pressure dropped continuously from stagnation pressure at the inlet to atmospheric pressure at the nozzle exit. No evidence of shock waves was found in any of the two-phase flow tests. The details of the flow behavior downstream from the throat warrant further investigation but are beyond the scope of this paper. The present results should, however, be useful for calculating the two-phase flow rate in convergent nozzles.

Conclusions

1 "Separated flow" theory, modified by a simple empirical "blockage factor," can usefully describe gas-liquid nozzle flows when the two phases are separated in a plenum chamber before entering the nozzle.

2 If mixing occurs in the plenum chamber and is not suppressed in the nozzle the results tend to move toward the predictions of "homogeneous" flow theory.

3 For values of the dimensionless liquid flow G_f^* less than 0.3 choking occurs close to the point where the isentropic gas Mach Number (calculated from the pressure ratio) reaches unity. However, there is a suggestion of slightly higher flow rates being achieved with $M_{gt} > 1$ at the nozzle throat, as predicted by separated flow theory.

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DISCUSSION

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1 The authors should be commended for a brave attempt at a new conceptual model for describing gas-liquid flows. In doing so, they have also provided a limiting case which, taken together with homogeneous flow theory, allows the quantitative behavior of such flows to be bracketed. Judging by the experimental results, their imagination has been rewarded.

2 The paper also brings to mind the following questions:

(*a*) Does their choking condition correspond (analytically) to a small wave which is stationary in laboratory coordinates?

(*b*) How much of the observed blockage was due to the side walls?

(*c*) Is there any merit to separating the gas and liquid streams by means of an oil or flimsy material barrier (initiating from the tongue) in order to separate blockage from entrainment?

(*d*) Which mechanism, blockage or entrainment, is primarily responsible for the deviation of the data from the simple theory?

Authors' Closure

The choking condition does indeed correspond analytically to the bringing to rest of a small wave at the nozzle throat. We can proceed by eliminating α between equations (1) and (2), differentiating and making use of equation (3). The result is:

$$M_g^2 \frac{G_f^*}{2\left(\dfrac{p_{of}-p}{p_{og}}\right)^{3/2}} + (1 - M_g^2)\left(1 + \frac{k-1}{2} M_g^2\right)^{\frac{k}{k-1}} \times \left(1 - \frac{G_f^*}{\left(\dfrac{p_{of}-p}{p_{og}}\right)^{1/2}}\right) = 0 \quad (8)$$

In addition we have from Bernoulli's equation,

$$\rho_f V_f^2 = 2\,(p_{of} - p) \quad (9)$$

and from gas dynamics

$$\rho_g V_g^2 = k p_{og} M_g^2 \left(1 + \frac{k-1}{2} M_g^2\right)^{-\frac{k}{k-1}} \quad (10)$$

Therefore equation (8) may be arranged with the help of (2) to give

$$\frac{(1-\alpha)}{\rho_f v_f^2} + \frac{\alpha}{\rho_g v_g^2}(1 - M_g^2) = 0 \quad (11)$$

Equation (11) is just the condition given by Wallis[4,5] for a stratified compressibility wave to be brought to rest (when one of the phases is incompressible). Clearly we must have $M_g^2 > 1$ at this point.

We made several attempts to separate the effects of side wall blockage, wave blockage, and entrainment. However, the results were inconclusive and could not be checked without independent measurements of these contributing factors. A more sophisticated experimental program, perhaps incorporating the "flimsy barrier" idea, might be useful for elucidating these mechanisms. For practical prediction purposes, however, it might be most useful to combine all these contributing effects in the single empirical parameter B.

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