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A Modified Sediment Transport Model for Natural Streams

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In an earlier investigation of the behaviour of tracer particles for determination of bed load transport in an alluvial stream (Thomsen 1980), specific records were taken of the particle velocities in the upper bed layer. These data aroused the interest for more detailed investigations.

The result of the measured surface particle velocities with radioactive tracers, performed in five locatities with different hydraulic conditions in natural alluvial rivers, has been used for determination of the relation U_G/U_f' vs. $\sqrt{\theta'/\theta_0}$.

The obtained results have been inserted in the parameters of Engelund and Fredsøe's sediment transport model (1976) and compared with experimental data (Guy et al. 1966).

Some reservations and methods for possible improvements of the sediment transport model are finally discussed.

Introduction

During the last few decades it has become of increasing interest to get more knowledge of sediment transport in rivers from geological, engineering, and pollution viewpoints. In Denmark especially the latter has been focussed upon, because many rivers are flowing through fertile farmland.

The volume of total sediment transport, i.e. suspended and bed load, can be measured either directly or determined theoretically. Besides being very time-consuming, measurements in the field may impose many technical problems, however, so sediment transport values are often found by means of semi-empirical formulae supported by foregoing detailed studies of the involved parameters.

Investigations of particle motion in flowing water were carried out by a.o. Einstein (1950) and Bagnold (1954). Their theories proved to be fruitful and have been taken up and further elaborated into several formulae and models on sediment transport, for example by Engelund and Fredsøe (1976), who, in their model, determine bed load transport on the basis of experiments carried out by Fernandez Luque et al. (1976) and Meland and Norrman (1966) on motion and transport velocities of single particles moving along the bed.

Determination of surface particle transport velocities in relation to the time-averaged bed shear stress has often been based on experiments in flumes of shallow depth and without bed-forms. The critical bed shear stress determined under such conditions will be lower, however, than when determined at same velocity and depth in channels with bed-forms, Rathbun and Goswani (1966). This is due to the fact that a greater bed shear stress causes bed-forms to delay the flow which will therefore need a steeper gradient to obtain same mean velocity as in tests without bed-forms, depth being the same.

It has been found, however, that the gradient and the velocity, which are necessary to move the surface particles in the topmost layer of a ripple/dune bed, may be less than those demanded to move grains over a flat bottom, because the bed-forms cause a flow separation and heavily fluctuating bed shear stresses, Raudkivi (1966). These are large enough to move grains also at lower velocities than those necessary for moving sediment particles over a flat bottom, Middleton (1978).

This knowledge was leading to the presumption that a determination of the velocity of surface particles in relation to the time-averaged bed shear stress in rivers with natural bed-forms might result in another function relation than found in the laboratory.

In previous measurements of velocities for radioactive sediment particles in the topmost layer of bed-forms in a natural stream, these were found to be lower than those found in laboratory tests with same bed shear stress, Thomsen (1980).

Objectives

For the investigation the following objectives were set up:

A) Measurements of migration velocities for grains under different hydraulic conditions in natural alluvial rivers in order to determine the relation U_G/U_f' vs. $\sqrt{\theta'/\theta_0}$.

B) Comparison between the Engelund and Fredsøe sediment transport model including the result of A and the Fort Collins data, such as described by Guy et al. (1966).

C) Estimation of the usefulness of the model, possible reservations and suggestions of improvements.

Methods and Documentation

General

The investigations were carried out in some alluvial rivers in Western Jutland. These were chosen because they offered good opportunities to make single measurements in rivers of suitably varying dimensions, and on the basis hereof to find how different hydraulic conditions may influence transport rates of sediments moving in the upper bed layer.

According to the considerations outlined above, the following test areas were selected:

1) The river Grindsted Å, about 12 km downstream the town Grindsted.
2) The river Brede Å at Bredebro.
3) The Karlsgårde Kanal, about 1 km downstream the lake Ansager Sø.
4) The river Skjern Å at the bridge Hedeby Bro at the town Skjern.
5) The river Ansager Å, 3 km upstream the town of Ansager (Thomsen 1980).

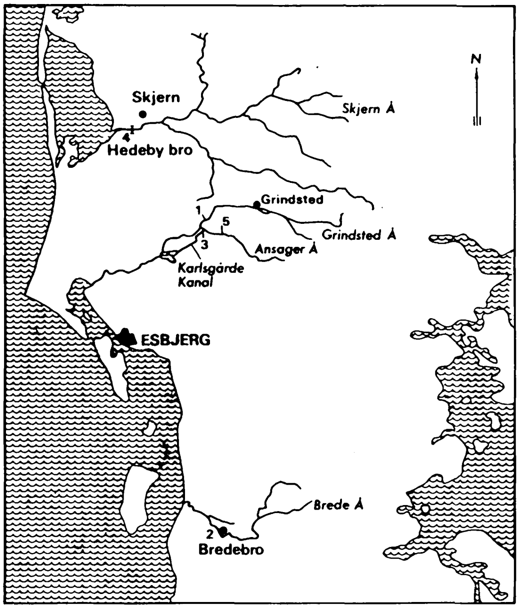

Fig. 1. Survey map showing the location of test areas.

By means of a »Kajak«-sampler, bottom sediments were first taken in the test areas and sieved with 0.50 ф intervals in order to determine the distribution of grain sizes in the bed load.

Tracer information

The sediment particles to be applied as tracers must be of a size to ensure that, under the given hydraulic conditions, they will be movable along the bottom. The critical hydraulic parameters to fulfill this criterion were calculated from previous measurements made at the same places by B. Hasholt (1972, 1977), and personal communication, Shields' criterion for motion (1936), and Engelund and Fredsøe's criteria for suspension of sediment particles (1976). In Fig. 2 the distribution of grain sizes is indicated for each test plot together with the grain fraction which has been used as tracer.

For each test plot natural sand from the river bottom has been collected, fractioned, and the selected fraction labelled with Au-198 at the Isotope Centre, Copenhagen, after a process developed by Petersen (1960).

Methodology

The following general remarks on the methodology are due for each test plot.

A 20-m long steel wire, with marks at 0.20 m intervals, was mounted along the river. Along this line the bottom topography was recorded so the transport route of the single tracer particles could be described. It was endeavoured that the line

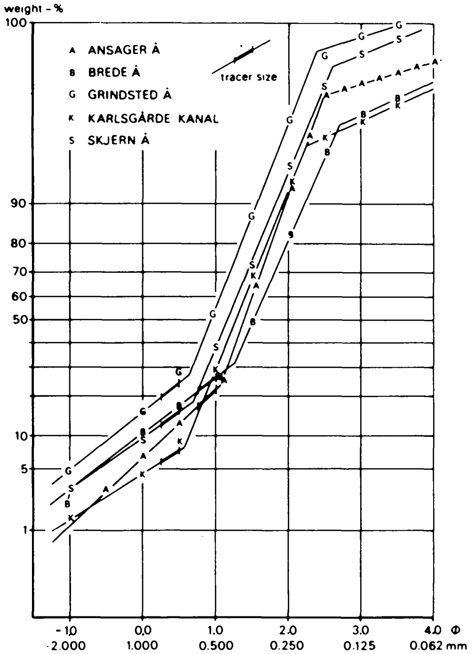

Fig. 2. The distribution of grain sizes for each test plot.

included a part element of a larger bed-form to ensure a continuous particle motion. Downstream from the injection place, a NaI-solid scintillation detector connected with a BASC was then set up; for controlling a continuous motion of tracer particles an extra detector was placed between the two positions. The detectors were connected with permanently working recorders, and placed in a way allowing free passage of sediment and tracer particles.

The dosing of tracers was made as an δ-injection through a tube placed on the bottom. The least possible amount of radioactive particles were applied.

Before the injection, the necessary hydraulic parameters (*V*, *D*, and *I*) were determined in the traditional way. The mean velocity of the flow was measured in the line where the velocity of surface particles was determined. During the whole period of investigation the water depths were recorded along the line.

Results

Surface Particle Velocities

Table 1 shows the surface particle velocities U_G recorded by the detectors when the first tracer particles were passing by.

The highest percentage deviation in velocity to the detectors I and II was found in the river Grindsted Å, where also the highest values of bed-load transport were measured. The higher uncertainty on the determination of surface particle velocities in Grindsted Å may be caused by the fact that not all of the set up hydraulic and sedimentological criteria on particle transport have been fulfilled; cf. Fig. 3.

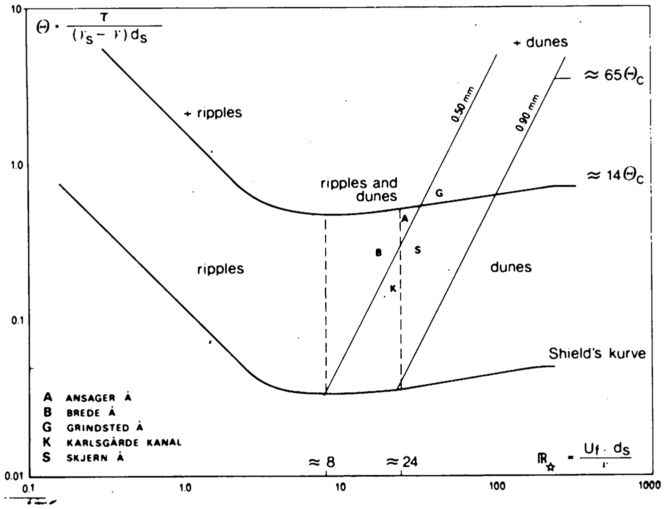

Fig. 3. Shields' curve, after Yalin (1977).

Table 1 – Records of surface particle velocities

Location	Recorded at Detector I	Recorded at Detector II
Grindsted Å	13.4 cm/s	15.0 cm/s
Karlsgårde Kanal	7.8 cm/s	7.1 cm/s
Brede Å	9.0 cm/s	9.2 cm/s
Skjern Å	15.0 cm/s	15.3 cm/s
Ansager Å		4.2 cm/s

Hydraulic Data

The measured and calculated hydraulic data are shown in Table 2. For determination of the reduced friction velocity $U_f' = \sqrt{g\, D'\, I}$, where I is the gradient of the energy line, the reduced depth D' is determined on the basis of Einstein's original definition (1950)

$$\frac{V}{U_f'} = 6 + 2.5 \ln \frac{D'}{k} \tag{1}$$

where k denotes the equivalent sand roughness found as $k = 2.5$ d, Engelund and Hansen (1972).

The dimensionless bed shear stress (Shields' parameter) is given by

$$\theta' = \frac{D'\, I}{(s-1)d} \tag{2}$$

where s is the relative density of the sediment.

Table 2 – Hydraulic data

Location	V	I	D	D'	U_f'	θ'
Grindsted Å	0.60 m/s	0.95‰	0.89 m	0.126 m	0.034 m/s	0.149
Karlsgårde K	0.48 m/s	0.15‰	1.34 m	0.323 m	0.022 m/s	0.076
Brede Å	0.42 m/s	0.27‰	0.92 m	0.177 m	0.022 m/s	0.083
Skjern Å	0.67 m/s	0.32‰	1.12 m	0.342 m	0.033 m/s	0.154
Ansager Å	0.36 m/s	0.53‰	0.77 m	0.083 m	0.021 m/s	0.068

Experimental Results

For each test plot the following experimental conditions must be fulfilled:

1) the flow should be turbulent and rough
2) the bed-forms should be dunes or dunes/ripples
3) the tracer particles should be in motion just along the bed.

ad 1) By means of v. Kármán's number, it is found whether the flow corresponds to hydraulic, rough bed, $K > \sim 10$, Engelund and Pedersen (1978). In hydraulic, rough turbulent flow

$$K = \frac{U_f k}{\nu} > \sim 70 \tag{3}$$

is valid (Yalin 1977).

Table 3 shows the calculated values for these criteria.

Table 3 – Hydraulic criteria

	criterion 1		criteria 1 & 2			
locality	K	$\frac{U_f k}{\nu}$	θ	$\mathbb{R}_*$	bed-form	D/d_{tr}
Grindsted Å	33	110	0.665	43.8	du.	$1.2\ 10^3$
Karlsgårde K.	16	54	0.158	21.1	ri. & du.	$1.7\ 10^3$
Brede Å	13	44	0.274	17.3	ri. & du.	$1.7\ 10^3$
Skjern Å	23	76	0.282	30.3	du.	$1.5\ 10^3$
Ansager Å	19	64	0.454	25.2	du.	$1.4\ 10^3$

ad 2) For each test area the bed-forms were determined by plotting the values of Shields' parameter (θ) and Reynold's figure for grain ($\mathbb{R}_*$)cf. Fig. 3. The diagram is only valid for high values of the non-dimensional depth ($D/d > 1{,}000$), Yalin (1977). With low values of the D/d ratio, the interval for $8 < \mathbb{R}_* < 24$ will disappear, because the transition phase from ripples to dunes will occur at $\mathbb{R}_* \simeq 8$ due to the fact that there will be no ripples with $d > \sim 0.5$ mm, while there will always be dunes with $d \simeq 0.5$ mm.

ad 3) The mobility of the tracer particles appears from the position of the hydraulic parameters in Fig. 3.

The motion of the tracer particles took place along the river bed. The type of motion depends upon the single particle's size in relation to the hydraulic parameters. Meland and Norrman (1966) distinguish between 3 »stages« of particle motion. 1) »stop and go« movement, which occurs with bed shear stresses slightly above the critical values. 2) another stage of continuous movement in contact with the bed with increasing values of the critical parameters and, finally, 3) when the grains start to be lifted up from the river bed. Also Francis (1973) distinguishes between 3 stages of movement, which are not quite identical with those of Meland and Norrman, however, as he includes parts of stage 2 in stage 1.

The surface particle velocities found in this investigation (Table 3) are lower than those found by a.o. Meland and Norrman (1966), Francis (1973), and Fernandez Luque et al. (1976); this confirms that the tracer particles studied here have not been suspended. The particle motion in the upper layer has here been induced by bed shear stresses corresponding to stage 2, as a continuous movement in contact with the bottom according to Meland and Norrman (1966) and Francis (1973).

The first surface particles detected at positions I and II at a distance of 2 respectively 4 m from the injection point were measured and proved to be identical cf. Table 1.

Moreover, according to the calculated hydraulic criteria, the possibility of stagnant periods during the transport from dosing place to detector can be precluded, cf. Fig. 3.*

On the basis of the present results and according to the principles established by Engelund and Fredsøe in their sediment transport model, a semi-empirical function is set up from the transport rate.

The function is based upon measurements of transport velocities of separate particles along the bed and the hydraulic conditions

$$\frac{U_G}{U_f'} = \alpha\left(1 - \sqrt{\theta_0/\theta'}\right) \qquad (4)$$

Representing the force transferred to the particles, the reduced values »'« have here been used. U_f' is the effective friction velocity, and $\alpha U_f'$ the effective flow velocity at a height of 1 or 2 grain diameters above the fixed bed

$$\theta_0 = \frac{4\beta}{3\alpha^2 c} \qquad (5)$$

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* In a private communication the author received some comments from F. Engelund. According to these continuous contact with the bed does not mean that the probability p is unity, but only that p is close to unity. The author's experiments are generally carried out for θ between 0.4 and 0.6, which corresponds to $\theta' \sim 0.2$ and $p \sim 0.7$ to 0.8.
What the author really measures is not U_B (which is the particle velocity during actual movement) but pU_B, which is the mean velocity for periods of both motion and rest. With values of p as discussed above it is understandable that the author finds U_B smaller than other scientists. The measurements are – according to Engelund's alternative interpretation – actually a nice confirmation of his original results.

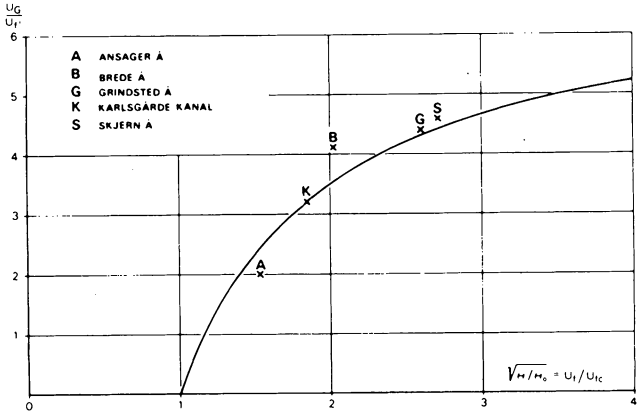

Fig. 4. Graph of the non-dimensional particle velocity U_G/U_f' vs. the non-dimensional dispersive stress $\sqrt{\theta'/\theta_0}$.

In this experiment θ_0 was found to be approximately half of θ_c, as also found by Fernandez Luque et al.

The general appearance of the equation below follows the one constructed by Engelund and Fredsøe on the basis of a comprehensive data material compiled by Meland and Norrman (1966) and Fernandez Luque et al. (1976). The lower velocity values found in the present investigation result in a change of α from 9.3 to 7.1

$$\frac{U_G}{U_f'} = 7(1 - 0.7\sqrt{\theta_c/\theta'}) \qquad (6)$$

From Fig. 4 it appears that the relation between the particle velocity and the effective shear velocity approximates a constant value of 7, which means that the surface particle velocity is approximating the velocity of the flow just above the bed, i.e. a particle velocity of $U_G = 7\ U_f'$.

If relating this to the logarithmic velocity profile for hydraulic, rough flow at the bed,

$$\frac{U}{U_f'} = 8.5 + 2.5 \ln\frac{y}{k} \qquad (7)$$

and presuming that the level where U is identical with U_G is $y = 0.35\ k$, (Einstein and El Samni 1949), we get $U \simeq 6\ U_f'$. The found relation between particle velocity and friction velocity is thus in good agreement with the expected values.

Correspondingly, in experiments with level beds and well-sorted material, such as those conducted by Engelund and Fredsøe, $y = k$ will be obtained and, accordingly, $U \simeq 9\ U_f'$. When dealing with natural streams, however, with bed-forms and heterogene material, the relation found here between particle- and friction velocity should be used when constructing a sediment transport model.

Comparison of the Model with Fort Collins Data

The results obtained here have been applied on Engelund and Fredsøe's transport model of principles.

For comparison, a calculation has been made of Fort Collins data run 21 with a mean fall diameter of 0.28 mm, corresponding to the one used by Engelund and Fredsøe

Fort Collins data run 21

mean fall diameter d_f = 0.28 mm
slope I = 0.00131
depth D = 0.326 m
mean velocity V = 0.725 m/s
temperature T = 16.5 °C
equivalent sand roughness k = 2.5 d

The following parameters were calculated according to Engelund and Fredsøe (1976)

D' = 0.116 m ; U_f' = 0.0386 m/s
θ' = 0.329 ; p = 0.859
Q_c = 0.05

Hereafter the non-dimensional rate of bed-load transport is found by

$$\Phi_B = 4_p(\sqrt{\theta'} - 0.7\sqrt{\theta_c}) = 1.43 \tag{8}$$

against the measured $\phi_B = 0.95$. Calculated on the basis of Engelund and Fredsøe's model $\phi_B = 1.79$.

The resulting non-dimensional shear stress for the suspended transport is expressed by

$$\theta' = \theta_c + \frac{\pi}{6}\beta p + 0.043\, s\, \theta' \lambda_b^2 \tag{9}$$

the lower limit of the integral in the equation for suspended transport is taken as

$$a = 2d \tag{10}$$

Hereafter the non-dimensional suspended transport can be found from

$$\Phi_s = 11.6\sqrt{\theta'}\, c_b\, \frac{a}{d}\left(\ln \frac{30.2\, D}{k}\, I_1 + I_2\right) \tag{11}$$

and parameters calculated we get

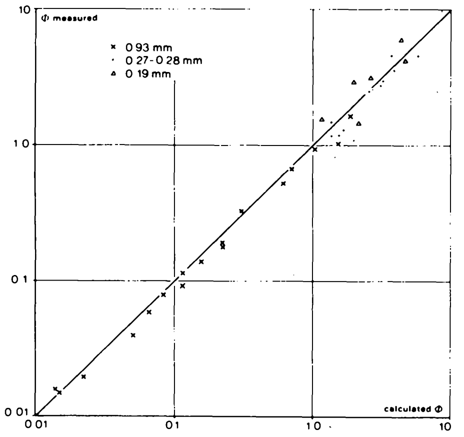

Fig. 5. Measured and calculated tranport rates.

$z = 1.49 \quad ; l_1 = 0.40 \quad ; l_2 = -2$

$c_b = 0.10$

Whereafter the suspended transport is calculated to $\phi_s = 2.41$
and the total transport $\phi_t = 3.84$
Measured $\phi_s = 2.50$ and $\phi_t = 3.84$

In Engelund and Fredsøe's model $\phi_s = 3.38$ which gives the total transport $\phi_t = 5.17$.

Correspondingly, the total non-dimensional sediment transport calculated from Engelund and Hansen (1972) is $\phi_t = 5.1$.

By means of the Fort Collins data a comparison was made between the non-dimensional total sediment transport rates calculated on the basis of quantitative measurements and on the basis of the here constructed model, respectively. The result appears from Fig. 5.

A corresponding graph was made by Engelund and Fredsøe (1976, Fig. 4), but with less agreement between measured and calculated values.

Evaluation of the Model

As shown in Fig. 5 there is a good correlation between measured and calculated sediment transport rates. The parameters changed in the model only comprise part of its theoretical basis. Parameters which might be uncertainly determined, and which are influencing the theory, are especially the expression of the transport – p, and the determination of the effective mean fall velocity. Future investigations for improving the description of these paramters will also improve the model. Likewise, detailed in situ measurements of the total sediment transport in natural streams will offer a more qualified basis for a calibration of the model.

One first method for improving the model would be to consider the sediment in a mumber of fractions, Einstein (1950) and Deigaard (1979), where the i'th fraction has the diameter d_i and has ϕ_i weight-percent of the bed load.

For each fraction the transport is then calculated in relation to its percentage occurrence along the bed, i.e. that the total sediment transport is calculated as

$$q_b = \sum_i q_{bi} \tag{12}$$

The suspended transport of each fraction is calculated on the basis of the bed load concentration of each fraction, c_{bi}. The fall velocity of the suspended material is already calculated as the effective fall velocity w_{ef}

$$w_{ef} = \frac{\sum \varphi_i \, w_i}{\sum \varphi_i} \tag{13}$$

where w_i is the fall velocity of the single fraction.

The calculations outlined above will be so time-consuming, however, that an electronic data processing of the whole sediment transport model will be necessary.

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List of symbols

a – the lower integration limit in the suspended transport equation
c – the drag coefficient
c_b – volumetric bed concentration
D – water depth
D' – the boundary layer thickness over a dune
d – grain diameter, at which 50% are finer
d_f – fall diameter
d_i – the grain diameter of the i'th fraction
d_s – sieve diameter

d_{tr} – diameter of the tracer particles
g – the acceleration of gravity
I – energy gradient (slope)
I_1 and I_2 integration values
K – $0.3\frac{U_f k}{\nu}$ von Kármán's number
k – equivalent sand roughness
p – the probability of movement of particles in a single layer
q_b – the specific bed load
q_{bi} – q_b of the i'th size fraction
$I\!R_*$ $\frac{U_f d}{\nu}$ Reynold's number of grain
s – the relative density of the sediment grains
T – water temperature
U – flow velocity
U_f – friction velocity
U_f' – friction velocity due to skin friction
U_G – migration velocity of the particle
V – the mean flow velocity
w_{ef} – the effective fall velocity
w_i – the fall velocity of the i'th size fraction
y – distance from bed level
z – $w/0.4\ U_f'$ the Rouse number
α – constant
β – dynamic friction coefficient
γ – specific gravity of the water
γ_s – specific gravity of the grains
θ – the dimensionless bed shear stress
θ' – the dimensionless bed shear stress due to skin friction
θ_c – the critical dimensionless bed shear stress (Shields' parameter)
θ_i – θ corresponding to the i'th size fraction
λ_b – the linear bed concentration
ν – kinematic viscosity of the water
τ – the bed shear stress
τ' – the bed shear stress due to skin friction
τ_c – the critical bed shear stress
ϕ_i – the percentage by weight of the i'th size fraction
ϕ_B – the dimensionless specific bed load
ϕ_S – the dimensionless specific suspended load
ϕ_t – the dimensionless specific total load

All units are given in accordance with the SI-system

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