Since the length of link AB does not enter into the calculations, it is a good plan to make it as long as possible so that the wear of the lining will have the least effect on the value of angle γ.

The friction forces help to apply the shoe to the drum and thus reduce the magnitude of the external force P. However, the proportions of the brake must not be such that there is danger of the shoe becoming self-locking during normal usage. The normal operating characteristics of this type of brake can be illustrated best by numerical examples.

Numerical Examples

Example 1. Find direction and magnitude of operating force P for $\alpha = 15$ deg, $a = 0.5r$, and $c = 0.8r$.

Application of the foregoing equations gives the values shown in Fig. 5(a) for the nondimensional ratios $N/brp_{\max}$ and $R/brp_{\max}$ when γ is taken as 16 deg. The figure is composite and shows results for values of the coefficient of friction from $\mu = 0.1$ to $\mu = 0.6$, inclusive. Values for actuating force factor $P/brp_{\max}$ are found as the closing line of the respective force polygon.

Fig. 5(b) shows the various values of $P/brp_{\max}$ plotted as a ray diagram from a common origin. This diagram illustrates the manner in which the force factor varies in both magnitude and direction with changes in the coefficient of friction μ. A minimum value, $P_{\min}/brp_{\max}$, is obtained for μ equal to about 0.34. For both larger and smaller values of μ a greater force is required to operate the brake.

When γ is taken as 20 deg, with other data unchanged, the ray polygon appears as in Fig. 6(a). The value of the minimum actuating force is reduced.

For γ equal to 24 deg, the value of $P_{\min}/brp_{\max}$ is approximately zero so there would be danger of the brake locking should the coefficient of friction, for any reason, change to about 0.44.

For $\gamma = 28$ deg, the forces for various values of μ are as shown in Fig. 6(b).

Example 2. Same data as for Example 1, namely, $\alpha = 15$ deg, $c = 0.8r$, except $a = 0.65r$.

For $\gamma = 20$ deg, $\gamma = 24$ deg, and $\gamma = 28$ deg, the force systems for $P/brp_{\max}$ are as shown in Figs. 6(c), (d), and (e), respectively. For $\gamma = 32$ deg the value for $P_{\min}/brp_{\max}$ is very close to zero.

Example 3. Same data as for Example 1, namely, $\alpha = 15$ deg, $c = 0.8r$, except $a = 0.8r$.

For $\gamma = 20$ deg, $\gamma = 24$ deg, $\gamma = 28$ deg, and $\gamma = 32$ deg the force systems for $P/brp_{\max}$ are as shown in Fig. 6(f), (g), (h), and (i), respectively.

Conclusions

Examination of the foregoing figures leads to the following conclusions:

1 Direction β of actuating force P is a function of the coefficient of friction μ. It is highly desirable that the lining material have a constant value for μ, under all operating conditions, in order to retain the validity of the results found by the foregoing design equations.

2 Value for $P_{\min}$ is reduced by use of a smaller value for a.

3 It is possible to find combinations of the constants such that $P_{\min}$ will be zero. The dimensions should be chosen so that $P_{\min}$ cannot vanish should there be an increase in the value of the coefficient of friction during operation.

When force P becomes zero, force polygon, Fig. 4, consists only of N, μN, and R. The corresponding coefficient of friction μ' and inclination γ' of link AB have the following relationship

$$\mu' = \tan \gamma' \quad [18]$$

The moment equation about A can be written with the P-term omitted which gives

$$a'N(1 + \mu') = \sqrt{2}\, T' \quad [19]$$

This equation applies only to a self-locking brake.

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Discussion

G. A. G. Fazekas.[3] This paper shows a rigorous analysis of forces acting upon a brake shoe that has two degrees of freedom of movement, but is restricted to the case when the pressure pattern is symmetrical with respect to the bisector of the lining arc. Essentially, the method amounts to finding the locus, direction, and magnitude of the shoe-tip load P, with all other factors remaining constant.

Once the position of P is determined, however, it does not seem possible to use the same technique for finding out how rapidly the torque or wear pattern would vary consequent upon some change in friction μ. For if μ deviates from its nominal value upon which the analysis was based—as is inevitable with present friction materials—automatically, the wear pattern ceases to be symmetrical and the present analysis is no longer applicable.

To come to details of the paper, the proof given for the pressure pattern is based upon postulating explicitly that all components of the brake are infinitely rigid, except the lining material which in turn obeys Hooke's law. Tacitly, it is further supposed, however, that the lining thickness is uniform and this is true for virgin linings only. Since roughly (pressure) $\propto$ (wear), after some use the lining thickness will be no longer constant, and the pressure pattern should change too correspondingly. Fortunately, though, of the various brake components, lining materials are about the most rigid ones. This explains why variation of lining thickness has comparatively little effect on pressure pattern as deduced from measurements of wear. Wear measurements suggest too that in most instances the sinusoidal pressure pattern is, nevertheless, reasonably approximated, and may be used therefore for purposes of analysis.

It should be noted that Equation [11] can be rewritten as

$$T = S\bar{r} + Q = SL$$

and it can be readily shown that L is given by

$$L = 4r\frac{\sin\ \alpha}{2\alpha + \sin 2\alpha}$$

where

$$2\alpha = \text{lining arc}$$

This enables one to carry out the graphical construction shown in

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Fig. 7 from which forces on the shoe and brake torque may be obtained to any desired degree of accuracy.

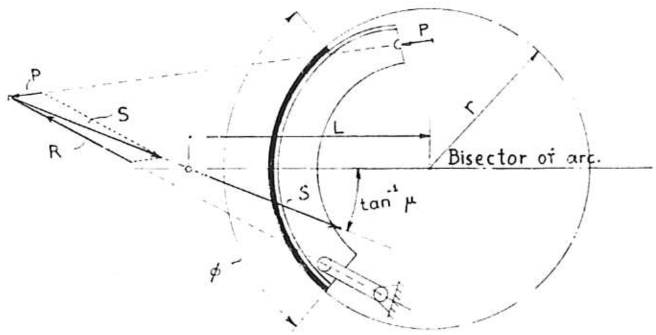

FIG. 7

It would be highly desirable indeed if μ would remain constant, as concluded by the author. Unfortunately, as yet, no such material seems to be available commercially. Even atmospheric conditions alone can induce what is referred to as the "morning sickness" of brakes. This is particularly severe with some highly self-energized brakes—having small a or its equivalent. In practice, it is this very susceptibility to changes of μ that sets ndirectly a limit to P.

Author's Closure

The author is grateful for the pertinent comments of Professor Fazekas.

His method of graphical analysis constitutes an alternative approach to the problem. His force designated S is the resultant of the author's forces N and S. Shifting (author's) S to location L has the effect of eliminating moment Q from the computations. Thus

$$Q = S(L - \bar{r})$$

That this is an identity is shown by substituting T/S for L and the value of $\bar{r}$ from Equation [9].