DISCUSSION/AUTHORS CLOSURE

Discussion of Thin-Walled Open-Profile Bars

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This paper yields incorrect and misleading information on the buckling theory of thin-walled bars. In a recent paper (Goto and Chen, 1989), we showed the validity of the Wagner hypothesis and pointed out the problems associated with Ojalvo’s derivation. In this discussion, we will provide a supplementary explanation to show more specifically why Ojalvo’s theory is incorrect.

First, we will show that Ojalvo’s criticism of our formulation of the finite displacement theory of thin-walled beams is incorrect. He objected to the equilibrium equations (3.22) and (3.23) of Washizu (1968) and hence the use of pseudo-stress, i.e., the second Piola-Kirchhoff stress tensor, in our variational formulation. In fact, identical equilibrium equations can be written using either the pseudo or the actual stress, although the expressions are different because of the difference in the definitions of the two stresses. With a proper transformation, it is not difficult to show that the two different expressions of equilibrium equations agree with each other. Thus, Ojalvo’s statement that the equilibrium equations written by the pseudo-stress have no basis in a conventional understanding of statics is not correct. Beam theory (Goto et al., 1985) under the conditions of finite displacements and finite strains precisely considers this difference between the pseudo-stress and the actual stress. For the equilibrium equations obtained by variational calculus, all the pseudo-stress resultants are transformed to the actual stress resultants. The validity of these equilibrium equations is also confirmed by the equilibrium approach in the referenced paper. Furthermore, under the assumption of small strains, the actual stress can be approximated well by the second Piola-Kirchhoff stress tensor. For almost all engineering structures, strains are negligibly small compared with unity, even in the case where structures have undergone extremely large displacements. Thus, the procedure adopted by Nishino et al. (1973, 1979) and Hasegawa et al. (1985) is also valid and there is no problem in these theories. These three theories lead naturally to Wagner’s K term.

Next, we will proceed to demonstrate that Ojalvo’s theory simply results from some mistakes and inconsistencies in his derivation.

For buckling under uniform compression, we have explained in some detail the problems associated with Ojalvo’s theory (Goto and Chen, 1989). So here we only discuss his other theory for bars with a longitudinal plane of symmetry.

Ojalvo presented an expression for the external potential in equation (12) where the following equation is used to represent the curvature of the centroid line about the x-axis:

\[-v'' = -\frac{1}{2}\frac{d}{dy}\left(\beta u'' + y\theta'' \right) + \frac{y''}{2}. \quad (20)\]

We will show in what follows that equation (20) is incorrect. Ojalvo derived the x-component of the relative rotation of the centroid line as \(-\beta(u'' + y\theta'')dy\), assuming that the \(\eta\)-component of the incremental displacement at the shear center is zero at the instant of buckling. This gives the first term of the right-hand side of equation (20). However, this assumption has no good justification and is generally inadequate. A correct procedure to obtain this curvature will be shown later.

The other mistake found in Ojalvo’s derivation is the second term in the right-hand side of equation (20). This term, as seen from Figs. 4 and 6, corresponds to a component of the curvature due to the centroidal displacement in the \(\eta\)-direction caused by the rotation about the shear center. Ojalvo calculated this component of the curvature about the \(\xi\)-axis by the equation

\[-(\phi_0(z + dz) - \phi_0(z))/dz^2 = -y_\phi'(\theta')^2/2, \quad (21)\]

where \(\phi_0(z)\) is the centroidal displacement in the direction of the \(\eta\)-axis at \(z\). However, the use of equation (21) to calculate the curvature is incorrect. In lieu of equation (21), the curvature, which is the second derivative of displacement \(\phi_0\), should be calculated by

\[-(\phi_0(z + dz) - \phi_0(z))/dz^2 = -y_\phi'(\theta')^2. \quad (22)\]

Here, we shall show a correct procedure to calculate \(-v''\). Since \(v\) can be expressed in the form \(v = v + y(1 - \cos \theta)\), the curvature \(-v''\) is given as follows up to the second order in terms of the torsional angle:

\[-v'' = v' + y(1 - 2\cos \theta)\theta'' \quad (23)\]

\(v'\) in equation (23) should be determined from the condition that the bending moment \(M_\beta\) remains constant at the instant of buckling. The curvature \(-v''\) so determined is given by

\[-v'' = -(1/2)(\beta_x + 2y_\phi)\theta'' \quad (24)\]

References


Author’s Closure

The criticisms voiced in the Goto, Chen, and Nishino Discussion may be separated into three categories: (1) It is believed that our expression for curvature about the \(x\)-axis, \(-\beta(u'' + y\theta'')\), used with the virtual work theorem for the singular purpose of evaluating the potential of external loads, is incorrect, (2) exception is taken to our remarks concerning the Washizu virtual work principle (theorem) and stationary potential energy principle (theorem), both for finite displacements. We believe these to be false theorems, and (3) exception is taken to our assertion that the Wagner effect has no place in a rational theory of bars.

As to the first criticism, the discussers object because the part \(-\beta(u'' + y\theta'')\) is written without consideration of a certain \(\eta\) component of the shear center displacement. Such \(\eta\) component does not enter because \(u'' + y\theta''\) is a curvature of the centroid line about the \(\eta\)-axis which, when multiplied by \(\theta\), gives the component of the curvature about the \(x\)-axis.

With regard to the remainder of the first criticism, we emphatically disassociate ourselves from equation (21) which is inaccurately attributed to us. Equation (22), suggested instead by the discussers, fails because it ignores that \(\eta\) is not a fixed direction such as \(y\). The part of our analysis which yields \(x\)-axis curvature \((\theta''y_\phi^2)/2\) is valid up to an approximation which ignores terms of higher order than those which are quadratic in displacements \(u, \theta, \phi\) and/or their derivatives.

Criticism voiced with respect to our statements concerning the Washizu theorems are unresponsive. We object to the Washizu theorems because they are derived without regard for Newtonian laws for statics. If we are correct, there can be no mathematical formulation of the type alluded to which can cure an infirmity in the physics of the problem. We doubt the