Comparing grey formulations of the velocity-area method and entropy method for discharge estimation with uncertainty

Stefano Alvisi, Silvia Barbetta, Marco Franchini, Florisa Melone and Tommaso Moramarco

ABSTRACT

Two methods, namely the velocity-area method and the entropy method, for assessing with uncertainty discharge measurements at gauged river sites are analysed and compared; uncertainty is represented through the grey number technique. Two different approaches for the 'greyification' of both methods are presented. In the first approach, the uncertainty affecting each measurement used to estimate the discharge is characterized by means of a grey number: all the grey uncertainty components are then combined through grey mathematics. In the second approach, greyification is applied to the relationship expressing the total uncertainty on the discharge measurement provided by the EN ISO 748 guidelines. Results of the application of the proposed methods to measurement data pertaining to three different gauged sections of the Tiber River, in central Italy, show that the first greyification approach leads to a broader discharge uncertainty estimate with respect to the second. Furthermore, as the greyification approach and the flow area quantification are the same, the velocity-area and entropy methods provide nearly the same estimate of the uncertainty affecting the discharge measurements, i.e., the grey discharges provided by the two methods are very similar. This testifies in favour of the entropy method, which is simpler than the other from an operative viewpoint.

Key words | discharge calculation, flow velocity, grey number, measurement uncertainty

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INTRODUCTION

From a hydrological point of view, discharge at a gauged river site is of the utmost importance, on the one hand, for planning the management of the water resources of river basins, as well as the control of floods, and on the other, for the calibration and validation of hydrologic and hydraulic models (e.g., Azamathulla & Zahiri 2012; Zahiri & Azamathulla 2012). For this reason, it is required that its value be accurate as much as possible. Overall, discharge is determined on the basis of direct velocity measurements, obtained by means of current meters, which are then used to calculate the mean flow velocity once the depth-averaged velocity has been estimated along verticals sampled in the flow area (Herschy 2009). This procedure is the basis for developing the stage-discharge relationship, commonly known as the rating curve, which represents the synthesis of all of the velocity measurements performed at a river site.

However, to have a reliable rating curve for a gauged site, the site needs to be easily accessible, equipped with hydrometric sensors for flow depth monitoring and suitable for direct velocity measurements also for higher stages. Unfortunately, this ideal site configuration is seldom feasible as, unlike flow depth measurements that are simple and relatively inexpensive, velocity measurements, besides being costly, are extremely difficult to perform at higher stages due to the danger that might be encountered when sampling velocity points in the lower portion of the flow area. This makes it

doi: 10.2166/hydro.2013.160

difficult to set-up an accurate stage-discharge relationship, particularly in the part corresponding to higher stages (e.g., Azamathulla *et al.* 2011). In this case, the sampling of the maximum flow velocity, which is located in the upper portion of the flow area, turns out to be very useful for obtaining an accurate estimate of discharge (Chiu 1987). Indeed, based on entropy theory (Shannon 1948; Jaynes 1957a, b; Chiu & Chiou 1986; Chiu 1987, 1988, 1989, 1991), it has been shown that the mean flow velocity can also be estimated from the value of maximum velocity, through a linear relationship identified by the entropic parameter *M*. Investigating some equipped sites along the Mississippi River, Xia (1997) noted that the *M* value was quite similar for sections located along straight branches, and was equal to 2.1; whereas for sites along bends the *M* value was equal to 4.8. Similar findings were obtained by Moramarco *et al.* (2004) and Moramarco & Singh (2010) by investigating a number of gauged river sections located along natural channels of the Upper Tiber basin.

The measurements of the quantities mentioned above, i.e., stage, velocity points on verticals, maximum flow velocity as well as the river cross-section geometry are characterized by a number of uncertainty sources which need to be quantified to get a reliable rating curve. Indeed, both the standard velocity-area method and the relationship between mean and maximum flow velocity, hereinafter referred to as the 'entropy method', are affected by uncertainty tied to measurements used for the discharge calculation. In particular, as far as the velocity-area method is concerned, two main types of uncertainty need to be taken into account: (1) the flow area, which depends on the number and the depth of sampled verticals, the distance between them and the number of velocity points sampled on vertical; and (2) the depth-averaged velocity along the vertical, which is linked to the exposure time of the velocity points measurement and the precision of the current meter. Guidelines of how to quantify each of these types of uncertainty can be found in EN ISO 748 (2007).

With the entropy method, the types of uncertainty are: (1) the flow area, which is here assumed to be estimated in the same way as for the velocity-area method; (2) the velocity points sampled in the upper portion of flow area, through which the maximum flow velocity is identified; and (3) the linear entropic relationship between mean and maximum flow velocity.

Therefore, the sources of error affecting the two approaches in mean flow velocity assessment are different, while they are the same as regards the flow area estimation. Hence, it seems reasonable to investigate whether these two different approaches provide estimates of discharge with the same level of uncertainty, once the method to investigate the uncertainty is established.

To this end, grey mathematics techniques are used to quantify the uncertainty (Deng 1982). The choice of the grey formulation to characterize the uncertainty of streamflow measurements and to mathematically transfer it to the discharge, arises from the consideration that this approach is well suited to representing and combining various sources of errors/uncertainty, not only that ascribable to the randomness of the phenomenon alone, but also that arising from other components such as systematic error or inadequate knowledge of the phenomenon itself, as in the measurement of stage and flow velocity (Shrestha & Simonovic 2010).

The paper is organized as follows. The section below outlines first the velocity-area method and then the entropy method for the crisp discharge estimation. The section after that provides an overview of the grey mathematics followed by a section describing the procedure used for the greyification of both the velocity-area method and the entropy method. This is followed by a section detailing the comparison of the two methods and the results achieved for three gauged river sections along the Tiber. Finally, the last section features the conclusions drawn from this investigation.

CRISP DISCHARGE ESTIMATES AT A GAUGED RIVER SITE

Velocity-area method

The velocity-area method is the most widely used approach for assessing stream discharge in natural rivers. It relies on measurement of stream point velocities, depths of flow and distances across the channel between sampled verticals. The velocity is measured by current meter at one or more points along each vertical, and then a mean value is estimated. The discharge, *Q*, is then evaluated through the mean-section method by summing the product of the depth-averaged velocity depth and width between verticals as:

$$Q = \sum_{i=1}^{m-1} (b_{i+1} - b_i) \cdot \frac{(d_{i+1} + d_i)}{2} \cdot \frac{\bar{v}_{i+1} \cdot d_{i+1} + \bar{v}_i \cdot d_i}{(d_{i+1} + d_i)} \quad (1)$$

where $\bar{v}_i$ and $\bar{v}_{i+1}$ are the depth-averaged velocities for vertical i and $i+1$, respectively, d_i and d_{i+1} are the flow depths observed at verticals i and $i+1$, respectively. b_i and b_{i+1} are the distances from an initial datum point to verticals i and $i+1$, and m is the number of sampled verticals. The mean-section method enables computation of the contribution of the end segments in the same manner as for the other segments. It is worth noting that the velocity-area method relies on the hypothesis that the velocity field does not change significantly during the measurement. Instead, when the velocity field changes significantly during the measurement, different methods, like the independent vertical method, are more appropriate for assessing the discharge, as suggested by EN ISO 748 (2007).

Entropy method

A fundamental variable of the open channel flow is the velocity and its distribution in a river cross section. Chiu (1988) investigated the flow velocity distribution through a probabilistic approach based on the concept of entropy. Considering the probabilistic formulation derived by Chiu, the relation between the mean velocity, v_m, and the maximum velocity, $v_{\max}$, occurring in a gauged river section for given stages and discharges can be expressed as:

$$v_m = \Phi(M) v_{\max} \quad (2)$$

where

$$\Phi(M) = \frac{v_m}{v_{\max}} = \frac{e^M}{e^M - 1} - \frac{1}{M} \quad (3)$$

and M is the dimensionless entropy parameter (Chiu 1989, 1991; Chiu & Said 1995) (see Appendix A for more details concerning the entropy method and its derivation; available online at http://www.iwaponline.com/jh/016/160.pdf). The value of M provides fundamental information on the main characteristics of the channel section, such as changes in bed form, slope and geometric shape (Chiu & Murray 1992).

Equation (2) shows that a sample of pairs (v_m, $v_{\max}$) can be used to assess $\Phi(M)$ and then estimate the entropy parameter, M. In particular, the relationship between the mean velocity and the maximum velocity expressed by Equation (2) was tested at some sections of the Mississipi River (Xia 1997), and at different gauged sites in the Tiber River basin in central Italy (Moramarco *et al.* 2004). Both studies showed that the relationship was perfectly linear, and that the M value can therefore be considered constant along a river reach. Alternatively, the entropy parameter M can be estimated by using a relationship between $\Phi(M)$ and the geometric and hydraulic characteristics of the gauged section as explained by Moramarco & Singh (2010).

Hence, once M is estimated at gauged sections and $v_{\max}$ is sampled through, for instance, the use of a current meter or radar sensors in the upper portion of the flow area where it typically occurs, Equation (2) can be applied to estimate the mean flow velocity. To this end it is worth noting that the *real* value of the maximum velocity, $v_{\max}$, is unknown, but in practice the maximum value of sampled velocity points in the upper portion of the flow area can be assumed as representative of it. In fact, recent works (Corato *et al.* 2011; Moramarco *et al.* 2011; Fulton & Ostrowski 2008, to quote a few) based on the entropy approach, showed that it is possible to achieve accurate estimates of the mean velocity, v_m, by sampling $v_{\max}$ only. Finally, the discharge can be assessed from the knowledge of the flow area, A, as:

$$Q = \Phi(M) v_{\max} A = v_m A \quad (4)$$

GREY NUMBERS

The grey number approach allows representation of the uncertainty associated with a given quantity by a number whose exact value is unknown but whose variation range is known (Liu & Lin 1998). A grey number $x^{\pm}$ can therefore be mathematically expressed as:

$$x^{\pm} = [x^-, x^+] = \{x \in x^{\pm} | x^- \leq x \leq x^+\} \quad (5)$$

where x, x^- and x^+ are real numbers, and x^- and x^+ represent the lower and upper boundaries of the interval. It is worth emphasizing that the technique of grey numbers is a non-probabilistic approach for the characterization of uncertainty, and, in fact, once a grey $x^{\pm}$ is assigned, no information is available about the distribution of the x value within the range, $[x^-, x^+]$, i.e., no assumption is made on the probability distribution within the range (Liu & Lin 2006; Alvisi & Franchini 2010).

As part of the grey number technique, the main mathematical operations between grey numbers, as well as the concept of function f of grey numbers, are defined. In particular, given two grey numbers $x^\pm = [x^-, x^+]$ and $y^\pm = [y^-, y^+]$ the main operations are defined as follows (Wang & Wu 1998):

$$\text{addition: } [x^-, x^+] + [y^-, y^+] \triangleq [x^- + y^-, x^+ + y^+] \quad (6)$$

$$\text{subtraction: } [x^-, x^+] - [y^-, y^+] \triangleq [x^- - y^+, x^+ - y^-] \quad (7)$$

$$\text{multiplication: } [x^-, x^+] \times [y^-, y^+] \triangleq [\min\{x^-y^-, x^-y^+, x^+y^-, x^+y^+\}, \max\{x^-y^-, x^-y^+, x^+y^-, x^+y^+\}] \quad (8)$$

$$\text{and division: } [x^-, x^+] \div [y^-, y^+] \triangleq [x^-, x^+] \times \left[\frac{1}{y^+}, \frac{1}{y^-}\right] \quad (9)$$

where x^-, x^+, y^- and y^+ are real numbers, $\triangleq$ stands for 'being defined as', and, for the division, must be $0 \notin [y^-, y^+]$.

A function f of n_g grey numbers $x_1^\pm, x_2^\pm, \ldots x_{n_g}^\pm$ is, instead, defined as (Yang et al. 2004):

$$\left[f\left(x_1^\pm, x_2^\pm, \ldots, x_{n_g}^\pm\right)\right]^\pm = \left[\left[f\left(x_1^\pm, x_2^\pm, \ldots, x_{n_g}^\pm\right)\right]^-, \left[f\left(x_1^\pm, x_2^\pm, \ldots, x_{n_g}^\pm\right)\right]^+\right] \quad (10)$$

where the lower $\left[f\left(x_1^\pm, x_2^\pm, \ldots, x_{n_g}^\pm\right)\right]^-$ and the upper $\left[f\left(x_1^\pm, x_2^\pm, \ldots, x_{n_g}^\pm\right)\right]^+$ limits of the grey function are obtained by searching for (a) the set of real values $x_1, x_2, \ldots x_{n_g}$, with $x_i^- \le x_i \le x_i^+$ that minimizes f, and (b) the set of real values (usually different from the previous) that maximizes it.

GREY UNCERTAINTY IN DISCHARGE ASSESSMENT AT GAUGED SITES

The methods described earlier address the crisp estimate of the discharge, starting from the river cross-section geometry and, in the case of the velocity-area method, from several velocity points sampled along verticals in the flow area, while, in the case of the entropy method, from the measurement of the maximum flow velocity. As previously noted, each of these measurements is affected by errors/uncertainty. Assuming characterizing the uncertainty on individual measurements through grey numbers, the subsequent sections show how to quantify the total uncertainty on the discharge, both when this is assessed through the velocity-area method and when it is evaluated using the entropy approach, *the flow area estimate being the same*.

In both cases, each component of uncertainty that affects the measures is greyified by introducing a grey number $X^\pm$, which represents a range with the central value 0 and whose boundaries, X^- and X^+ are the considered error component/uncertainty expressed as percentage, i.e., $X^\pm = [X^-, X^+] = [-5/100, +5/100]$. The numerical quantification of these individual components of uncertainty will be provided in the case study section with reference to the specific case study considered.

Grey discharge estimate by the velocity-area method

The grey estimate of discharge through the velocity-area method can be addressed on the basis of two different approaches, both derived from that proposed by Shrestha & Simonovic (2010), that is (a) by aggregating the individual components of uncertainty through grey mathematics (hereinafter referred to as type I greyification) and (b) by greyifing the overall uncertainty indicated by EN ISO 748 (2007) (hereinafter referred to as type II greyification).

In both approaches, the sources of uncertainty in the estimation of the discharge are the number of verticals, their position, their depth, the number of velocity measurements on each vertical, the exposure time for the measurement of each velocity point, and the accuracy of the equipment used to measure the velocity.

In detail, type I greyification characterizes each of the components of uncertainty previously mentioned by a grey number; these different components of uncertainty are then combined by means of grey mathematics to provide an estimate of the grey discharge. It is worth noting that, by combining the different components of uncertainty by means of grey mathematics, the different sources of uncertainties are, in practice, handled as uncorrelated given the very nature of grey mathematics; indeed, this approach is somehow consistent with EN ISO 748 (2007) guidelines where, within a probabilistic framework, the single sources of uncertainty for estimating the overall uncertainty affecting the discharge are combined, disregarding the terms of covariance (see also Herschy 2009).

Specifically, the uncertainty on the location of the ith vertical is expressed as:

$$b_i^{\pm} = b_i\left(1 + X_{b,i}^{\pm}\right) \tag{11}$$

where $X_{b_i}^{\pm}$ is the grey uncertainty on the measure of abscissa b_i.

The uncertainty on the depth of the ith vertical is expressed as:

$$d_i^{\pm} = d_i\left(1 + X_{d,i}^{\pm}\right) \tag{12}$$

where $X_{d,i}^{\pm}$ is the grey uncertainty on the measurement of depth d_i.

Finally, the uncertainty on the depth-averaged velocity $\bar{v}_i$ along the ith vertical is:

$$\bar{v}_i^{\pm} = \bar{v}_i\left(1 + \sqrt{\left(X_{p,i}^{\pm}\right)^2 + \left(X_{c,i}^{\pm}\right)^2 + \left(X_{e,i}^{\pm}\right)^2}\right) \tag{13}$$

where $X_{p,i}^{\pm}$, $X_{c,i}^{\pm}$, $X_{e,i}^{\pm}$ are the grey uncertainties pertaining to the numbers of velocity points along the vertical, the current meter measurement and the exposure time, respectively. Overall, going back to Equation (1) and replacing each crisp term with the corresponding grey number, the following is obtained (Equation (14)):

or, to simplify,

$$(Q^{\pm})^* = \sum_{i=1}^{m-1}\left(b_{i+1}\left(1 + X_{b,i}^{\pm}\right) - b_i\left(1 + X_{b,j}^{\pm}\right)\right)\cdot \frac{1}{2}\cdot\left[\bar{v}_{i+1}\left(1 + \sqrt{\left(X_{p,j+1}^{\pm}\right)^2 + \left(X_{c,i+1}^{\pm}\right)^2 + \left(X_{e,j+1}^{\pm}\right)^2}\right)\cdot d_{i+1}\left(1 + X_{d,i+1}^{\pm}\right) + \bar{v}_i\left(1 + \sqrt{\left(X_{p,i}^{\pm}\right)^2 + \left(X_{c,i}^{\pm}\right)^2 + \left(X_{e,i}^{\pm}\right)^2}\right)\cdot d_i\left(1 + X_{d,j}^{\pm}\right)\right] \tag{15}$$

Finally, taking the grey uncertainty $X_m^{\pm}$ on vertical numbers into account (see Shrestha & Simonovic 2010) the grey discharge $Q^{\pm}$ is:

$$\begin{aligned} Q^{\pm} &= (1 + X_m^{\pm})(Q^{\pm}) \\ &= (1 + X_m^{\pm})\sum_{i=1}^{m-1}\left(b_{i+1}\left(1 + X_{b,j}^{\pm}\right) - b_i\left(1 + X_{b,i}^{\pm}\right)\right)\cdot\frac{1}{2}\cdot \left[\bar{v}_{i+1}\left(1 + \sqrt{\left(X_{p,j+1}^{\pm}\right)^2 + \left(X_{c,i+1}^{\pm}\right)^2 + \left(X_{e,j+1}^{\pm}\right)^2}\right)\right. \\ &\quad \left.\cdot d_{i+1}\left(1 + X_{d,i+1}^{\pm}\right) + \bar{v}_i\left(1 + \sqrt{\left(X_{p,i}^{\pm}\right)^2 + \left(X_{c,i}^{\pm}\right)^2 + \left(X_{e,i}^{\pm}\right)^2}\right)\cdot d_i\left(1 + X_{d,i}^{\pm}\right)\right] \end{aligned} \tag{16}$$

In the case of type II greyification, on the other hand, the total percentage uncertainty on the discharge, X_Q, is estimated considering the following relationship, wherein each component of uncertainty X represents the (crisp) percentage uncertainty corresponding to an assigned confidence

$$(Q^{\pm})^* = \sum_{i=1}^{m-1}\left(b_{i+1}\left(1 + X_{b,i}^{\pm}\right) - b_i\left(1 + X_{b,i}^{\pm}\right)\right)\cdot\frac{\left(d_{i+1}\left(1 + X_{d,j+1}^{\pm}\right) + d_i\left(1 + X_{d,j}^{\pm}\right)\right)}{2}\cdot \frac{\left[\bar{v}_{i+1}\left(1 + \sqrt{\left(X_{p,i+1}^{\pm}\right)^2 + \left(X_{c,j+1}^{\pm}\right)^2 + \left(X_{e,j+1}^{\pm}\right)^2}\right)\cdot d_{i+1}\left(1 + X_{d,j+1}^{\pm}\right) + \bar{v}_i\left(1 + \sqrt{\left(X_{p,j}^{\pm}\right)^2 + \left(X_{c,j}^{\pm}\right)^2 + \left(X_{e,j}^{\pm}\right)^2}\right)\cdot d_i\left(1 + X_{d,j}^{\pm}\right)\right]}{\left(d_{i+1}\left(1 + X_{d,i+1}^{\pm}\right) + d_i\left(1 + X_{d,i}^{\pm}\right)\right)} \tag{14}$$

level (see also EN ISO 748 (2007), p. 25):

$$X_Q = \left[X_m^2 + \frac{\sum_{i=1}^{m-1}\left(b_{i+1} - b_i \cdot \frac{\bar{v}_{i+1}\cdot d_{i+1} + \bar{v}_i \cdot d_i}{2}\right)^2 \left(X_{b,i}^2 + X_{d,i}^2 + X_{p,j}^2 + \frac{1}{n}\left(X_{c,i}^2 + X_{e,i}^2\right)\right)}{\sum_{i=1}^{m-1}\left(b_{i+1} - b_i \cdot \frac{\bar{v}_{i+1}\cdot d_{i+1} + \bar{v}_i \cdot d_i}{2}\right)^2} \right]^{1/2} \tag{17}$$

The overall percentage uncertainty X_Q on the discharge is then greyified in a similar manner to that previously considered for the individual components of uncertainty, i.e., by introducing the grey number $X_Q^{\pm}$ corresponding to an interval with the central value 0 and whose boundaries X_Q^- and X_Q^+ are the percentage error/uncertainty equal to $-X_Q$ and $+X_Q$, respectively:

$$X_Q^{\pm} = \pm X_Q = \pm \left[X_m^2 + \frac{\sum_{i=1}^{m-1}\left((b_{i+1} - b_i) \cdot \frac{\bar{v}_{i+1}\cdot d_{i+1} + \bar{v}_i \cdot d_i}{2}\right)^2 \left(X_{b,i}^2 + X_{d,i}^2 + X_{p,j}^2 + \frac{1}{n}\left(X_{c,i}^2 + X_{e,i}^2\right)\right)}{\sum_{i=1}^{m-1}\left((b_{i+1} - b_i) \cdot \frac{\bar{v}_{i+1}\cdot d_{i+1} + \bar{v}_i \cdot d_i}{2}\right)^2} \right]^{1/2} \tag{18}$$

Therefore, the grey discharge is given by:

$$Q^{\pm} = Q\left(1 + X_Q^{\pm}\right) \tag{19}$$

In short, by using type I greyification, all the sources of uncertainty affecting the discharge measurement are greyified and combined according to Equation (16); by using type II greyification the crisp sources of uncertainty are combined according to Equation (17) and then greyified by using Equation (18).

Grey discharge estimate by the entropy method

The estimate of the grey discharge is performed first by estimating the average velocity $v_m^{\pm}$ using the following grey formula of the entropy method (see also Equation (2)):

$$v_m^{\pm} = \Phi^{\pm}(M) \cdot v_{\max}^{\pm} \tag{20}$$

where $\Phi^{\pm}(M)$ is a grey parameter and $v_{\max}^{\pm}$ is the grey maximum flow velocity. $v_{\max}^{\pm}$ is estimated from the maximum crisp flow velocity, considering that it is surveyed using several measures of velocity points within a narrow strip in the central portion of the upper flow area and selecting the greatest (Moramarco et al. 2011). There are two sources of uncertainty in the grey estimate of the maximum flow velocity, that is: (a) the exposure time $X_e^{\pm}$ and (b) the current meter measurement, $X_c^{\pm}$. Therefore, the maximum grey flow velocity is:

$$v_{\max}^{\pm} = v_{\max}\left(1 + \sqrt{\left(X_c^{\pm}\right)^2 + \left(X_e^{\pm}\right)^2}\right) \tag{21}$$

where $v_{\max}$ is the measured maximum crisp flow velocity.

Once the mean grey flow velocity $v_m^{\pm}$ is estimated by Equation (20), the grey discharge $Q^{\pm}$ is finally assessed by multiplying $v_m^{\pm}$ and the corresponding grey flow area $A^{\pm}$:

$$Q^{\pm} = v_m^{\pm} \cdot A^{\pm} \tag{22}$$

where $A^{\pm}$ is given as in the velocity-area method (see Equations (11), (12) and (14)):

$$A^{\pm} = \sum_{i=1}^{m-1}\left(b_{i+1}\left(1 + X_{b,i+1}^{\pm}\right) - b_i\left(1 + X_{b,i}^{\pm}\right)\right) \cdot \frac{d_{i+1}\left(1 + X_{d,i+1}^{\pm}\right) + d_i\left(1 + X_{d,j}^{\pm}\right)}{2} \tag{23}$$

The approach described so far is related to direct application of the grey formulation of the entropy method; however, as previously noted, to apply the entropy approach (see Equation (20)) one needs to assess $\Phi^{\pm}(M)$. In practice, $\Phi^{\pm}(M)$ is estimated from a sample of n_{obs} observed pairs of mean and maximum flow velocity, wherein for each pair the grey uncertainty is assessed thus obtaining pairs of grey values $v^{\pm}_{\max,i}$ and $v^{\pm}_{m,i}$ with $i = 1{:}n_{\text{obs}}$. Once these pairs are known, $\Phi^{\pm}(M)$ is estimated through grey linear regression, which represents a variant of the fuzzy linear regression originally proposed by Tanaka *et al*. (1982) and subsequently amended by Hojati *et al*. (2005) (see also Westerberg *et al*. 2011). In practice, by using the grey linear regression an entropic parameter, which is itself a grey number, is obtained; this grey number includes uncertainties due to the mean and maximum flow velocities used during the calibration and, in turn, allows for the estimate with uncertainty of the mean flow velocity starting from a maximum grey flow velocity.

In more detail, referring to Figure 1, $\Phi^{\pm}(M)$ is estimated by looking for the lower, $\Phi^{-}(M)$, and upper, $\Phi^{\pm}(M)$, extremes, so that

$$\sum_{i=1}^{n_{\text{obs}}} \left(d_i^{\text{LR}} + d_i^{\text{UL}}\right) \tag{24}$$

is minimum. d_i^{LR} (LR stands for 'Lower-Right') is the distance between the reference point of the 'observed' crisp pair $\left(v^{+}_{\max,i}, v^{-}_{m,i}\right)$, i.e., the vertex at the lower right of ith rectangle defined by the pair of grey values $v^{\pm}_{\max,i}$ and $v^{\pm}_{m,i}$, and the corresponding point assessed as $\Phi^{-}(M)\cdot v^{+}_{\max,i}$

$$d_i^{\text{LR}} = \left|v^{-}_{m,i} - \Phi^{-}(M)\cdot v^{+}_{\max,i}\right| \tag{25}$$

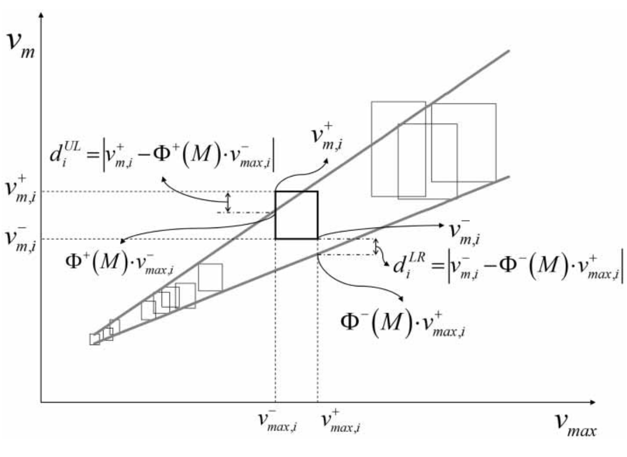

Figure 1 | Estimate through grey linear regression of the parameter $\Phi^{\pm}(M)$.

Analogously, d_i^{UL} (UL stands for 'Upper-Left') is the distance between the reference point of the 'observed' crisp pair $\left(v^{-}_{\max,i}, v^{+}_{m,i}\right)$, i.e., the vertex at the upper left of ith rectangle defined by the pair of grey values $v^{\pm}_{\max,i}$ and $v^{\pm}_{m,i}$, and the corresponding point assessed as $\Phi^{+}(M)\cdot v^{-}_{\max,i}$, i.e.:

$$d_i^{\text{UL}} = \left|v^{+}_{m,i} - \Phi^{+}(M)\cdot v^{-}_{\max,i}\right| \tag{26}$$

In practice, to estimate $\Phi^{\pm}(M)$, the maximum grey velocities $v^{\pm}_{\max,i}$ are obtained from the measured crisp values $v_{\max,i}$, greyified according to Equation (21); while the mean grey velocities $v^{\pm}_{m,i}$ are inferred from the grey discharge estimated by the velocity-area method, greyified following the type I or type II procedures described in the section Grey discharge estimated by the velocity-area method, and then dividing the grey discharge thereby obtained by the corresponding grey flow area, estimated by Equation (23), yielding:

$$v^{\pm}_{m} = \frac{Q^{\pm}}{A^{\pm}} \tag{27}$$

To sum up, the direct application of the grey formulation of the entropy method implies that the maximum grey flow velocity is estimated by using Equation (21) and is used to estimate the grey average velocity by using Equation (20), where $\Phi^{\pm}(M)$ has been previously estimated through grey regression (see Equations (24)–(26)); finally, the grey discharge is assessed through Equation (22).

CASE STUDY

Three gauged river sections located along the Tiber River in central Italy were selected to compare the uncertainty in discharge estimation through the grey velocity-area method and the grey entropy method. The selected hydrometric sites of Santa Lucia, Ponte Felcino and Ponte Nuovo are gauged for carrying out flow velocity measurements by current meter from cableways. The location of the investigated sites is shown in Figure 2, while Table 1 shows the number and main properties of the available flow velocity measurements, reported along with the mean section width. As is evident, the Ponte Nuovo site is characterized by the most

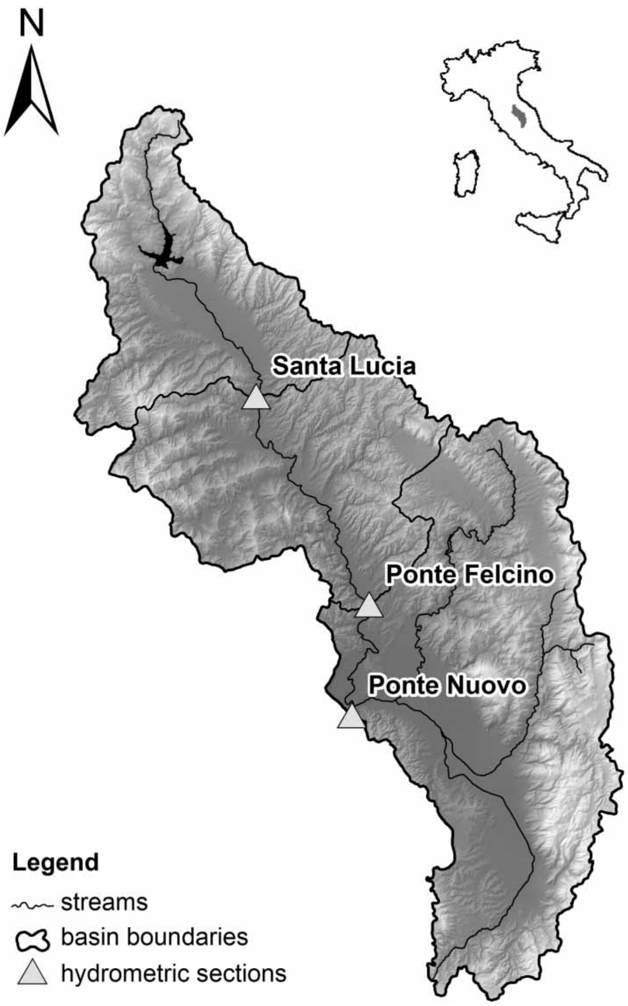

Figure 2 | Location of the hydrometric sections selected for this study.

Table 1 | Selected hydrometric sections: number of available velocity measurements, n_{obs}; minimum stage, h_{min}, and maximum stage, h_{max}, observed during velocity measurements; minimum discharge, Q_{min}, and maximum discharge, Q_{max}, derived from velocity measurements through velocity-area method. The mean section width is also shown

Gauged section	Mean width (m)	n_{obs}	h_{min} (m)	h_{max} (m)	Q_{min} (m^3/s)	Q_{max} (m^3/s)
Ponte Nuovo	48	22	0.44	5.70	5.76	541.6
Ponte Felcino	37	9	0.99	4.60	2.54	411.54
Santa Lucia	23	11	0.46	3.56	1.71	195.24

comprehensive sample of velocity measurements, which are carried out at a wide range of water levels, from 0.44 m up to 5.76 m. For this hydrometric section, the maximum observed stage was equal to about 7.8 m which occurred during the very severe flood of November 2005 and during which it was not possible to perform discharge measurements. As regards the Santa Lucia and Ponte Felcino sections, a smaller sample of flow measurements is available for medium–high water level values. In no case were significant changes of the water level during the flow measurements observed, in accordance with the main hypothesis of the velocity-area method.

As regards the grey uncertainty quantification of each component of the discharge estimation (for both methods), this is addressed by assuming that the lower/upper boundary of the grey number representative of the uncertainty (see for instance $X^{\pm}_{b,j}$ in Equation (11)) is equal to ± the corresponding percentage error assessed at the confidence level 95%, as defined by EN ISO 748 (2007) (pp. 40–43). By way of example, Table 2 shows the numerical values of each component of the grey uncertainty for the velocity-area method applied to the Ponte Nuovo gauged section, for which around $n_p = 10$ velocity point measurements were carried out on each of the $n_m = 10$ verticals, all having abscissa $b < 100$ m; sampling of the velocity points is performed at depth d, which is always less than 6 m, with exposure times of 30 s and measured velocity greater than 0.4 m/s.

With the aim of representing the results, and in particular the grey discharge and stages in the Q-h plane, the crisp stage measurements are also greyified, considering two sources of uncertainty, namely the grey uncertainty of the equipment error, $X^{\pm}_{ins}$, and the uncertainty due to the determination of the mean reference gauge height, corresponding to the measured discharge $X^{\pm}_{ref}$. As detailed information about the accuracy of stage measuring equipment is unavailable, it was assumed that $X^{\pm}_{ins} = \pm 1\,\text{cm}$ $X^{\pm}_{ref} = \pm 1\,\text{cm}$, as proposed by Shrestha & Simonovic (2010). It is worth noting that both uncertainty components are independent of the measured stage, h, and, hence do not represent a percentage fraction (see the section Grey uncertainty in discharge assessment at gauged sites). Based on this, the total uncertainty is surmised as (see also Shrestha & Simonovic 2010):

$$h^{\pm} = h + \left[X^{\pm}_{ins} + X^{\pm}_{ref}\right] \tag{28}$$

which, considering the values adopted, yields $h^{\pm} = h \pm 2\,\text{cm}$.

Table 2 | Equipped section – Ponte Nuovo. Numerical value of each component forming the grey uncertainty in the velocity-area method, obtained from the corresponding components of uncertainty evaluated with a confidence level of 95%, by the EN ISO 748 (2007) standards (see Annex E of EN ISO 748 (2007))

Uncertainty component		Assumptions	95% confidence level	Grey number
X_b	Uncertainty in width	$b < 100$ m	$0.15 \times 2 = 0.3\%$	[−0.003, 0.003]
X_d	Uncertainty in depth	$0.4 < d < 6$ m	$0.65 \times 2 = 1.3\%$	[−0.013, 0.013]
X_p	Uncertainty in the measurement of mean velocity due to limited number n_p of points in the vertical	$n_p \cong 10$	$0.5 \times 2 = 1\%$	[−0.01, 0.01]
X_c	Uncertainty in point velocity measurement due to current meter rating error	Individual rating; velocity above 0.5 m/s	$0.5 \times 2 = 1\%$	[−0.01, 0.01]
X_e	Uncertainty in point velocity measurements due to limited times of exposure t_{exp}	$t_{\mathrm{exp}} = 30$ s	$4 \times 2 = 8\%$	[−0.08, 0.08]
X_m	Uncertainty in measurement of mean velocity due to limited number n_m of verticals	$n_m = 10$	$4.5 \times 2 = 9\%$	[−0.09, 0.09]

For the estimate of the grey parameter $\Phi^{\pm}(M)$, obtained by minimizing Equation (24), the algorithm SCE-UA (Shuffled Complex Evolution – University of Arizona; Duan *et al.* 1992) was used and refined with the MATLAB™ *fmincon* function based on sequential quadratic programming (Powell 1983; Schitlowski 1985). In particular, a grey $\Phi^{\pm}(M)$ value was estimated through grey regression for each of the three selected hydrometric sections, by using, for each regression, the corresponding set of n_{obs} couples of grey mean and maximum velocities observed in the section (see Table 1, column 3, n_{obs}).

In the following section, discharge estimates yielded by the two approaches (velocity-area and entropy method) combined with type I and type II greyification are illustrated and compared.

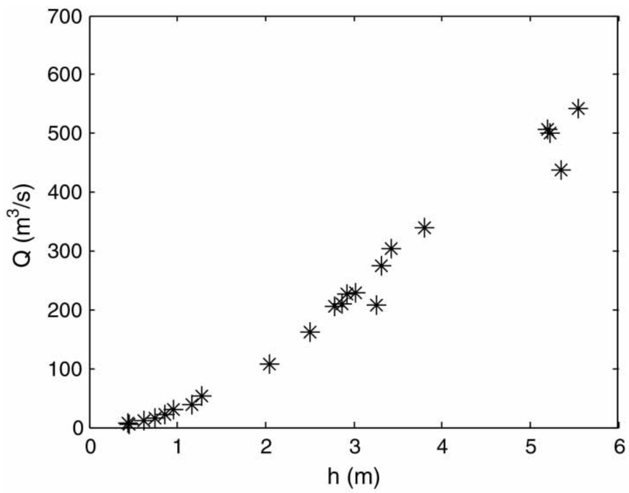

Figure 3 | Equipped section – Ponte Nuovo: crisp stages *h* and discharges *Q* of the measurement set indicated in Table 1.

Results and discussion

Considering the Ponte Nuovo gauged section and the set of available data detailed in Table 1, Figure 3 shows the crisp measurements of stage and discharge obtained by applying the velocity-area method (see Equation (1)). Figure 4 illustrates the corresponding grey values of discharge. In particular, Figure 4(a) shows the results obtained using type I greyification based on the aggregation of the single components of uncertainty (see Equation (16)), while in Figure 4(b) the discharge referring to type II greyification (see Equations (18) and (19)) is plotted. By inspecting Figure 4, it is evident that the type I greyification leads to a 'wider' grey discharge, i.e., a greater estimate of the overall uncertainty, than that obtained through the type II greyification. For instance, with reference to the maximum value of the observed crisp discharge ($Q = 541$ m^3/s), corresponding to $h = 5.54$ m (see Figure 3), type I greyification yields a grey discharge of $Q^{\pm} = [440, 656]$, corresponding to a width of grey number of 216 m^3/s; instead, when type II greyification is employed, $Q^{\pm} = [486, 596]$ is estimated, corresponding to a considerably narrower width equal to 110 m^3/s.

Detailed analysis of these grey numbers reveals that type I greyification leads to the estimation of a grey discharge whose boundaries ($Q^{-} = 440\,\mathrm{m^3/s}$, $Q^{+} = 656\,\mathrm{m^3/s}$) are not symmetrical with respect to the corresponding crisp value ($Q = 541$ m^3/s). Indeed, the lower boundary, Q^{-}, is equal to Q 0.813, which corresponds to a decrease with respect to the

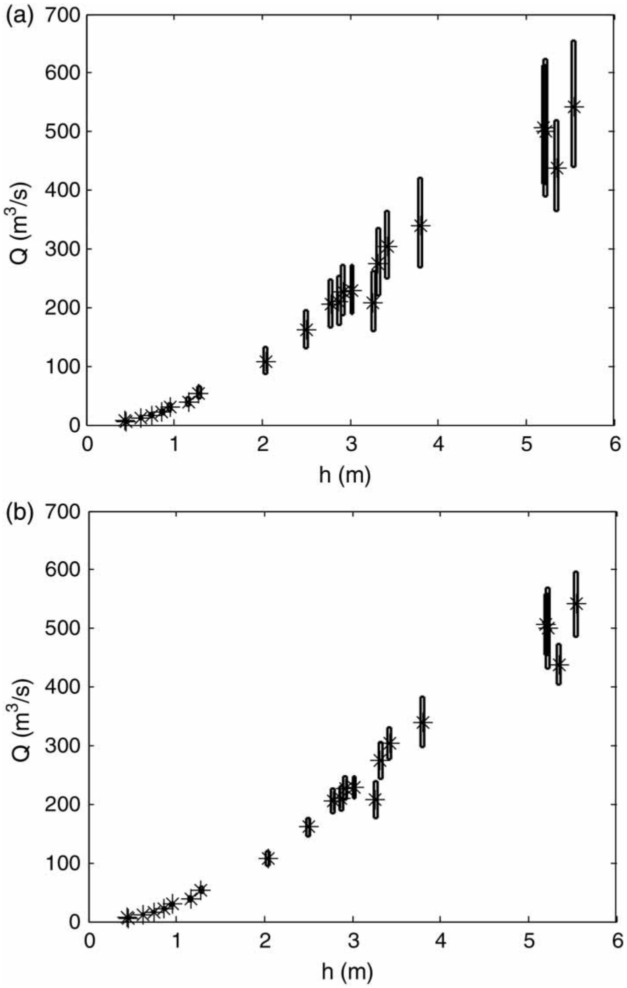

Figure 4 | Equipped section – Ponte Nuovo: grey stages and discharges obtained through (a) type I greyification approach and (b) type II greyification approach.

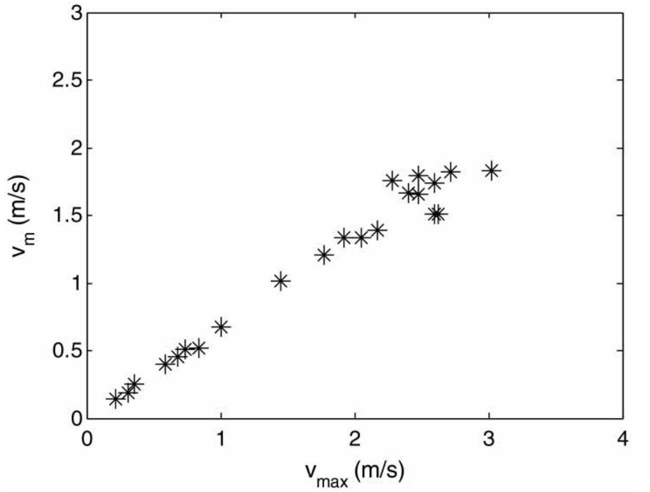

Figure 5 | Equipped section – Ponte Nuovo: crisp maximum and mean velocities belonging to the measurement set indicated in Table 1 (see also Figure 3).

crisp discharge of 17.7%, while the upper boundary Q^+ is equal to Q 1213, which corresponds to an increase with respect to the crisp discharge of about 21%. This result is the consequence of the grey mathematics in the context of type I greyification and, in particular, of the product operation expressed by Equation (8). To make this concept clearer, an example is shown. The following product is considered:

$$x^{\pm} = x \cdot \left(1 + X_{\alpha}^{\pm}\right) \cdot \left(1 + X_{\beta}^{\pm}\right) \tag{29}$$

where x is a generic measurement and $X_{\alpha}^{\pm}$ and $X_{\beta}^{\pm}$ are two components of the grey uncertainty equal to $[-0.2, +0.2]$ and $[-0.5, +0.5]$, respectively. Application of the definition of grey product (see Equation (8)) yields $x^{\pm} = x \cdot (1 \pm 0.2) \cdot (1 \pm 0.5) = x \cdot [0.8, 1.2] \cdot [0.5, 1.5] = x \cdot [0.4, 1.8]$, i.e., x^- refers to a decrease with respect to the crisp value x of 60% and x^+ to an increase of 80%. If, on the other hand, the type II greyification approach was applied, we would obtain an estimate of the grey discharge whose boundaries ($Q^- = 486\,\mathrm{m}^3/\mathrm{s}$, $Q^+ = 596\,\mathrm{m}^3/\mathrm{s}$) are symmetrical with respect to the corresponding crisp value ($Q = 541\,\mathrm{m}^3/\mathrm{s}$). This is to be expected considering that in this second case, by means of Equation (17), a total percentage uncertainty of the crisp discharge X_Q is estimated and in turn greyified as $X_Q^{\pm} = \pm X_Q$. In the specific case, the result is $X_Q^{\pm} = \pm X_Q = \pm 0.102$, which corresponds to a variation of $\pm 10.2\%$ and, as a consequence, the grey discharge $Q^{\pm} = Q\left(1 + X_Q^{\pm}\right) = Q(1 \pm 0.102)$ is centred with respect to the crisp discharge value.

Incidentally, both the considerations on the extent of the uncertainty, i.e., on the width of the grey number representative of the discharge, and those on the symmetry of such a grey number with respect to the crisp discharge value are in full agreement with that observed by Shrestha & Simonovic (2010), with reference to the application of the fuzzy technique for the characterization of uncertainty.

Considering the application of the entropy method, Figure 5 shows the crisp values of maximum and mean flow velocity for the set of measurements plotted in Figure 3, while Figures 6(a) and 6(b) show the corresponding greyified values. In particular, the greyification of the maximum velocity has been carried out according to Equation (21),

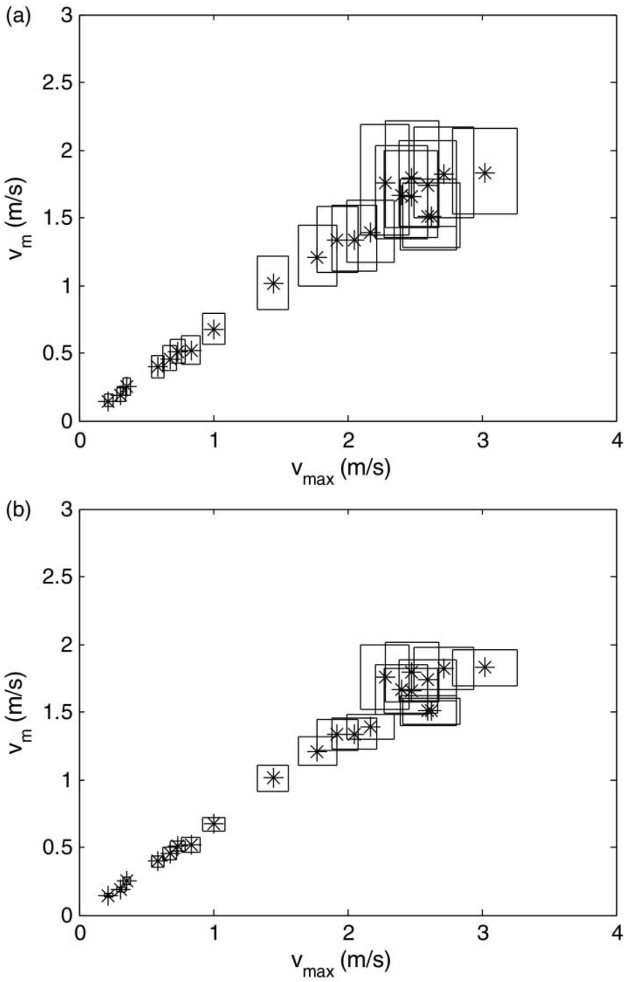

Figure 6 | Equipped section – Ponte Nuovo: grey quantification of the uncertainty on the maximum and mean velocities. The uncertainty pertaining to the mean velocities is obtained through the use of the velocity-area method greyified (a) through the type I approach and (b) through the type II approach.

Table 3 | Values of the parameter $\Phi^{\pm}(M)$ estimated by the grey linear regression for the three equipped sections considered in this study

	Greyification approach	
Section	**Type I**	**Type II**
Ponte Nuovo	[0.51, 0.87]	[0.56, 0.79]
Ponte Felcino	[0.50, 0.89]	[0.55, 0.81]
Santa Lucia	[0.50, 0.88]	[0.55, 0.80]

while the mean velocity was greyified on the basis of Equation (27) using, in the first case, the grey discharge estimated by velocity-area method, greyified by the type I approach (Figure 6(a)), and in the second case, the grey discharge once again estimated by the velocity-area method but greyified by the type II approach (Figure 6(b)).

In both cases, the grey flow area is obtained by Equation (23). Consequently, by comparing Figures 6(a) and 6(b) it can be found that the grey maximum velocities are the same, while, similarly to what has already been previously observed with reference to the grey discharge, type I greyification yields larger grey mean velocities compared to those obtained through type II greyification. This is reflected in a different value of the parameter $\Phi^{\pm}(M)$, estimated by the grey linear regression, which is equal to [0.51, 0.87] and [0.56, 0.79] in type I and II, respectively (see also Table 3). Figures 7(a) and 7(b) show, in addition to the values of

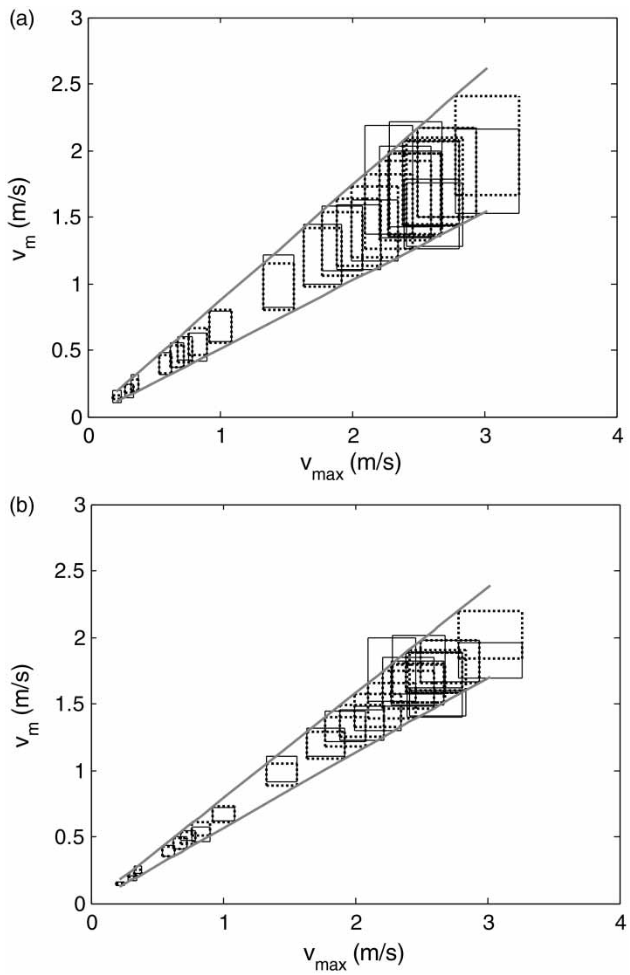

Figure 7 | Equipped section – Ponte Nuovo: greyified entropy method. Comparison between the mean velocities obtained through the velocity-area method greyified (a) through the type I approach and (b) through the type II approach (black solid line) with those estimated on the basis of the grey maximum velocity given by Equation (20) (black dashed line).

maximum and mean velocity, greyified through the two approaches shown in Figures 6(a) and 6(b), the curves obtained by the grey linear regression and the corresponding grey values of mean velocity, estimated through Equation (20) as a function of the grey maximum velocity.

Finally, for each of the two greyification approaches, starting from the grey mean velocities thereby obtained, by applying Equation (22) the grey discharges were estimated considering the grey flow area calculated through Equation (23). The grey discharge values obtained are shown in Figures 8(a) and 8(b) and compared with those obtained using the velocity-area method (see Figure 4). Inspection of the two figures makes it clear that, for the same greyification

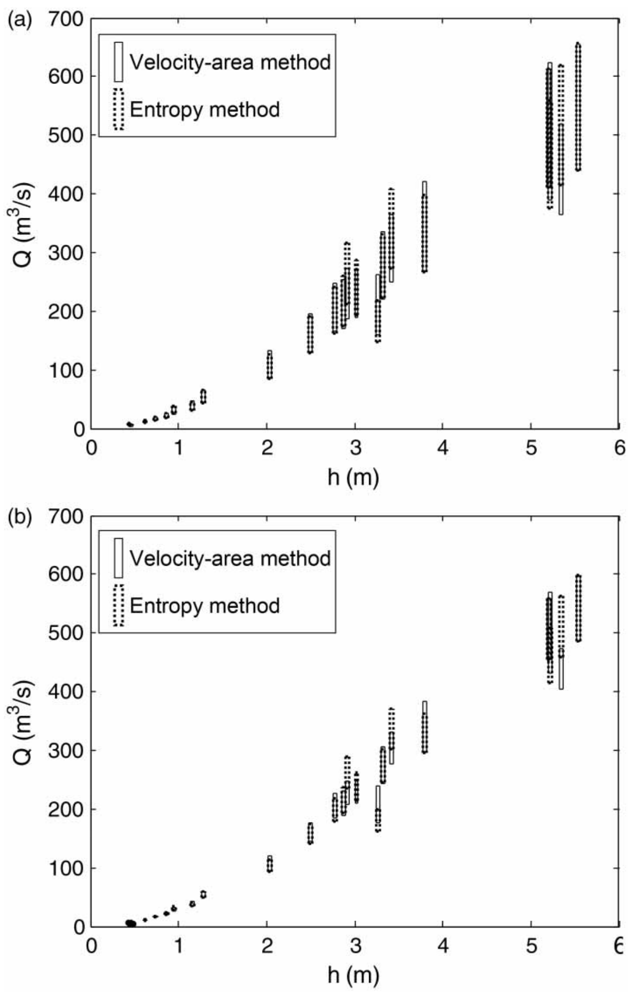

Figure 8 | Equipped section – Ponte Nuovo: comparison between the grey discharges estimated through the velocity-area method greyified (a) through the type I approach and (b) through the type II approach, with the corresponding grey discharges estimated through the grey entropy method.

method, the grey discharge obtained by the grey formulation of the entropy method is quite similar to that obtained through the formulation of the velocity-area method.

Table 4 shows the mean absolute error and the percentage error affecting the grey discharges estimated through the entropy method with respect to those estimated using the velocity-area method; in the same table, the average percentage error on the estimation of the lower and upper limit of the grey numbers of discharges estimated through the entropy method is shown along with the one estimated through the velocity-area method. For the formal definition of these errors and percentages, we refer the reader to Appendix B (available online at http://www.iwaponline.com/jh/016/160.pdf).

It is worth noting that the first two types of errors indicate how much the widths of the grey numbers, representing the uncertainty, differ, regardless of their mutual 'position', while the errors on the upper and lower limits estimation indicate how much these grey numbers tend to overlap/deviate between each other.

From the results obtained for the Ponte Nuovo gauged section, detailed in Table 3, it can be seen that the two methods, velocity-area method and entropy method, lead to estimates of the grey uncertainty on the discharge that, in terms of width, differ on average about 10 m^3/s and 8 m^3/s, depending on whether type I or type II greyification is adopted. Considering the error on the upper and lower boundaries of the grey numbers, one can also observe that, on average, they differ by a few percent: this means that the two methods not only lead to similar widths of the grey numbers, but these numbers also tend to overlap.

Table 4 | Mean values of the absolute (AWE) and percentage (PWE) error on the amplitude and percentage error on the estimate of the lower and upper boundaries (PQ^-E and PQ^+E) of the grey discharges obtained through the entropy method with respect to those obtained through the velocity-area method, when the two approaches of greyification were applied to the three equipped sections considered in this study

Section	Greyification approach	*AWE* (m^3/s)	*PWE* (%)	PQ^-E (%)	PQ^+E (%)
Ponte Nuovo	Type I	10.6	15.2	6.3	6.4
	Type II	8.2	19.8	4.7	6.1
Ponte Felcino	Type I	4.7	12.5	12.3	7.6
	Type II	2.4	14.3	9.4	6.5
Santa Lucia	Type I	1.3	11.5	8.1	3.5
	Type II	1.1	13.6	5.3	3.5

The procedures described above were repeated for the other two gauged sections, Ponte Felcino and Saint Lucia, which were, however, characterized by a smaller sample size of measurements. In Table 3, the values of the parameter $\Phi^{\pm}(M)$ estimated by the grey linear regressions for these two sections are reported together with those of the Ponte Nuovo section. As can be observed, when the greyification type is the same, i.e., type I or II, the grey numbers $\Phi^{\pm}(M)$ obtained for the three sections are very similar, as already observed, but in a crisp case (i.e., disregarding uncertainty) by Moramarco *et al*. (2004).

Figures 9 and 10 show the grey discharge values obtained using the velocity-area method and the entropy

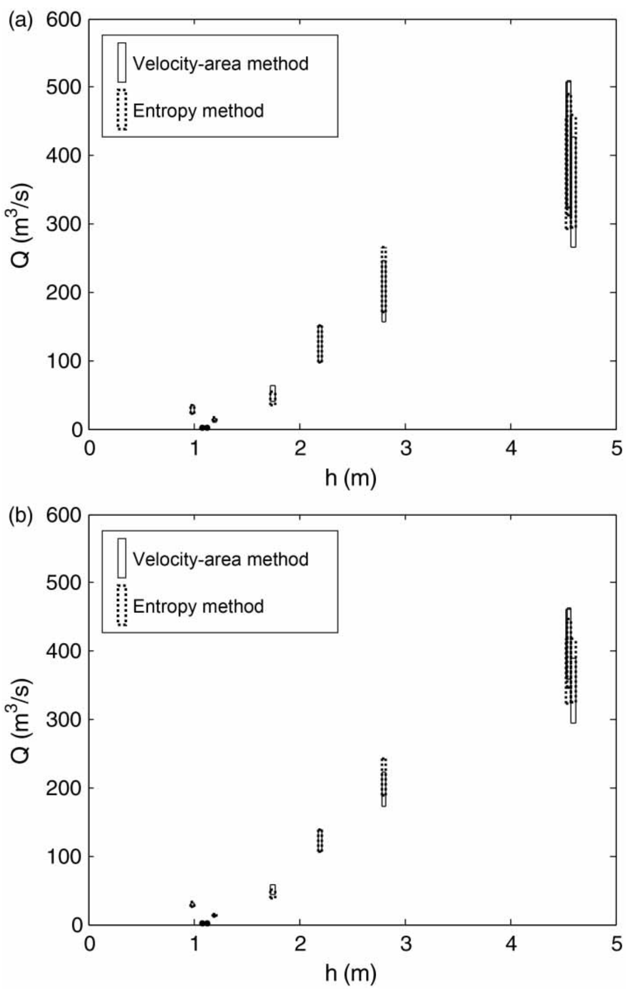

Figure 9 | Equipped section – Ponte Felcino: comparison between the grey discharges estimated through the velocity-area method greyified (a) through the type I approach and (b) through the type II approach, with the corresponding grey discharges estimated through the grey entropy method.

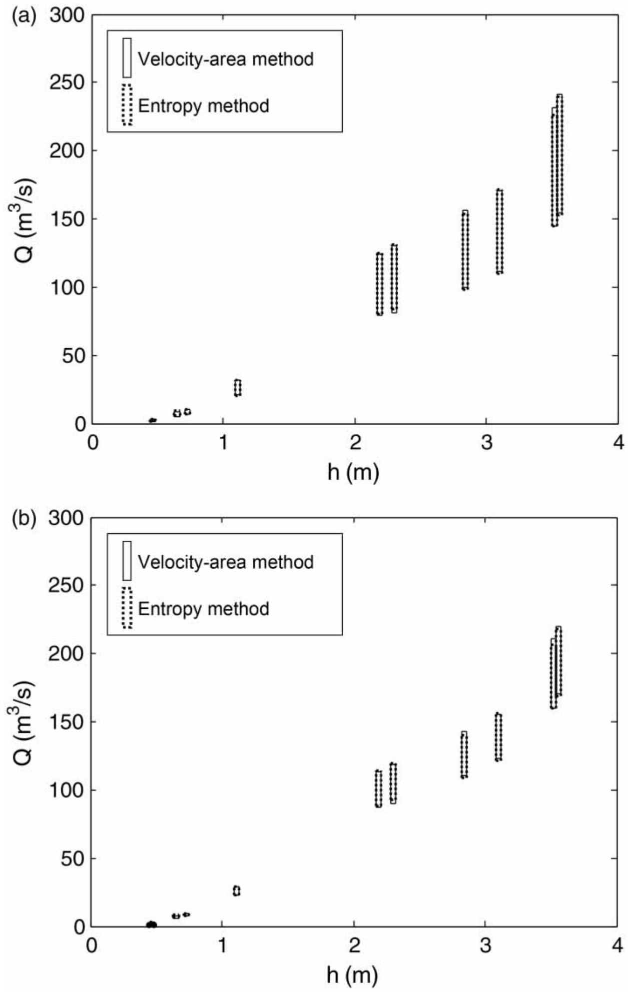

Figure 10 | Equipped section – Santa Lucia: comparison between the grey discharges estimated through the velocity-area method greyified (a) through the type I approach and (b) through the type II approach, with the corresponding grey discharges estimated through the grey entropy method.

method, for each of the two greyification approaches. In particular, one can observe that, using the same greyification type, also in the case of measurements carried out at the Ponte Felcino section (Figure 9), a good match was found between the grey numbers of the discharge obtained using the velocity-area method and those obtained by the entropy one, with errors on the width slightly lower than those obtained in the case of the Ponte Nuovo site and errors on the limits of the grey numbers slightly higher (see Table 4). In addition, one can also observe a significant increase in the mean percentage error in the estimation of the lower extreme of the grey number ($PQ^{-}E$) due to the presence of some measurements of very low flow (see Figure 9), at which lower boundaries of grey numbers very near to zero correspond; therefore, very large percentage errors correspond to small errors in magnitude.

In the case of measurements carried out at the Santa Lucia site (Figure 10), still assuming the same greyification approach, a good correspondence between the grey discharge estimated through the velocity-area method and the entropy method is observed, mainly with reference to the accuracy in estimating the grey number width (see Table 3, columns 1 and 2). Overall, the results obtained for these two cross sections confirm, therefore, what has been found with reference to the Ponte Nuovo site, namely, for a given greyification approach, the possibility of achieving through the application of the entropy method an estimation of the total uncertainty of the discharge which is equivalent to that provided by the velocity-area method. We can therefore conclude that the grey formulation of the entropy method represents a valid alternative to the grey formulation of the velocity-area method to estimate with uncertainty discharge measurements, since it provides similar results but is operatively simpler and, in addition, can be easily applied for high flood monitoring when sampling velocity points in the whole flow area is impossible or very difficult.

CONCLUSIONS

The discharge measurements provided by grey formulations of the velocity-area and entropy methods are analysed and compared. Both the velocity-area and entropy methods were greyified through two different approaches: the first one based on the aggregation through the grey mathematics of all the uncertainty components affecting the discharge measurement, each one characterized by a grey number (type I greyification), and the second based on the greyification of the total uncertainty, provided by the EN ISO 748 guidelines (type II greyification).

Analysis of the results obtained through the application to the data sets from three equipped sections of the Tiber River in central Italy revealed that, given its very nature, type I greyification leads to a wider estimate of the uncertainty affecting the discharge measurement than that yielded by type II greyification. Furthermore, the grey discharge number provided by the type I greyification is not symmetrical with respect to the corresponding crisp discharge measurement, unlike the one provided by type II greyification, in agreement with that observed, but with reference to the application of the fuzzy technique by Shrestha & Simonovic (2010).

It was also observed that, being the greyification approach the same, the grey-based velocity-area method and entropy method provide very similar estimates of the total uncertainty affecting the discharge measurement. We can therefore conclude that the grey formulation of the entropy method represents a valid alternative to the grey formulation of the velocity-area method to estimate discharge measurements with uncertainty, since it provides similar results but is operatively simpler.

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First received 24 March 2013; accepted in revised form 28 September 2013. Available online 23 November 2013