we put away these states. A part of the \( a_r \)'s is listed in Table I. The number of density, given by (A.7) is

\[ \Omega(E, V) = \sum_{r=0}^\infty \frac{2}{25} K \left( E^2 - \frac{1}{\alpha'} r \right)^{25/2} a_r \Theta \left( E - \sqrt{\frac{1}{\alpha'} r} \right). \]  

(2.5)

Notice that Eqs. (2.4) and (2.5) are sum of phase space of particles with various masses. As shown in Ref. 7), we have an approximation of Eq. (2.1),

\[ d(E, V) \approx a_r \quad \text{for large } E, \]  

(2.6)

Table I. The \( a_r \)'s are collected. Each number corresponds to that of degeneracies at \( E = \sqrt{n/\alpha'} \).

<table>
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<th>( n )</th>
<th>( a_r )</th>
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<td>1.431840\times10^8</td>
</tr>
<tr>
<td>10</td>
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</table>

Fig. 1. Temperature for open bosonic strings is shown. \( E \) is energy for one string.
where "a" is changed to a quantity with continuous parameter \( r = a' \). By using an asymptotic form of the number of partitions as in Refs. 2)~6), \( a_r \) is rewritten as\(^{11}\)

\[
d(E, V) \approx d_0 (E \sqrt{a'})^{-a} e^{bE \sqrt{a'}} \tag{2.7}
\]

where \( d_0 \) is a normalization factor and \( a = 26, b \sqrt{a'} = 2\sqrt{2\pi} \) when we have set \( a' = 1/2. \) This approximate result is equivalent to the calculations of Refs. 2)~6). An approximation of \( \Omega^{(1)}(E, V) \) is obtained from Eq. (2.7) by integrating over \( E, \) but it diverges at \( E = 0. \) A lower cutoff must be introduced into integrations to avoid this divergence. In the early works this cutoff was explained as a lower bound of applicability of the approximation. Our method does not need the cutoff. As discussed in Ref. 7), Eq. (2.7) is not a good approximation in the low energy regions but our calculations are valid in all regions within the ability of the model.

Using Eqs. (2.8) and (2.4), temperature is obtained and shown in Fig. 1. In the same way, temperature is given by Eqs. (2.8) and (2.7) in the asymptotic region as

\[
\frac{1}{T} \approx b - \frac{a}{E} \tag{2.8}
\]

where \( E \) is the energy for one string and \( 1/b \) is called ultimate or critical temperature because it is a value of temperature, \( T, \) when \( E \to \infty. \) The coefficient "\( a \)" is positive and the temperature goes toward the ultimate temperature from higher temperature. In contrast to the approximate results our exact one in Fig. 1 shows that temperature goes toward the ultimate temperature from lower temperature. In the region where \( E > 5 \) the temperature looks like a constant via energy, but according to numerical calculations, in this region the temperature is increasing monotonically and extremely slowly nearer toward the ultimate temperature, \( 1/(2\sqrt{2\pi}). \)

In a case of closed strings temperature is not a good thermodynamical variable, because the most probable state of a system cannot be specified by the temperature.\(^7\) This strange situation is due to the fact that the energy of a closed string is a multivalued function of temperature due to the definition (A·1). If we adopt the definition (A·1), which is an ordinary definition of temperature in thermodynamics, there exist states with higher temperatures than ultimate temperatures.\(^7\) In contrast to close strings the system of open bosonic strings looks like an ordinary thermodynamical system. The temperature is a good thermodynamical variable to distinguish states of a system. Any state with higher temperatures than ultimate temperature does not exist. This result is consistent with a case of open strings of Ref. 5) in which the author adopted a canonical ensemble analysis.

§ 3. Thermodynamics of strings in the presence of background fields

We calculate state densities in the same way as in the last section. Firstly we consider the equations of motion and boundary conditions of charged and neutral strings to obtain Green's functions or 0-th component of Virasoro algebra.\(^{10}\) The dimension of space-time is 26 throughout.

An action of the open bosonic string in background fields is given as
where
\begin{equation}
S_0 = -\frac{1}{2\pi\alpha'} \int d^2\sigma \frac{1}{2} \partial_\sigma X^\mu \partial^\sigma X_\mu \tag{3.2}
\end{equation}
and
\begin{equation}
S_1 = -\frac{1}{2\pi\alpha'} \int dt g_\alpha A_\mu(X) \delta(\sigma) X^\mu
- \frac{1}{2\pi\alpha'} \int dt g_\pi A_\mu(X) \delta(\sigma-\pi) X^\mu, \tag{3.3}
\end{equation}
where \(A_\alpha\) is an external vector field and, \(g_\alpha\) and \(g_\pi\) are coupling constants at each end of a string. The equations of motion for \(X_\mu\) and external field are
\begin{equation}
\left(-\frac{\partial^2}{\partial \tau^2} + \frac{\partial^2}{\partial \sigma^2}\right) X^\mu = 0 \tag{3.4}
\end{equation}
and
\begin{equation}
\partial_\sigma X^\mu - F^\mu_{\nu\sigma} \partial_\tau X^\nu = 0 \quad \text{at } \sigma = 0 \text{ and } \pi, \tag{3.5}
\end{equation}
where \(F^\mu_{\nu\sigma} = (\partial A_{\nu}/\partial X^\sigma) - (\partial A_{\sigma}/\partial X^\nu)\), and \(g_\alpha\) and \(g_\pi\) are included in \(A_\mu\).

According to superstring theories all interactions and matters are made out of strings. We adopt this idea and expect that classical background fields are made from classical part of \(X_\mu\). \(^{10}\) \(X_\mu\) may be decomposed into the classical and quantum parts as \(X_\mu + \xi_\mu\) where \(X_\mu\) is a classical part and \(\xi_\mu\) is a quantum one. \(X_\mu\) and \(A_\mu\) satisfy classical equations of motion, Eqs. (3.4) and (3.5). Following the above assumptions we can obtain an action as
\begin{equation}
S(X + \xi) = S_0(X) + S_1(X) - \frac{1}{2\pi\alpha'} \int d^2\sigma \frac{1}{2} \partial_\sigma \xi^\mu \partial^\sigma \xi_\mu
- \frac{1}{2\pi\alpha'} \int dt \left[ \frac{1}{2} \frac{\partial}{\partial X^\nu} F_{\mu\nu}(X) \xi_\nu \xi^\nu \partial_\tau X^\mu + \frac{1}{2} F_{\mu\nu}(X) \xi_\nu \partial_\tau \xi^\mu
+ \frac{1}{6} \frac{\partial}{\partial X^\nu} F_{\mu\nu}(X) \xi_\nu \xi^\nu \partial_\tau \xi^\mu + \cdots \right]_{\sigma=0,\pi}, \tag{3.6}
\end{equation}
where we have assumed the field \(F_{\mu\nu}\) is smooth enough. The equations of motion for \(\xi_\mu\) are
\begin{equation}
\left(-\frac{\partial^2}{\partial \tau^2} + \frac{\partial^2}{\partial \sigma^2}\right) \xi^\mu = 0 \tag{3.7}
\end{equation}
and
\begin{equation}
\partial_\sigma \xi^\mu - F^\mu_{\nu\sigma} \partial_\tau \xi^\nu + g_\sigma \frac{\partial}{\partial X^\tau} F^\nu_{\phantom{\nu}\sigma\nu} \partial_\tau X^\nu \xi^\lambda + \cdots = 0 \quad \text{at } \sigma = 0, \pi, \tag{3.8}
\end{equation}
where \(F^\mu_{\nu\sigma}\) depends only on \(X_\mu\) and hereafter we shall write the coupling constants explicitly. In the following we set \(F^\mu_{\nu\sigma}\) to be a constant because the equations with
constant background fields are solved exactly,\(^{10}\) and the constant background fields may be a good approximation in the equilibrium system. \(F_{\mu\nu}\) is an antisymmetric real tensor. Some elements of \(F_{\mu\nu}\) can be absorbed into wave functions \(X_\mu\) by orthogonal transformations. The absorbed elements have no physical meaning. After orthogonal transformations \(F_{\mu\nu}\) is written as

\[
F_{\mu\nu} = \begin{pmatrix}
0 & -f_0 \\
f_0 & 0 \\
0 & -f_1 \\
f_1 & 0 \\
\end{pmatrix},
\]

(3.9)

where all \(f\)'s are real. In Ref. 10 it was claimed that open strings in the presence of strong electric fields are unstable. We are discussing a problem of a stable system, so we set \(f_0=0\). Namely we consider the system in magnetic fields.

In order to solve the equations of motion we choose a special coordinate\(^{10}\) as

\[
\frac{1}{\sqrt{2}}(\xi_0 + \xi_1) = \xi_0^+,
\]

\[
\frac{1}{\sqrt{2}}(\xi_0 - \xi_1) = \xi_0^- , \quad \text{for 0-th and 1st components}
\]

(3.10)

and

\[
\frac{1}{\sqrt{2}}(\xi_j + i\xi_{j+1}) = \xi_{m+},
\]

\[
\frac{1}{\sqrt{2}}(\xi_j - i\xi_{j+1}) = \xi_{m-} , \quad \text{for } j=2m \text{ where } m \text{ is a positive integer}.
\]

(3.11)

By using these coordinate variables, the equations of motion are split into equations for each 0 and \(m\) component with 2 dimensions. Because of \(f_0=0\), \(\xi_{0\pm}\) is given in light-cone gauge, as usual.

Equation (3.7) gives a normal mode expansion and the coefficients are determined by Eq. (3.8). Due to the constancy of background fields, Eq. (3.8) is rewritten as

\[
\partial_\sigma \xi^\mu - F^{\mu}_{\nu} \partial_\nu \xi^\nu + g_{\sigma \rho} \frac{\partial}{\partial X^\rho} F^\nu_{\rho} \partial_\tau X^\nu \xi^\lambda = 0 ,
\]

(3.12)

and reduces to the following equations,

\[
\frac{\partial}{\partial \sigma} \xi^+ + ig_0 f \frac{\partial}{\partial \tau} \xi^+ = 0 ,
\]

\[
\frac{\partial}{\partial \sigma} \xi^- - ig_0 f \frac{\partial}{\partial \tau} \xi^- = 0 \quad \text{for } \sigma = 0
\]

(3.13)
and
\[ \frac{\partial}{\partial \alpha} \xi^+ - ig_\sigma f \frac{\partial}{\partial \alpha} \xi^+ = 0, \]
\[ \frac{\partial}{\partial \alpha} \xi^- + ig_\sigma f \frac{\partial}{\partial \alpha} \xi^- = 0 \quad \text{for } \sigma = \pi, \]

where the subscripts, \( m \), of \( f \) and \( \xi \) are omitted because sets of equations for each component have the identical form and do not affect the other components. We give a definition of \( \tau \) independent inner product as
\[ (\Psi_m, \Psi_n) = \frac{1}{\pi} \int_0^\pi [i \Psi_m \Psi_n^* - g_\sigma f \Psi_m \Psi_n \delta(\sigma) + g_\alpha f \Psi_m \Psi_n \delta(\sigma - \pi)], \]

where the \( \Psi_n \)'s are solutions of the equations of motion of \( \xi \) and \( \Psi_n^* \partial_\tau \Psi_n = \Psi_n^* \partial_\tau \Psi_n - \partial_\tau \Psi_n^* \Psi_n \). The \( \Psi_n \)'s are written as
\[ \Psi_n = \frac{1}{|n-\epsilon|} \cos((n-\epsilon)\sigma + \gamma) \exp[i(n+\epsilon)\tau] \quad \text{for } \xi^+, \]

and \( \Psi_n^* \) for \( \xi^- \), where \( \tan \gamma_0 = -\tan \gamma = g_\sigma f, \tan \gamma_\pi = -g_\alpha f \) and \( \epsilon = \gamma_0 + \gamma_\pi \). Notice that if \( g_\sigma = -g_\sigma, \epsilon = 0 \), and we call strings with this boundary conditions neutral strings and the others charged strings. In addition to these functions, there exist 0-mode functions as
\[ \Psi_0 = \xi_0^+ = \text{const} \quad \text{for charged strings} \]
and
\[ \Psi_0 = \xi_0^+ + \left[ \tau - g_\sigma f \left( \sigma - \frac{1}{2} \pi \right) \right] P^- \quad \text{for neutral strings}, \]

where \( g_\sigma = -g_\sigma \) for neutral strings. The 0-mode of \( \xi^- \) is \( \Psi_n^* \). \( P^- \) corresponds to the momentum and in the case of charged strings there exists no term corresponding to the momentum. By using the above function \( \Psi_n \)'s, \( \xi^+ \) and \( \xi^- \) are written as
\[ \xi^+ = \xi_0^+ + i \sqrt{2 \alpha} \left[ \sum_{n=1}^\infty a_n \Psi_n(\tau, \sigma) - \sum_{n=1}^\infty b_n^+ \Psi_n(\tau, \sigma) \right] \]
(3.17)

for the charged string, and
\[ \xi^+ = \xi_0^+ + 2 \alpha \left[ \tau - g_\sigma f \left( \sigma - \frac{1}{2} \pi \right) \right] P^- + i \sqrt{2 \alpha} \sum_{n=1}^\infty a_n \Psi_n(\tau, \sigma) - b_n^+ \Psi_n(\tau, \sigma) \]
(3.18)

for the neutral string where \( a \) and \( b^+ \) are creation and annihilation operators. \( \xi^- \) is written as complex conjugate of the above expansions. By using the above expansions and the mass-shell condition, we obtain the 0-th component of Virasoro algebra as
\[ L_0 = -P_0^2 \partial' + P_1^2 \partial' + \sum_{n=1}^{24-2d} \left[ P_i^2 \partial' + \sum_{n} nC_n C_n' \right] \]
for charged strings, and
\[
L_0 = -P_0^2 \alpha' + P_1^2 \alpha' + \sum_{t=1}^{2d-2d} \left[ P_t^2 \alpha' + \sum_{n=1}^{\infty} n C_{n}^{i+} C_{n}^{i} \right]
\]
\[
+ \sum_{i=1}^{2d} \frac{P_i^2}{1 + g_j^2 f_j^2} \alpha' + \sum_{i=1}^{2d} \sum_{n=1}^{\infty} \left[ n(a_n^{i+} a_n^{i} + b_n^{i+} b_n^{i}) \right]
\]
for neutral strings, where \( d \) is the number of non-zero 2x2 matrices in the stress-tensor in expression (3·9) or 2d is the number of dimensions of the background fields. \( C \) and \( C^+ \) are creation and annihilation operators in the absence of background fields. \( g_j \) is a coupling constant for background field in \( j \)-th dimension.

We calculate state densities of strings in the presence or absence of background fields by using the method as shown in § 2. Green's functions are very similar to that used in § 2 except for number of dimension of momentum for charged strings and the overall factor, \( 1/(1 + g^2 f^2) \), of momentum for neutral strings. These differences are reflected state densities. In Eqs. (2·1) and (2·2) \( \text{Tr } G_{E} \) corresponds to \( 1/(L_0 - 1) \) and \(-1\) is caused by ambiguity of normal ordering. In the same way as that in § 3 we obtain the following states densities,

\[
d(E) = K' \sum_{n=1}^{\infty} A_n 2E(E^2 - (\text{mass})^2)^{(25 - 2d)/2 - 1}
\]
for charged strings, where \( K' \) is a phase volume in \( 25 - 2d \) dimensions and \( A_n \), number of degeneracies, and \( (\text{mass})^2 \) are given by the following mass operator,

\[
a'(\text{mass})^2 = \sum_{i=1}^{2d} n C_{n}^{i+} C_{n}^{i} + \sum_{i=1}^{d} \left[ \epsilon_i b_0^{i+} b_0^{i} + (n - \epsilon_i) b_n^{i+} b_n^{i} + (n - \epsilon_i) a_n^{i+} a_n^{i} + \frac{1}{2} \epsilon_i (1 - \epsilon_i) \right] - 1
\]

and

\[
d(E) = \prod (1 + (g_j^2 f_j^2)) K \sum_{n=1}^{\infty} A_n 2E(E^2 - \frac{n}{\alpha'})^{3/2}
\]
for neutral strings where \( A_n \) is the number of degeneracies and equal to \( a_n \) in § 2. The above two state densities contain tachyon states. These results are consistent with the partition function in Ref. 10).

We impose different boundary conditions on a neutral string and charged string and, therefore, we expect that states of these two types of strings belong to different Hilbert spaces. According to string theories,\(^{10}\) it is allowed that there exists an interaction between charged and neutral strings and, therefore, types of strings in the final state can be different from that of strings in the initial state. In order to examine stability of a system of strings phase volume of strings is important. The phase volume of neutral strings is larger than that of charged strings because of lack of some components of momentum and, therefore, neutral strings are dominant.
From the above reasons we expect that all charged strings change into neutral strings by the interaction and any system of strings comes to consist of only neutral strings finally. In the low energy region where $E < 1$, charged strings are expected to be dominant. We cannot say that our method is valid in this region, because in this region only the lowest mass modes appear and the ordinary field theories are valid.

For neutral strings, temperature is given by the same form as that of strings in the absence of background fields. Temperature of charged strings, however, has very different behaviors as

$$\frac{1}{T} = \frac{d(E)}{\Omega(E)}$$

$$= \frac{\sum_n A_n E (E^2 + \text{mass}^2)^{(2-2d)/2}}{\sum_n A_n (E^2 + \text{mass}^2)^{(25-2d)/2}} (25-2d),$$

where mass$^2$ depends on $n$. This expression is very complicated and, therefore, we appeal to the approximate result by exponentiation of $A_n$.\(^{11}\) A leading term of $A$ is the partition due to the first term in Eq. (4·22) and that is written as

$$d(E) \approx \exp \left[ 2\pi \sqrt{\frac{2A-2d}{12}} - E \right].$$

From the above equation we notice that the ultimate temperature becomes higher due to background fields. This is caused by reduction of the number of momenta. Roughly speaking, due to the background magnetic fields motion of a string is allowed in a region perpendicular to the background magnetic field. The energy is distributed to the momenta of allowed directions of motion, so temperature of charged strings is larger in all energy regions.

Magnetization, $M$, of a string is given by

$$\frac{\partial \ln \sigma(E, f)}{\partial f} = \frac{M}{T} = \frac{-1}{\Omega(E, f)} \left[ \sum_n A_n K (E^2 + \text{mass}^2)^{(23-2d)/2} \right]$$

$$\times \sum_{i=1}^{d} (\sin \gamma_o + \sin \gamma_w) \{ b_{a^i} b^i_0 + \sum_n (-a_n a_0^* + b_n b_0^*) \}.$$\(^{(3·26)}\)

We notice that this equation is written as a sum of magnetization of free particles and when $\gamma_0 = - \gamma_w$, $M$ vanishes. This condition is the same as the neutral string condition, but on these two strings completely different boundary conditions are imposed. According to the above discussion we expect that in the presence of background fields neutral strings are dominant in a system of strings.

§ 4. Discussion and conclusion

In § 2 we have shown that the ultimate temperature of a system consisting of open bosonic strings is reached in a very low energy region which is much lower than the asymptotic region. This makes sharp contrast to Refs. 3), 4) and 6) in which it was predicted to be reached in the asymptotic region. In contrast to many predic-
tions in early works, the temperatures of such a system as considered in § 3, increase monotonically to reach the ultimate temperature as the average energy of the strings. These results together with Ref. 7) suggest that the approximation of exponentiation of the degeneracies\(^3\),\(^6\) may not work, because the behavior of temperature is determined by the next to leading order, as shown in Eqs. (3·7) and (3·8). We conclude that only the value of ultimate temperature is correct when this approximation is used, but that the way of approaches.

The number of dimensions of momentum directly affects the calculations of the phase space of a system of open strings in the presence of background fields, so the phase volume of charged strings is smaller than that of neutral strings in a high energy region. Usually background fields appearing in string theories are also constructed by strings and, therefore, we can discuss the stability of the system by studying the stability of background fields. Equation (3·26) suggests that a system of neutral strings is stabler than a system of charged strings. If charged and neutral strings coexist initially in a system, the system is unstable and neutral strings are dominant as the system becomes stable. For neutral strings the number of dimensions of momentum is equal to that of strings in the absence of background fields, and the effects coming from background fields appear as an overall factor, \(1 + g^2 f^2\). This factor is exactly equivalent to what was obtained in Refs. 5) and 10) for canonical ensemble. We conclude that the canonical ensemble analysis is contained in our state density formalism.

In § 4 the 0-th and 1st components of background fields have been assumed to vanish in order to eliminate possible unstable states. This assumption allowed us to take the light-cone gauge. Without this assumption, we could not have performed these calculations. This means also that we cannot treat strings in electric background fields in the present formalism. This is connected to the quantization problem.

Appendix

We consider a system of energy \(E\) enclosed in a volume \(V\), consisting of strings and external fields \(F\)\(^7\). State density \(\sigma\) depends on \(E\), \(V\) and \(F\). The temperature \(T\) and pressure \(P\) are given by the expressions,

\[
\frac{\partial \ln \sigma(E, V, F)}{\partial E} = \frac{1}{T}
\]

and

\[
\frac{\partial \ln \sigma(E, V, F)}{\partial V} = \frac{P}{T}.
\]

In quantum theory state density can be written as

\[
\sigma(E, V, F) = \text{Tr} \delta(E - H).
\]

In the following we use some approximations, which are standard in statistical mechanics. One of them is ideal gas approximation where elements of gas have the
same mass spectrum as that of strings. This is known as the string gas model. A principle of statistical mechanics is that every substate of the system has equal probability to be occupied. Hence the most probable state is the one which occupies the largest phase volume, so the contribution from it dominates the behavior of the state density. Assuming that a state density for the system is written as a sum of state densities for \(n\) strings state, \(\sigma(n, E, V, F)\), it can be approximated by \(\sigma(n, E, V, F)\) of the most probable state. Hereafter we calculate \(\sigma(n, E, V, F)\) instead of \(\sigma(E, V, F)\) and consider that \(n, E, F\) and \(V\) are independent thermodynamical variables. We define the number of state by

\[
\Omega(E, V, F) = \int_0^E \sigma(x, V, F) dx .
\]  
(A·4)

For the ideal gas \(\Omega(E, V, F)\) is written by

\[
\Omega(E, V, F) = \frac{1}{n!} \prod_{i=1}^n \Omega_i(E_i, V, F) .
\]  
(A·5)

The dominant contribution to \(\Omega(E, V, F)\) comes from the region where \(E_i\) are all of order \(E/n\). This provides the following estimate,

\[
\Omega(E, V, F) = \frac{1}{n!} \left[ \Omega^{(1)} \left( \frac{E}{n}, V, F \right) \right]^n ,
\]  
(A·6)

where \(\Omega^{(1)}\) is number of density for one string and given by

\[
\Omega^{(1)}(E, V, F) = \int_0^E d(x, V, F) dx ,
\]  
(A·7)

where \(d(E, V, F)\) is a state density for one string state. By using the above quantities temperature is given by

\[
\frac{1}{T} \approx \frac{d \left( \frac{E}{n}, V, F \right)}{\Omega^{(1)} \left( \frac{E}{n}, V \right)} ,
\]  
(A·8)

where higher order terms in \(1/n\) are neglected. Other thermodynamical quantities are given in the same way.

References

2) J. Ellis, talk given at Second Nobel Symp. on Elementary Particle Physics (Marstrand, Sweden), Preprint CERN-TH.4474/86 (1986); lectures given at Lake Louise Conf. (Canada), Preprint CERN-TH. 4391/86 (1986).
4) R. Hagedorn, in Cargése Lectures in Physics (Gordon and Breach, 1973), vol. 6 and references therein.