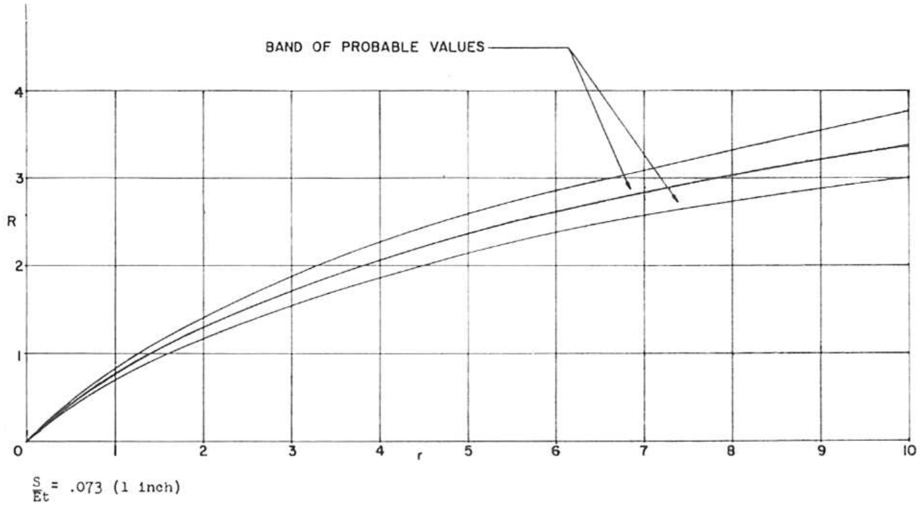

FIG. 17 PARTICULAR SPRING-BACK CORRECTION CURVE (FROM FIG. 15)

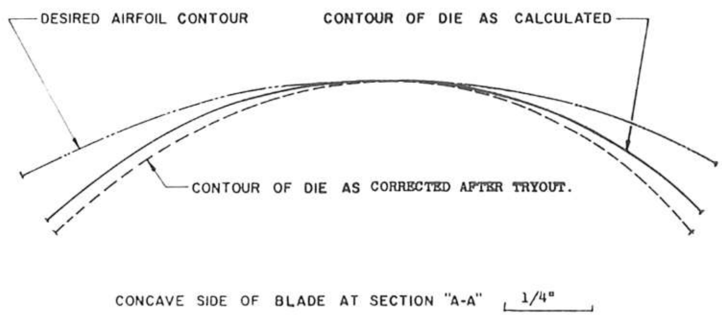

FIG. 18 CONCAVE SIDE OF BLADE AT SECTION "A-A"

(*a*) Experimental corrections to the tool will still be necessary to compensate for local and end effects, as well as for twist, etc.

(*b*) The difference between the high and low curves in Fig. 17 is not appreciable.

(*c*) The lion's share of the spring-back correction is accomplished quickly and cheaply which would otherwise be an unpleasant cut-and-try process.

Finally, it can be seen that these results are limited to two-dimensional problems and that they can be readily extended:

(*a*) To predict failure during forming.

(*b*) To predict spring back during stretch-forming operations.

Discussion

M. E. CIESLICKI.[4] In Fig. 10 of the paper the values used for S and E do not correspond to the data used for design in the Jet Engine Department of General Electric. We use $S = 62{,}000$ and $E = 35.3 \times 10^6$. Using these values, the values of $(RS)/(Et)$ will be reduced by 25 per cent. This would put the theoretical curve somewhat below the experimental points plotted.

S. H. CRANDALL.[5] In presenting extensive test data for large spring backs the author has made a valuable contribution. He shows that in spite of the many complicating factors in sheet-metal forming the data correlate remarkably well with a simplified version of Schroeder's[3] theory.

Regarding this theory the writer would like to make two suggestions. First, in the elastic spring back of a curved sheet the sheet is bending as a plate rather than as a beam and hence the plate stiffness should be used instead of the beam stiffness. When the author's analysis is repeated on the plate basis the only difference in the final equation is that the factor $(RS)/(Et)$ is replaced by $RS(1 - \eta^2)/Et$ where η is Poisson's ratio. For most metals this correction is of the order of 10 per cent.

The calculations in both Schroeder's and the author's analyses can be simplified by taking advantage of the fact that the spring back is elastic. It is only necessary to obtain the bending moment M which exists across the section when the sheet is in the tool. The spring back is then just the result of the elastic change in curvature due to an equal and opposite M. Thus

$$\frac{1}{R} - \frac{1}{r} = \frac{12M}{Et^3} \text{ or } \frac{12M}{Et^3}\ (1 - \nu^2)$$

according to whether beam or plate theory is used.

The difference between Schroeder's and the author's analyses lies in the assumption of the shape of the stress distribution making up the bending moment in the tool. The author assumes (*a*) a flat-topped yield for all metals whereas Schroeder would use (*b*) the actual stress-strain curve for each material. The advantage of (*a*) is that it permits all materials to be represented on a single nondimensional chart. In any particular case where good

[4] Supervisor, J-79 Manufacturing and Materials Control, General Electric Company, Aircraft Gas Turbine Division, Cincinnati, Ohio.

[5] Associate Professor of Mechanical Engineering, Massachusetts Institute of Technology, Cambridge, Mass. Mem. ASME.

stress-strain data were available the labor of going to (*b*) would not be excessive.

M. F. Spotts.[7] The author has performed a useful service in presenting an equation for the first approximation of the die radius R when the final radius of curvature r is specified for a formed metal part. Considering the simplified nature of the assumed stress-strain curve, the experimental check with the results of the equation is very good indeed.

It might be mentioned however that the equation can be obtained without the need of integration in the following manner. In accordance with the assumed stress-strain curve, the stress distribution over a cross section of the material in the die is shown as O-A-B in Fig. 19(*A*), herewith. Resultant forces and moment

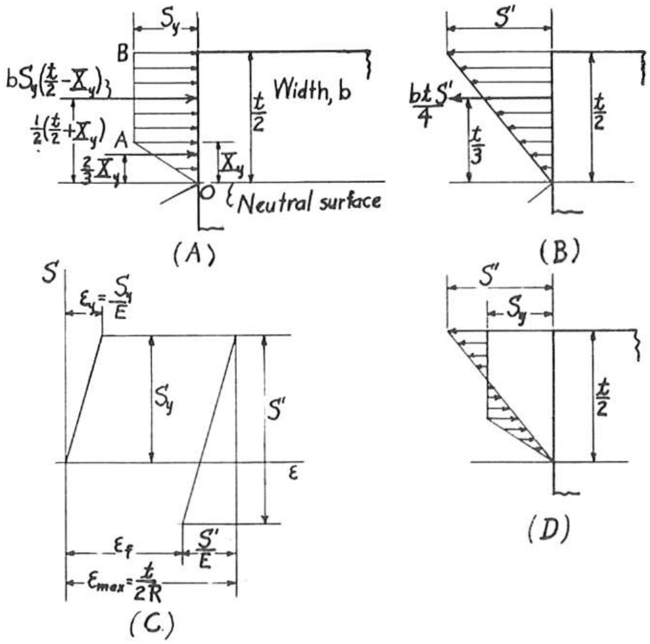

Fig. 19

arms for the triangular and rectangular stress distributions are indicated. The bending moment M for the cross section then is

$$M = 2\left[\frac{1}{2}\, bS_yX_y\, \frac{2}{3}\, X_y + bS_y\left(\frac{t}{2} - X_y\right)\frac{1}{2}\left(\frac{t}{2} + X_y\right)\right]$$

$$= \frac{bS_y}{12}(3t^2 - 4X_y^2) \qquad [1]$$

The elementary beam equation $S = EX/R$ is valid for the elastic range. For point A in Fig. 19 this becomes

$$S_y = \frac{EX_y}{R} \qquad [2]$$

Substitution into Equation [1] gives

$$\frac{M}{EI} = \frac{1}{R}\left[3\,\frac{X_y}{t} - 4\left(\frac{X_y}{t}\right)^3\right] \qquad [3]$$

where $I = bt^3/12$.

When the part is removed from the die, moments M are released. This is equivalent to superposing the elastic stress distribution of Fig. 19(*B*) for which

$$\frac{1}{r_0} = \frac{M}{EI}$$

where $1/r_0$ is the change in curvature resulting from the release of moments M.

The final curvature

$$\frac{1}{r} = \frac{1}{R} - \frac{1}{r_0} = \frac{1}{R} - \frac{M}{EI}$$

$$\frac{1}{r} = \frac{1}{R} - \frac{1}{R}\left[3\,\frac{X_y}{t} - 4\left(\frac{X_y}{t}\right)^3\right] \qquad [4]$$

Substitution for X_y from Equation [2] gives

$$\frac{R}{r} = 4\left(\frac{RS_y}{Et}\right)^3 - 3\left(\frac{RS_y}{Et}\right) + 1 \qquad [5]$$

This is the author's equation which permits die radius R to be calculated.

The foregoing result also can be obtained from consideration of the stress-strain diagram of Fig. 19(*C*). Elementary beam equation $\epsilon = X/R$ is valid for both elastic and plastic deformation. Maximum elongations for upper and lower surfaces when the material is in the die is given by

$$\epsilon_{max} = \frac{t}{2R} \qquad [6]$$

Superposition of stress S' upon release from the die gives an elongation on the surface of S'/E in the opposite direction. The final elongation ϵ_f for the top surface is

$$\epsilon_f = \frac{t}{2R} - \frac{S'}{E} \qquad [7]$$

Stresses S' produce the moment

$$M = 2\,\frac{bt\ S'}{4}\cdot\frac{t}{3} = \frac{bt^2\ S'}{6}$$

After substitution of Equation [1] this becomes

$$S' = \frac{S_y}{2}\left(3 - 4\,\frac{X_y^2}{t^2}\right) \qquad [8]$$

The foregoing, together with the value of $1/R$ from Equation [2], should now be substituted into Equation [7]

$$\epsilon_f = \frac{t}{2}\,\frac{S_y}{EX_y} - \frac{S_y}{2E}\left(3 - 4\,\frac{X_y^2}{t^2}\right)$$

$$= \frac{S_y}{2E}\left[\frac{t}{X_y} - 3 + 4\,\frac{X_y^2}{t^2}\right] \qquad [9]$$

Equation [6], as applied to the material after removal from the die, becomes

$$\epsilon_f = \frac{t}{2r} \qquad [10]$$

After Equation [9] is substituted, the result can be easily reduced to Equation [5]. Residual stresses in the material are illustrated by Fig. 19(*D*).

Author's Closure

Professor Spotts has kindly added a more geometrical approach to the mathematical derivation which always lends clarity and versatility.

Professor Crandall also has contributed an elegant derivation. As was pointed out by other discussions, there is nothing novel

[7] Professor of Mechanical Engineering, Northwestern University, Evanston, Ill. Mem. ASME.

in the theory. Rather, it is the purpose of this paper to show a simple method for plotting experimental data and thus making it generally useful. In addition, metals are not produced with sufficient consistency to make a high degree of precision in spring-back prediction possible. Thus the appreciable additional labor of resorting to stress-strain diagram integration is probably not worth while.

The author believes that practical application of this method has been delayed by the complication of previous methods and hopes that the foregoing simplifications will prove of use to industry.

In view of the interest shown by manufacturers of coil springs, Professor Crandall has also co-operated with the author in deriving a theoretical spring-back formula for round bars and wires. The same basic assumptions have been made. In addition

$$D = \text{cross section at diameter of wire}$$

$$K = \frac{RS}{ED}$$

Then: $$\frac{R}{r} = 1 - \frac{2}{\pi}\sin^{-1}(2K) - \frac{4K}{3\pi}(5-8K^2)\sqrt{(1-4K^2)}$$

This formula gives a curve similar to the theoretical curve of Fig. 5 and is plotted in Fig. 20.

It is regretted that no experimental data are yet available. Hence this formula presents only a skeleton on which test data can be accumulated and a useful empirical curve drawn.

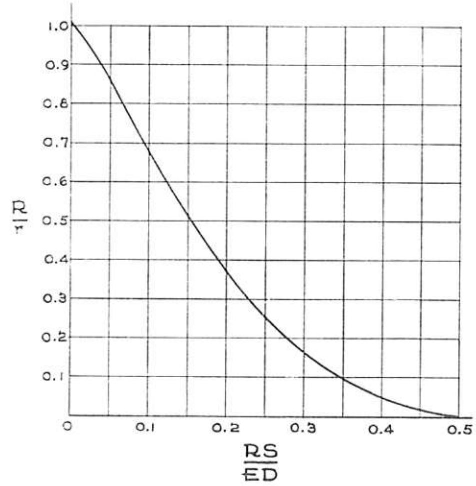

Fig. 20 Spring-Back Factor for Parts Having a Circular Cross Section