Efficiency of Temporal Integration of Sinusoidal Flicker

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PURPOSE. Detection efficiency for flickering stimuli of constant duration decreases with increasing temporal frequency. Increasing frequency in this case also implies increasing number of flicker cycles. The current study was conducted to investigate whether this result could be due to the limited ability of the central detector to integrate flicker cycles.

METHODS. Flicker sensitivity was measured at 1 to 20 Hz in strong external temporal noise with increasing stimulus duration.

RESULTS. Sensitivity increased with stimulus duration in a nonsaturating manner up to the longest exposure times used, indicating probability summation. When expressed in terms of detection efficiency (η) as a function of number of cycles presented (n) all data could be modeled as a single decreasing function of the form $\eta = 0.29n^{-0.70}$.

CONCLUSIONS. The results show that the number of cycles, not time, is the determinant of probability summation of flicker. The results are consistent with the idea that the central detector is a suboptimal matched filter spanning less than one cycle. (*Invest Ophthalmol Vis Sci.* 2003;44:5049–5055) DOI: 10.1167/iovs.02-1082

The performance of a central (cortical) contrast detector can be studied by adding dominant external noise to the stimulus. The narrow-bandwidth signal and detection-limiting external noise are both filtered similarly by the early transfer functions over the entire frequency range. The threshold signal-to-noise ratio at the level of the detector is thus equal to the physical signal-to-noise ratio of the external stimulus. Therefore, as long as the external noise is the dominant source of noise, the detection threshold measured is determined by the amount of external noise, not by the effect of early visual filters.[1–4] The signal-to-noise ratio at detection threshold can then be used to relate the performance measured directly to that of the "ideal observer" by determining detection efficiency.[5] This approach has frequently been used to obtain the detection efficiency for spatial and spatiotemporal signals[1,6,7] and more recently for purely temporal signals.[4,8,9]

Rovamo et al.[4,8] used external white noise of various spectral densities to test a general model for flicker detection. The detector could be described as a suboptimal matched filter with detection efficiency that decreased with increasing temporal frequency. Because stimulus duration was constant in these experiments, the number of flicker cycles presented was greater the higher the temporal frequency, and the authors speculated that this could explain the decrease in efficiency. Failure to integrate the signal effectively over several cycles would result in a decrease in detection efficiency, analogous to what has been observed in spatial vision.[10–13]

Alternatively, it is known that prolonging a flickering stimulus increases sensitivity, at least through the mechanism of probability summation.[14,15] Flicker sensitivity recorded without external noise has been found to increase with increasing exposure duration up to the longest durations tested. Log sensitivity as a function of log stimulus duration increases more or less linearly with slopes of approximately 0.19 to 0.25.[14,15] According to Watson's model[15] for probability summation, this slope should be the reciprocal of the steepness parameter β of the psychometric or frequency-of-seeing (FOS) function. Indeed, FOS functions recorded with similar stimuli have roughly the predicted steepness, corresponding to a β of approximately 4 to 5.[15] The only deviation from the slope of 0.19 to 0.25 was the steeper slope found at short exposure durations (100 ms), which may be due to the effects of the early modulation transfer functions (MTFs).[15]

In the experiments reported herein we measured flicker sensitivity as function of stimulus duration at a number of temporal frequencies (1–20 Hz). In all experiments, strong external, purely temporal white noise was added to the stimulus to reveal the temporal integration properties of the central detector. Only when the properties of the detector are understood can the effects of the early stages of signal processing in the visual system be accurately described. The main purpose of the present study was to test whether the decrease in detection efficiency with increasing flicker frequency for stimuli of constant duration is due to the limited integration of the signal over several stimulus cycles.

METHODS

Apparatus

Sinusoidally flickering stimuli were generated with computer (Pentium, System P60; Dell Computer Corp., Round Rock, TX) with a VGA graphics board (ProDesigner VGA+; Orchid Technologies, Daly City, CA). The graphics board generated 640 × 480 pixels, each 0.42 × 0.42 mm^2.

Stimuli were presented on a 16-in. RGB monitor with fast phosphor (9080i; Eizo Nanao, Ishikawa, Japan), used in white mode. At the frame rate of 60 Hz the display appeared steady in foveal vision. The nonlinear luminance response of the screen was linearized by using the inverse of its luminance response. The mean luminance was set to 50 cd/m^2, corresponding to a scotopic luminance of 130 cd/m^2. A summation device[16] was used to combine the red, green, and blue outputs of the VGA board to obtain a monochrome signal of 256 intensity levels from a palette of 16,384 intensity levels. The amplitudes of the flickering signals were calibrated with a phototransistor (TIL81; Texas Instruments Inc., Dallas, TX). There was no attenuation in amplitude up to 30 Hz (for further details, see Ref.4).

Stimuli

Sinusoidal flicker was produced within a sharp-edged, circular field of uniform luminance. Viewing distance was constant at 115 cm. The two stimulus diameters used, 0.25° and 4°, thus produced stimulus areas of 0.049 deg^2 and 12.6 deg^2, respectively. The stimulus field was sur-

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rounded by a circular equiluminous field limited to a diameter of 10° with a black cardboard mask. The flicker frequencies used were 1, 3, 10, 15, and 20 Hz. Flicker sensitivity was measured as a function of stimulus duration, which varied from 50 to 3000 ms, or from 1 to 60 cycles.

Contrast energies (E) of flickering stimuli were calculated by numerical integration across time by adding up the products of the frame duration and frame contrast (c) squared to remove its sign. Thus,

$$E = \Sigma c^2(t)\Delta t, \tag{1}$$

where $c(t) = [L(t) - L_0]/L_0$; $L(t)$ is the signal; L_0 is the average luminance; and Δt is the duration of each frame (1/60 seconds). The root-mean-square (RMS) contrast is

$$c_{\text{RMS}} = (E/t)^{0.5} \tag{2}$$

where t is the stimulus duration. For simple sinusoidal flicker, RMS contrast is equal to the Michelson contrast divided by $\sqrt{2}$. Flicker sensitivity (S) was taken as the inverse of RMS contrast, i.e. $S = 1/c_{\text{RMS}}$.

Experiments were performed with white temporal noise added to the stimulus. Spatially uniform, pure temporal noise was produced by adding a random number to each stimulus frame. The numbers were drawn independently from a Gaussian luminance distribution with zero mean. The luminance distribution was truncated at ±2.5 SD units.

The RMS contrast of temporal noise was 0.15 in all other conditions except for subject LJ at 1 Hz, for which a contrast of 0.3 was used. The contrast was chosen so that the external noise was dominant—that is, the sensitivity measured in external noise was at least three times lower than without noise. The RMS contrast of noise is equal to the standard deviation of the Gaussian distribution of random luminance values normalized by the mean luminance. The spectral density of temporal noise at the temporal frequencies where noise is white was calculated[1] as

$$N_t = c_n^{\,2}\Delta t \tag{3}$$

where c_n is the RMS contrast of noise and Δt is the duration of each frame, (i.e., 16.7 ms). In our experiments the spectral density of temporal noise was thus 15.0×10^{-4} seconds at 1 Hz for subject LJ and 3.75×10^{-4} seconds in all other conditions. The 60-Hz frame rate ensured that noise could be considered white at all temporal frequencies studied[17] and even the highest flicker frequency of 20 Hz was within the 30-Hz noise band.

The efficiency (η) of flicker detection is calculated as

$$\eta = d'^2\, N_t/E_{\text{th}} \tag{4}$$

where E_{th} is contrast energy of flicker at detection threshold and d' is the detectability index—in our case, 1.4.

Procedures

Experiments were performed monocularly in a room where the display was the only light source. Before each experiment, the subject adapted for 5 minutes to the average luminance of the screen. The subject's head was supported by a chin rest. Central fixation (without fixation point) was used in all experiments.

One eye was covered with a black eye patch. The pupil of the other eye was dilated to 8 mm with 1 to 4 drops of 10% phenylephrine hydrochloride (Metaoxedrine; Smith & Nephew Pharmaceuticals, Ltd., Romford, UK), which leaves accommodation unaffected. The average retinal illumination produced by our display was 2500 photopic trolands, which corresponds to 6500 scotopic trolands.

Thresholds were determined with a two-alternative, forced-choice algorithm with the four-down/one-up rule.[18] The contrast step was constant at 0.1 log units throughout the algorithm. The threshold contrast at the probability of 84% correct was obtained as an arithmetic mean of eight contrast reversals.

Each trial consisted of two exposures, separated by an interval. Both exposures were accompanied by a sound signal. Only one exposure contained the signal, but both exposures contained an uncorrelated sample of white temporal noise. The subject pressed one of two keys on an ordinary computer keyboard to indicate the exposure that contained the signal. An auditory feedback signal indicated whether the response was correct.

Every data point is the median of at least three threshold measurements. Median was used because, for a small number of samples (three to six threshold measurements), median is less affected by occasional outlying values, and hence it is statistically more robust estimate of the true threshold. The goodness of the least-squares line fits (GoF) was calculated as

$$\text{GoF} = 100(1 - k\varepsilon) = 100\{1 - k[1/n\, \Sigma(\log \eta_{\text{est}} - \log \eta)^2]^{0.5}\} \tag{5}$$

where n is the number of data points, η refers to data, and η_{est} to predicted value. Logarithmic values were used for calculating the RMS error (ϵ), as data were plotted on a logarithmic scale. The value of k is 1 for sensitivity and 0.5 for efficiency, because efficiency is based on contrast squared. If the average error between log η and log η_{est} is $\Delta\eta$, then GoF = 100 [1 − k abs($\Delta\eta$)]. For example, if k = 0.5 and $\Delta\eta$ = ±0.30, then GoF = 0.85, which appears to be the lower limit for visually acceptable fit. The reason for using GoF instead of r, the coefficient of determination, is that for fits with shallow slopes, both the explained variation and thus also the value of r tend to be small, whereas GoF still gives reasonable values (for further details, see Ref. 9).

Subjects

Two experienced subjects, 46 and 18 years of age, participated in the experiments. AR (46 years) was an uncorrected hypermetrope (+1.50 DS in both eyes). Subject LJ (18 years) was an emmetrope. The monocular visual acuity was 6/5 for AR and 6/4 for LJ. Informed consent was obtained from both subjects before the experiments, in accordance with the Declaration of Helsinki.

Results

Figure 1 shows log contrast sensitivity as a function of log stimulus duration, the latter expressed in terms of the number of cycles, for each of the five temporal frequencies used (1, 3, 10, 15, and 20 Hz). The stimulus duration in milliseconds is plotted along the horizontal axis at the top. The error bars refer to the standard error of the mean of logarithmic sensitivities at each stimulus duration. For the sake of clarity, the standard error of the mean is only shown for the highest (+SEM) and lowest (−SEM) data point at each stimulus duration.

At all frequencies, sensitivity increased monotonically and similarly for both subjects up to the longest exposure duration measured. This is one hallmark of probability summation and agrees with the results of temporal integration of flicker in the absence of external temporal noise.[14,15] The sensitivity at 1 Hz for LJ was lower than for AR, because a higher noise contrast was used (see the Methods section) to guarantee that external noise was dominant.

Two different sizes of stimulus field were used. Sensitivities were similar for both stimulus sizes, confirming that the dominance of external, purely temporal noise abolishes the area dependence of sensitivity.[9] The only difference that nearly reached statistical significance ($P < 0.05$, Mann-Whitney test/Wilcoxon two-sample test; CoHort Software, Monterey, CA) between areas was at 10 Hz for subject AR.

Even at the shortest exposure durations of 50 and 100 ms at 20 Hz, and 67 and 133 ms at 15 Hz (corresponding to one and

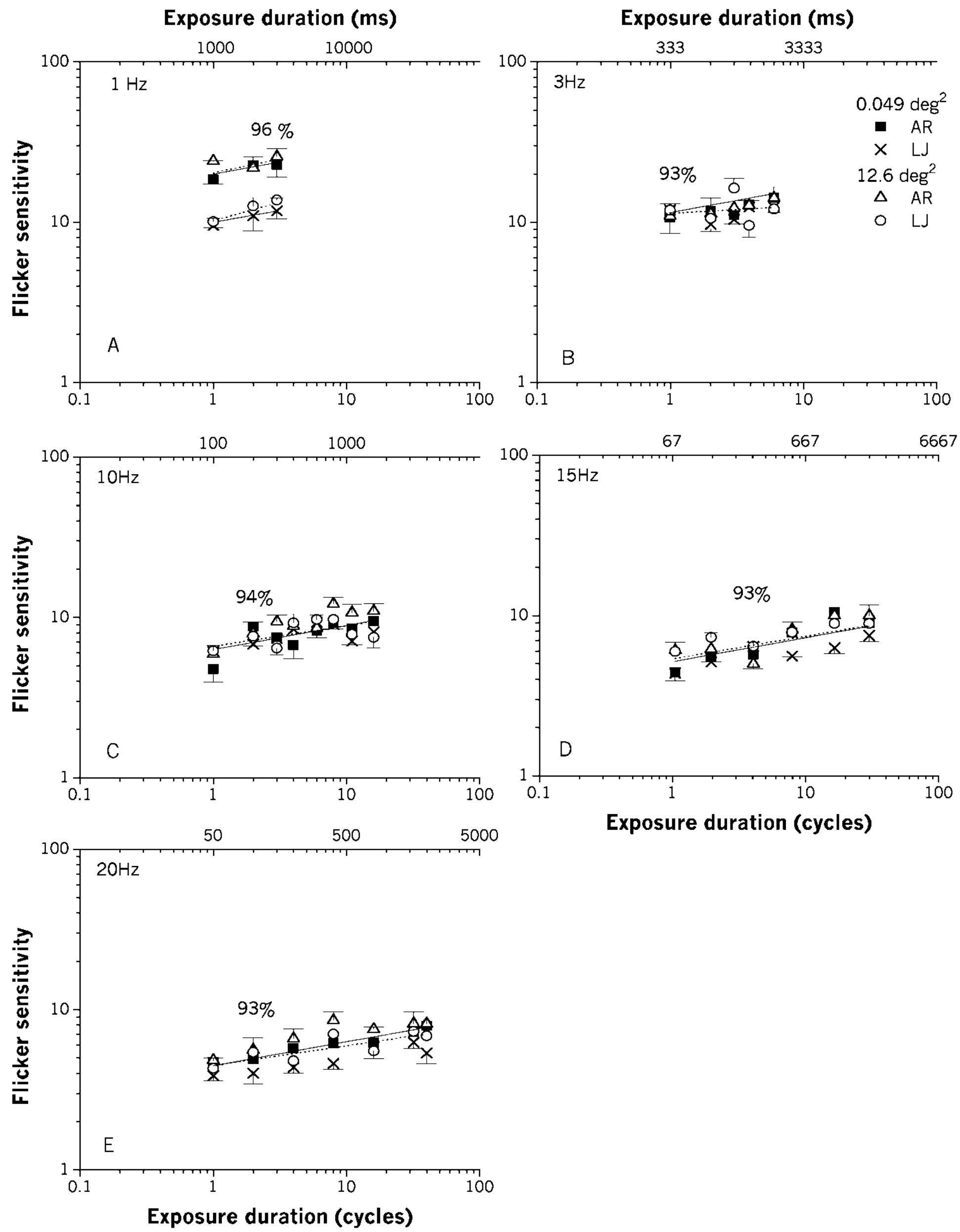

FIGURE 1. Flicker sensitivity as a function of stimulus duration expressed in number of cycles measured in strong external temporal noise. Stimulus duration in milliseconds is plotted on the horizontal axis at the *top* of the graph. Subjects and stimulus sizes as indicated. Continuous lines were calculated using equation 8, and its parameters were derived from the least-squares line fits to the data in Figure 3. The percentages *beside* the data indicate the goodness of fit calculated according to equation 5. The standard error of the mean, calculated for logarithms of sensitivities measured (i.e., raw data), are plotted for the highest (+SE) and lowest (−SE) data point only. *Dotted line*: least-squares line fit to the raw data.

two cycles) the data fell on a single slope, indicating probability summation down to the shortest period, unlike in the experiments of temporal integration of flicker in the absence of noise.[15] This supports Watson's[15] view that the initial steeper slope in his data was due to the early visual filters, the effect of which is not visible in our data measured in strong external noise.

Flicker sensitivity at each stimulus duration (expressed in cycles) decreased with increasing temporal frequency. This is in qualitative agreement with earlier results showing that the familiar band-pass shape of flicker sensitivity functions measured without noise is transformed to a low-pass shape as the external noise becomes dominant.[4,8]

Figure 2 demonstrates the low-pass shape of log sensitivity as a function of log temporal frequency. The slope of decrease seems to be −0.5, implying that one cycle is detected with constant efficiency over the whole frequency range tested (1–20 Hz).

The similarity of increase in sensitivity in Figure 1 across temporal frequencies implies that probability summation is similar at all temporal frequencies and the efficiency at each number of cycles remains constant as a function of temporal frequency, just as it does for the 1-cycle data in Figure 2. This means that the efficiency of detection as a function of the number of cycles at all temporal frequencies and stimulus areas could be described by a single equation of the type

$$\log \eta(n) = \log \eta_1 - K \log n \qquad (6)$$

(i.e., $\eta(n) = \eta_1 n^{-K}$) where η_1 is efficiency at one cycle ($n = 1$); $K = 1 - 2/\beta$, which is the slope of linear decrease, and β is the steepness of the psychometric function (for the derivation of the equation on the basis of probability summation, see Appendix A). In Figure 3 the data from Figure 1 are thus combined in one frame and plotted in terms of detection efficiency, calculated according to equation 4.

Log efficiency (η) as a function of log number (n) of cycles was similar at all temporal frequencies studied, forming a single cloud of data. The least-squares fit of equation 6 to the data in

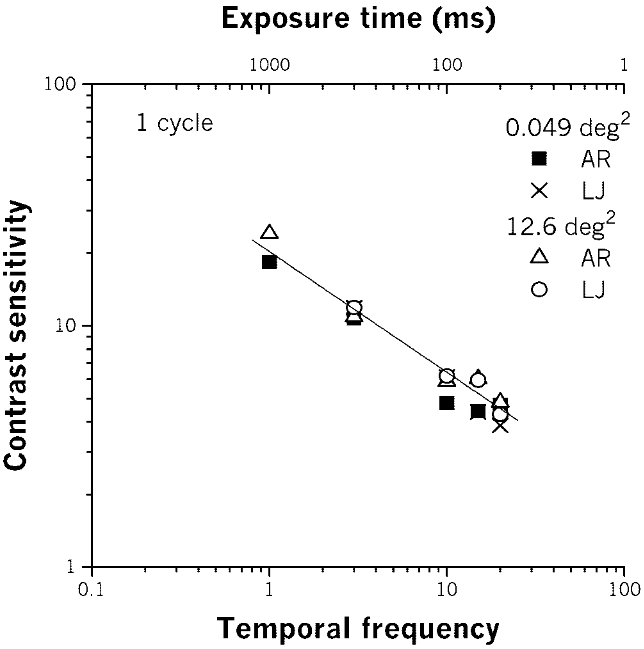

FIGURE 2. Sensitivity to a single cycle of temporal contrast modulation as a function of the frequency of the sinusoidal waveform. Stimulus duration in milliseconds is plotted on the horizontal axis at the *top*.

Figure 3 was found to be $\eta(n) = 0.29n^{-0.70}$. The goodness of fit, calculated according to equation 5, was 93%. The value of β was thus 6.7, and maximum efficiency at 1 cycle (η_1), 0.29. The fact that the efficiency of detection is less than unity, even for single-cycle stimuli, means that temporal integration was incomplete for all our stimuli. The dotted line in Figure 3 shows the least-squares line fit to the raw data (i.e., individual thresholds measured for each data point) combined for all stimulus conditions and both subjects. The fit to raw data is very similar to the fit to median data (Fig. 3, solid line).

Using the relationship $n = tf$ between the number of flicker cycles (n) presented, time (t) exposed and flicker frequency (f) studied, equation 6 can be expressed as

$$\eta = \eta_1\, t^{2/\beta-1} f^{2/\beta-1} \tag{7}$$

indicating that efficiency as a function of temporal frequency (f) at a constant exposure time (t) in seconds also decreases with a slope of −0.70. This result is fairly close to our previous estimate of −0.58.[4]

In Figure 4 the log efficiency of detection is plotted for each temporal frequency separately, along with the least-squares fit from Figure 3, shown as continuous lines. As Figure 4 shows, a single function of $\eta(n) = 0.29n^{-0.70}$ describes well the linear decrease of log efficiency as a function of log number of cycles at all temporal frequencies and stimulus areas. The goodness of fit, calculated according to equation 5, varied between 93% and 96%. When equation 6 was fitted separately to each frame in Figure 4, the slope (and standard deviation) was found to be fairly constant. It was −0.64 (0.11), −0.83 (0.09), −0.70 (0.06), −0.66 (0.06), and −0.74 (0.05) at 1, 3, 10, 15, and 20 Hz, respectively.

Using equations 2, 4, and 6 and the relation $n = tf$, sensitivity $S = 1/c_{RMS}$ can be described using two variables out of three (n, t, f)

$$S = (t\,\eta/d'^2 N_t)^{0.5} = (\eta_1/d'^2 N_t)^{0.5}\,(n^{1/\beta}/f^{0.5}) = (\eta_1/d'^2 N_t)^{0.5}\,(t^{1/\beta}/f^{0.5-1/\beta}) \tag{8}$$

where η_1 is efficiency at $n = 1$, d' is the detectability index (1.4), and N_t is the spectral density of external temporal noise (3.75×10^{-4} seconds for all other conditions except for LJ at 1 Hz, when it was 15.0×10^{-4} seconds). The slope of increase of log(S) as function of log(n) or log(t) was $1/\beta = 0.149$ at all temporal frequencies, which is indicated by $n^{1/\beta}$ in the middle part of equation 8 and $t^{1/\beta}$ in the rightmost part of equation 8.

The middle part of equation 8 describes flicker sensitivity as a function of exposure time expressed in cycles (as in Fig. 1). The continuous lines in Figure 1 were derived using the least-squares fit of Figure 3 and equation 8. The goodness of fit, calculated according to equation 5, varied 93% to 96%. The dotted lines in Figure 1 are the least-squares line fits to the raw data of each frame except at 1 Hz, for which the data of AR and LJ were fitted separately. The slopes (and standard deviation) were 0.17 (0.10) and 0.23 (0.11) at 1 Hz for AR and LJ, respectively, and 0.05 (0.04), 0.13 (0.02), 0.14 (0.02), and 0.12 (0.02) for 3, 10, 15, and 20 Hz, respectively. At 10 to 20 Hz, the slope is quite similar. At 1 to 3 Hz, its inaccuracy, reflected in the large standard deviation in comparison to the absolute slope, is due to the shallow increase in sensitivity combined with the small range of exposure durations.

The middle part of equation 8 also describes the square-root dependence of sensitivity on frequency (as in Fig. 2). The goodness of its fit to the data of Figure 2, calculated according to equation 5, was 94%.

The rightmost part of equation 8 predicts that sensitivity as a function of frequency should decrease with a slope of $1/\beta - 0.5 \approx -0.35$ when exposure duration is constant and external noise dominates. This is broadly consistent with the slope (~ −0.3) of the monotonically decreasing function of log sensitivity versus log frequency found in previous studies when external noise is strong and retinal illuminance is high.[4,8,9]

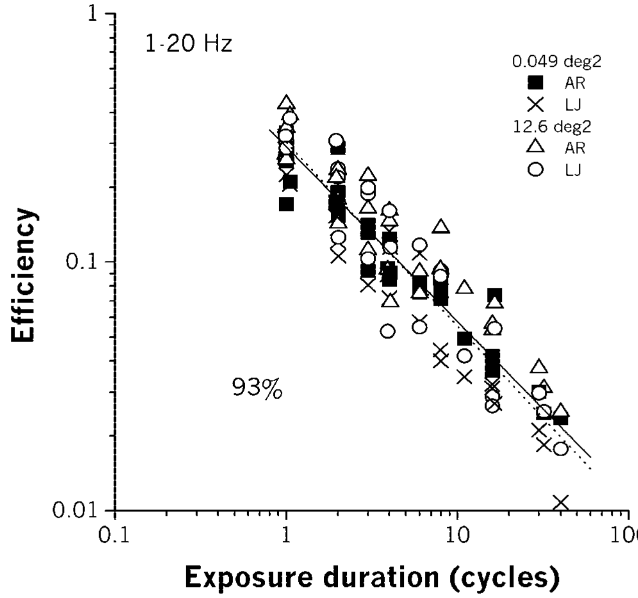

FIGURE 3. Detection efficiency as a function of stimulus duration expressed in number of cycles combined across subjects and all stimulus conditions. Subjects and stimulus sizes as indicated. The continuous line is a least-squares fit of equation 6 to the median efficiencies (i.e., data points). The best fitting function was found to be $\eta = 0.29n^{-0.70}$, SD of the slope was 0.026, and the goodness of fit, calculated according to equation 5, was 93%. *Dotted line*: corresponding fit to the raw data. The best fitting slope was −0.74 with SD of 0.022.

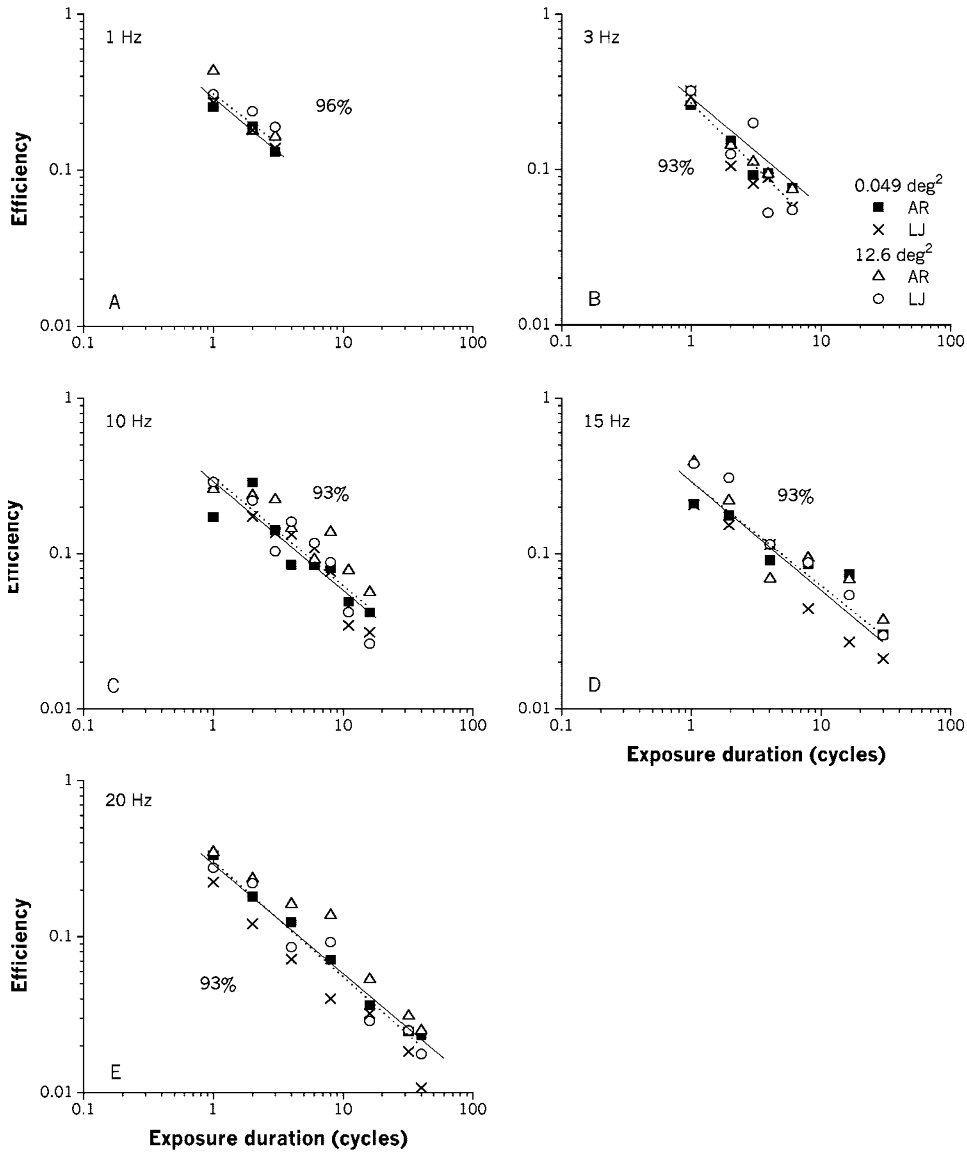

FIGURE 4. Detection efficiency as a function of stimulus duration expressed in number of cycles. Subjects and stimulus sizes are indicated. *Solid line*: least-squares fit to the combined data of Figure 3. The percentages *beside* the data indicate the goodness of fit calculated according to equation 5. *Dotted lines*: equation 6 fitted separately to the data in each graph.

DISCUSSION

Flicker sensitivity in dominant white external temporal noise increased with exposure duration up to the longest exposure times used, in agreement with previous studies of temporal integration of flicker without noise.[14,15] The nonsaturation of temporal integration suggests that the underlying process is probability summation over time. Our data suggest, however, that probability summation of cycles, not time, takes place. Probability summation implies that detection is limited by noise that is uncorrelated from one stimulus instant (e.g., stimulus cycle) to another, and thus ensures that within each instant there is some probability that threshold will be exceeded. With increasing exposure duration, the number of instants and thus the chances that the signal is detected increases. The slope of increase in sensitivity with exposure duration measured without external noise has been estimated[14,15] to be approximately 0.19 to 0.25 at exposure durations longer than 100 ms. We found an approximate slope of 0.145 in strong external noise.

We can think of at least two possible explanations for this difference. In Watson's[15] experiments without noise, the sensitivity increase was steeper at the shortest stimulus durations than at longer durations, and Watson suggested that the deviation could be because, with wider stimulus bandwidth (as is inevitably the case for short durations) the gain of the modulation transfer functions was not constant. It is worth noting that any such nonlinearity connected with early gains that might act to steepen the slope would not be seen in our experiments using strong external noise.

An alternative explanation concerns the different experimental protocols applied to determine sensitivity. Sensitivity benefits more from presentation of many cycles (increasing the opportunities of detection) when the uncertainty range of transition from nonseeing to seeing is wider—that is, the psychometric function is shallower and thus its steepness parameter (β) is smaller. If this transition is sharper in our 2-AFC protocol than with the yes/no staircase,[15] the result may be a weaker dependence of sensitivity on stimulus duration, as observed in the current study.

Although sensitivity rose, detection efficiency decreased with stimulus duration. The decrease on log–log coordinates was roughly linear. When duration was expressed as the number of cycles presented, a single function could be used to describe all the data across all the temporal frequencies studied. Thus, for stimuli with the same number of cycles, efficiency is constant against temporal frequency. By contrast, log efficiency at any given temporal frequency was found to decrease with a slope of approximately −0.7 as a function of log duration (irrespective of whether duration is expressed in number of cycles or seconds). A corollary is that detection efficiency for stimuli of constant duration in seconds decreases in a similar manner with increasing temporal frequency, as found in earlier studies.[4,8,9] The present experiments thus support the idea that the decrease in efficiency in the prior studies reflects the absence of any other means of temporal integration than probability summation across flicker cycles.

That detection efficiency at the exposure duration corresponding to one flicker cycle was significantly lower than the ideal (i.e., unity) can be interpreted to imply that human observers are suboptimal ideal observers. There are several reasons for the suboptimal performance,[6,7] such as noisy matched filter template, or inaccuracy of the template due to slight distortion of the flicker waveform, small error in flicker frequency, or small deviation in stimulus location in space or time, to mention a few. Additive neural noise can be neglected as insignificant in the present study, because strong external noise is the limiting factor for detection. Hence, neural noise cannot account for reduced efficiency.

In the perceptual template model (PTM)[19] additive neural noise and decision process are preceded by a nonlinear transducer function and multiplicative noise that is a function of both signal and external noise, whereas in the model of Eckstein et al. (EAW)[20] there is decisional uncertainty as a free parameter, but no transducer function, whereas multiplicative noise depends only on external noise. In principle, these models are capable of describing the current flicker data, but our model is simpler. However, in our data the contrasts were not extremely high, which may explain[19] why distinction between additive and multiplicative noises was not necessary.

It may seem surprising that the matched filter detecting single cycles is equally efficient down to the lowest temporal frequency (1 Hz), which corresponds to a period of 1 second. However, the present experiments can only say that the integration interval is proportional to the period of the stimulus sine wave, unlike previously hypothesized in models of temporal integration in noise[21–23] that suggested constant time for linear temporal integration. In these studies, however, the experimental task was to detect a stationary spatial image with pulsed presentation in the presence of noise. It is not surprising therefore that temporal integration of these studies resembles that of spatial or spatiotemporal stimuli,[24,25] which is different from the integration of purely temporal signals such as ours. In view of the general independence of the on and off systems,[26,27] it seems likely that the integration intervals could be located at the fast luminance changes between maxima and minima of each flicker period.

In experiments with square light pulses of various durations, Barlow[28] found that the situation in which temporal summation continues throughout the whole stimulus duration (giving a square-root dependence of sensitivity on stimulus duration) occurs only with very small stimulus fields. This type of temporal summation was limited or absent for large fields. In our experiments, we saw square root dependence for both large and small stimulus fields up to at least 0.5 seconds (Fig. 2). We think this is because detection of both the small and the large field is limited by dominant purely temporal noise, and so there is no real difference in spatial integration between small and large fields. This can be explained easily in terms of the physical signal-to-noise ratio, which remains constant irrespective of the area for flickering stimuli when embedded in purely temporal noise with no spatial luminance variation. When noise is sufficiently strong to be the main determinant of the detection threshold, constant physical signal-to-noise ratio at all stimulus areas keeps the detection threshold constant.

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APPENDIX A

Probability summation over time means that each instance provides an independent opportunity for detection. As the number of instances increases with exposure duration, so does the probability of detection. When presentation is extended in time, the detection threshold gradually falls over a wide time range,[15,29] so that threshold is proportional to exposure duration raised to a small negative exponent.[30] According to Tyler and Chen[31] Weibull analysis is an accurate theory for the description of systems with Gaussian additive noise if $4 < \beta < 8$, so that the additive noise distribution implied by Weibull analysis is approximately Gaussian, and the detection threshold sits at or above the maximum of the responses of all the monitored channels to additive noise.

On this basis, the flicker detection threshold can be described as

$$c_n = c_1 n^{-1/\beta} \quad (A1)$$

where c_n is the stimulus contrast of all n cycles presented at detection threshold, c_1 is the stimulus contrast of one cycle presented at detection threshold, and $1/\beta$ is the small exponent. Equation A1 can be readily transformed to

$$\log c_1 - \log c_n = 1/\beta \log n. \quad (A2)$$

Detection efficiency is $\eta_n = (d'^2 N_t)/E_{th(n)}$, where n is the number of cycles presented, d' is the detectability index, N_t is the spectral density of external noise, and $E_{th(n)}$ is the energy threshold. Thus, $\eta_n E_{th(n)} = d'^2 N_t$. Similarly for $n = 1$, $\eta_1 E_{th(1)} = d'^2 N_t$. They can be combined to $\eta_n E_{th(n)} = \eta_1 E_{th(1)}$, which can be written in a logarithmic form as

$$\log \eta_n = \log \eta_1 + \log E_{th(1)} - \log E_{th(n)}. \quad (A3)$$

Energy threshold $E_{th(1)} = c_1^2 t_1$ and $E_{th(n)} = c_n^2 t_n = c_n^2 n t_1$, where t_1 is the exposure duration of one cycle and t_n that of of n cycles. The latter part of equation A3 (i.e., $\log E_{th(1)} - \log E_{th(n)}$) thus reduces to

$$\log E_{th(1)} - \log E_{th(n)} = 2(\log c_1 - \log c_n) - \log n. \quad (A4)$$

Combining equations A2, A3, and A4 yields

$$\log \eta_n = \log \eta_1 - (1 - 2/\beta) \log n, \quad (A5)$$

which is $\eta_n = \eta_1 n^{(1-2/\beta)}$.