§ 5. Discussions I

If one considers only one pure interaction to describe the beta-decay,.......

If experiments are actually carried out to determine the electron-proton angular correlation (§ 3) and the proton momentum spectrum (§ 4), it will be possible to determine which one of five interactions is responsible for the neutron decay process. Though these experiments may be considerably difficult to be carried out with present technique, it seems to us that they are not altogether impossible. So in this section we will summarize our theoretical predictions and give some discussions about our results. The spectra of emitted protons at angles $\varphi = 90^\circ$ and $\varphi = 180^\circ$ are shown in Figs. 2a and 2b. The curves of angular correlation for five interactions are given in Fig. 3.

First of all, the beta-spectrum for the $P.$ interaction is markedly in disagreement with the experiment, as has been shown in Fig. 1. The proton momentum spectra at $\varphi = 90^\circ$ and $\varphi = 180^\circ$ show also peculiar behavior in the $P.$ interaction. However the angular correlation $\Phi_{00}(\varphi)$ (3.8) for the $P.$ interaction is rather similar to the $V.$ interaction, and it is hardly distinguished from each other by the experiment. Now the question arises whether the experiments will be able to distinguish between first four interactions, $S.$, $V.$, $T.$ and $A.$ Its possibility just depends on the experimental accuracy, as has been often mentioned. (See Fig. 3.)

The proton spectra (§ 4) irrespective of the emitted angle of electrons are shown in Fig. 4. When comparing experiments with our theoretical results the situation is similar to that of the angular correlation.

A clue for the choice of the type of interaction is given also by data of nuclear beta-decay processes. It is experimentally known that the $He^6 \rightarrow Li^6$ transition is an allowed

* Part I was published in Prog. Theor. Phys. 7 (1952), 469.
Fig. 2. Proton momentum spectra given by \((3 \cdot 1')\) for five interactions. Fig. 2a and 2b show spectra obtained at \(\varphi = 90^\circ\) and \(\varphi = 180^\circ\), respectively.

one and involves the change of the nuclear spin by unity \((J=1)\).\(^{15}\)\(^{16}\) The selection rules of the allowed transition given by the S. and V. interactions, known as Fermi's selection rules, do not permit any change of the spin units. The experimental result of \(\text{He}^6\) certainly contradicts with these selection rules. Therefore, if one considers only one pure interaction to describe this result, the interaction needs to be either T. or A.; as has been pointed out by Gamow and Teller, the change of nuclear spin of one unit is permitted in these interactions even for the allowed transition. Unfortunately, the allowed electron spectrum does not give any unique information as regards the choice between
these alternatives, because both interactions give the same spectrum. But in principle the electron-proton correlation and the proton spectrum would be able to give a decision at this point.

§ 6. Discussions II

If one considers mixed interactions,......

Our discussion on the experimental results above has been made under a rather special assumption that the true interaction must be determined by only one of five invariants. Recently there is a tendency among physicists to believe that a linear combination of five invariants is necessary rather than a pure interaction in the nuclear beta-decay process. 61 17) 8)

![Fig. 3. Angular correlations of electrons and protons obtained from (3.7) in the cases of five interactions. These curves are normalized at \( \varphi = 90^\circ \). For each interactions, the absolute values calculated at \( \varphi = 90^\circ \) are given as follows;]

\[
\begin{align*}
\phi_{11}(90^\circ) &= 0.30, \\
\phi_{22}(90^\circ) &= 0.15, \\
\phi_{33}(90^\circ) &= 0.20, \\
\phi_{44}(90^\circ) &= 0.25 \\
\phi_{55}(90^\circ) &= 0.33.
\end{align*}
\]
For example, Petshek and Marshak\textsuperscript{19} have studied how to explain the peculiar form of RaE spectrum by a linear combination of T. and P. In the case of the neutron decay, we can not say that the shape of allowed spectrum is completely in agreement with the experiments, if we take the fact seriously that the experimental results below 300 Kev. are smaller than the values theoretically expected under the assumption of the allowed transition. Of course, as has been described by Robson, these data have certainly considerable errors so that we can not believe these values as they are. But it is not meaningless to ask the question whether or not the present experimental data can be interpreted by the mixed interaction above, Robson's result in the lower energy region being taken seriously.
effects of this linear combination were first calculated by Fierz\textsuperscript{20} and later by Rozental\textsuperscript{21} and by de Groot and Tolhoek.\textsuperscript{17} Since no numerical results, especially no calculation for the neutron decay, have not yet been given, we shall discuss about the possibility of such mixtures. But we shall content ourselves only with studying the tendency, describing all four particles by the plane waves, as has been noted in § 2.

In this mixed interaction, the total transition probability is given by using $\sum_{\nu} G_{\nu} G_{\nu} S_{\nu \nu}$ in place of $G_{\nu}^2 S_{\nu \nu}$ in (2.3), where $\sum_{\nu}$ means the sum over 1 to 5 corresponding to five interactions. $S_{\nu \nu}$'s are given in Table II and are identical with the results obtained by Fierz and Tolhoek and other authors.

In order to show the effects of the interference terms, it is enough to study only the cases in which two interactions are combined. The comparison between the theoretical and experimental beta-spectra will be done by the Kurie plot method. Namely, we shall plot the dependence of $(\sum_{\nu} N(\rho)/\rho^2)^{1/2}$ on the electron energy ($\rho_0$), where $\sum_{\nu} N(\rho)$ is obtained from the following expression corresponding to (2.4):

$$P = \frac{G_{\nu}^2 \rho_0^2}{2\pi^2} \int_0^{\sqrt{\rho_0^2 - m^2}} d\rho \sum_{\nu} N(\rho).$$

(5.1)

Table II. $S_{\nu \nu}$'s in the linear combination of five interactions. The factors in the brackets indicate the order of magnitude of expressions. The thick letter means the term which changes the results by the interference effect; and 0 means that the cross correction term vanishes. $S_{\nu \nu}$ does not change when the suffixes $\nu$ and $\gamma$ are interchanged, e.g., $S_{\gamma 21} = S_{\gamma 21}$. $S_{\nu \nu}$'s have already been given in Table I.

<table>
<thead>
<tr>
<th>S</th>
<th>V</th>
<th>T</th>
<th>A</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_{\nu 1}(1)$</td>
<td>$S_{\nu 2}(1)$</td>
<td>$S_{\nu 3}(1)$</td>
<td>$S_{\nu 4}(1)$</td>
<td>$S_{\nu 5}(1)$</td>
</tr>
<tr>
<td>$S_{\nu 2}(m/M)$</td>
<td>$S_{\nu 3}(m/M)$</td>
<td>$S_{\nu 4}(m/M)$</td>
<td>$S_{\nu 5}(m/M)$</td>
<td></td>
</tr>
<tr>
<td>$S_{\nu 5}(m^2/M^2)$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[
\begin{align*}
S_{\nu 2} &= \frac{1}{6} S_{\nu 3} = 16 M_N m [(F \cdot q) - M_\nu \cdot \rho] \\
&\quad - 32 M_N M_\nu m (\epsilon - \rho_0), \\
S_{\nu 3} &= - S_{\nu 4} = - S_{\nu 3} = 32 M_N [g(F \cdot \rho) - \rho_0 (F \cdot q)] \\
&\quad + 32 M_N (\epsilon - \rho_0) [\epsilon \rho_0 + m^2 - 2 \rho_0^2 - \rho (\epsilon - 2 \rho_0) \cos \theta] \\
&\quad + 32 M_N [(\epsilon - 2 \rho_0) (\epsilon^2 + m^2 - 4 \rho^2) + 2 + m^2 (\epsilon - \rho_0)], \\
S_{\nu 4} &= 6 \cdot S_{\nu 4} = 96 M_N m [(F \cdot q) + M_\nu, q] \\
&\quad - 96 M_N m (\epsilon - \rho_0) [(\epsilon - \rho_0) + \rho \cos \theta] \\
&\quad - 96 M_N m \left[ (\epsilon - \rho_0) - \frac{(\epsilon^2 - m^2 - F^2)}{2} \right].
\end{align*}
\]
In this expression the integrand is defined by

\[
\sum_{\nu} N(\rho) = N_{\nu\nu}(\rho) + 2g_{\eta\nu}N_{\nu\eta}(\rho) + g_{\eta\eta}N_{\eta\eta}(\rho),
\]

(5.2)

where

\[
N_{\nu\nu}(\rho) = \frac{1}{\alpha_{\nu} c_{\nu}} \int_{-1}^{1} d(\cos \theta) \frac{S_{\nu\nu}}{64M_\nu \rho_{\nu}(\varepsilon - \rho_{\nu}) M_\rho},
\]

(5.3)

and \(g_{\nu\eta}\)'s are the mixing ratios of the \(\nu\) and \(\eta\) interactions, which are defined by

\[
g_{\nu\eta} = G_{\eta} a_{\eta}/G_{\nu} a_{\nu}.
\]

(5.4)

We shall further discuss about the angular correlations and the proton-spectra. Now we shall give their expressions for various cases successively:

(i) For comparison's sake we first give the Kurie plot for the allowed transition, that is, the spectrum for the pure S., V., T. and A. interactions without mixing:

![Fig. 5. Energy spectra of emitted electrons for the diagonal terms (\(N_{\alpha}\) and \(N_{55}\)), which are obtained from the squares of five interactions, and their cross correction terms (\(N_{\nu\eta}\)). Their expressions are given as the following table; \(N_{\alpha}(\rho) : (2.6)\) \(N_{55}(\rho) : (2.7)\) \(N_{35}(\rho) : (5.17)\) \(N_{45}(\rho) : (5.22)\).]
The expressions of $(N_a(p)/p^2)^{1/2}$ and $(\sum N(p)/p^2)^{1/2}$ are given by (5·5) and (5·7), respectively.

\[ \sqrt{N_a(p)/p^2} = (e - \rho_0). \] (5·5)

The straight line corresponding to this Kurie plot is shown in Fig. 6 with points showing the experimental data.

(ii) In the cases of interference of four interactions except $P$, the terms having the same order of magnitude as the allowed one are the terms arising from the interference between $S$ and $V$ and between $T$ and $V$. (See Table II). Thus we have the following two possibilities.

(iiia) In the case where the $V$ interaction is mixed with $S$, the interference term $(N_{12}(p))$ is obtained by inserting $S_{12}$ into (5·3);

\[ N_{12}(p) = (e - \rho_0)\frac{p^2 \cdot m}{\rho_0}. \] (5·6)

Therefore, we shall obtain the following expression corresponding to $(\sum N(p)/p^2)^{1/2}$;

\[ \sqrt{\sum N(p)/p^2} = \sqrt{(1 + g_{13}) - 2g_{13}(m/\rho_0)} (e - \rho_0). \] (5·7)

We plot the graph of (5·6) in Fig. 5. The graph of (5·7) are plotted.
in Fig. 6 for five values of \( g'_{12} \), in order to show how the spectrum changes with mixing ratio \( g'_{12} \). The values used for \( g'_{12} \) are 1, \((2 \pm \sqrt{3})\), and \(-(2 \pm \sqrt{3})\). We see that the curve for \( g'_{12} = (2 \pm \sqrt{3}) \) agrees with the experimental values obtained by Robson.

The angular correlation, in this mixture, is given by the following expression:

\[
P = \frac{G_1^2 \alpha_1^2}{2\pi^2} \int d(\cos \varphi) \sum_{\nu} \Phi(\varphi),
\]

where

\[
\begin{align*}
12 &= (2 + \sqrt{3}) \quad (\nu = 1; \eta = 2) \\
32 &= -(2 - \sqrt{3}) \quad (\nu = 3; \eta = 0) \\
34 &= -(2 + \sqrt{3}) \quad (\nu = 3; \eta = 4) \\
13 &= (2 - \sqrt{3}) \quad (\nu = 1; \eta = 2) \\
\end{align*}
\]

Fig. 7. Angular correlations of electrons and protons in the cases of linear combinations: S. with V., and T. with A. These curves are normalized at \( \varphi = 90^\circ \). The expressions of \( \Sigma_{12} \Phi(\varphi) \) and \( \Sigma_{34} \Phi(\varphi) \) are given by (5·9) and (5·14), respectively.

Fig. 8. Proton momentum spectra irrespective of the emitted angle in the cases of linear combinations; S. with V., and T. with A. These curves are normalized at \( P = m \). The expressions of \( \Sigma_{12} M(P) \) and \( \Sigma_{34} M(P) \) are given by (5·12) and (5·15), respectively.
\[ \sum_{19} \Phi(\varphi) = \langle I_a + \epsilon_{12}^2 (2I_a - I_b) - 2\epsilon_{19} I_d, \right \rangle, \]  
and

\[ I_d = \frac{1}{2} \int dp \frac{(F \cdot p/\sqrt{D'})}{m(e-p_0)}. \]  

These expressions correspond to (3.7) in § 3.

We shall further give the proton spectrum in this mixture. It is given by the following expression;

\[ p = \frac{G_{19}^2}{2\pi^2} \int \frac{dF \sum_{19} M(F)}{F^2 - m^2}, \]

where

\[ \sum_{12} M(F) = 1 + \frac{\epsilon_{12}^2}{4} (2I_a - I_b) - 2\epsilon_{19} I_d. \]

These expressions correspond to (4.3) in § 4. \[ \sum_{19} \Phi(5.9) \] and \[ \sum_{12} M(5.12) \] are, respectively, shown in Figs. 7 and 8 for two values of \( \epsilon_{12} \), i.e. \( \epsilon_{12} = (2 \pm \sqrt{3}) \).

(iib) In the case where T. and A. interactions are mixed, the expression \( N_{14} \) is the same with \( \epsilon_{19} \). Therefore, we get the same beta-spectrum as (iia): one has simply to substitute \( \epsilon_{19} \) by \( \epsilon_{14} \). But the expressions for the angular correlation and the proton-spectrum are different. We find that

\[ \sum_{19} \Phi(5.14) \] and \[ \sum_{19} M(5.15) \] are also respectively shown in Figs. 7 and 8 for two values of \( \epsilon_{19} = -(2 \pm \sqrt{3}) \).

(iii) Before entering to the remaining mixtures, we give for comparison's sake the Kurie plot for the pure P interaction. From (2.7), the desired expression is easily found:

\[ \sqrt{1 - \frac{N_{10}}{p^2}} \quad \{ \rho^2 + (e-p_0)^2 - (2\rho^2 (e-p_0)/3p_0) \}^{1/2} (e-p_0)/m. \]

The Kurie plot of (5.16) is shown in Fig. 9a. One can see clearly that it disagrees with the experiment.

(iv) \( SP_{55} \) and \( SP_{45} \) are apparently of the smaller order of magnitude by the factor \( (m/M) \) as compared with \( SP_5 \) for the allowed transition. \( SP_{55} \) given in Table II is still smaller, namely of the order of \( (m/M)^3 \). But these conclusions are based on the assumption that the P. coupling constant \( G_5 \) is of the same order of magnitude as the other \( G_5 \)'s. If it is assumed that \( G_5 \) has a suitable large value as that \( G_5 \) becomes comparable with the other \( G_5 \)'s,
these interference terms are sufficiently large so that they can be respectively used in the mixed theories consisting of two interactions, of P. and T. and of P. and A.

In the case of mixing the P. interaction with T., the expressions corresponding to (5.6), (5.7), (5.9) and (5.12) are as follows:

\[
N_{93}(\rho) \approx \frac{-1}{\sqrt{3}} \frac{\rho_0(\epsilon - \rho_p) - \rho^2 (\epsilon - \rho_p)^2 \rho^2}{m \rho_0}, \tag{5.17}
\]

\[
\sqrt{\frac{N_{93}(\rho)}{\rho^2}} \approx \frac{1}{\sqrt{3}} \frac{1 + G_{93}(\rho^2 + (\epsilon - \rho_p)^2 - \frac{2 \rho^2 (\epsilon - \rho_p)}{3 \rho_0})}{(\epsilon - \rho_p)} \tag{5.18}
\]

\[
\sum_{\text{93}} \Phi(\phi) \approx \left\{ \frac{1}{3} \left[ 4 I_a - I_b \right] + G_{93}^2 I_c - \frac{2}{\sqrt{3}} G_{93} [I_a + I_c] \right\}, \tag{5.19}
\]

\[
\sum_{\text{93}} M(F) \approx \left\{ \frac{1}{3} \left[ 4 I_a - I_c \right] + G_{93}^2 I_c \right\}, \tag{5.20}
\]

where

Fig. 9. Kurie plots in the cases of linear combinations: Figs. 9a and 9b show plots for the combinations of T. with P. and A. with P., respectively. These curves are normalized at \( \rho_0 = 1.9 \) m. The expressions of \( N_{93}(\rho)/\rho^2 \), \( \sum_{\text{93}} N(\rho)/\rho^2 \) and \( \sum_{\text{93}} N'_{\rho}(\rho)/\rho^2 \) are given by (5.16), (5.18) and (5.23), respectively.
(ivb) In the case of mixture of A. and P. interactions, the corresponding expressions are,

\[ N_{45}(\rho) = -\frac{1}{\sqrt{3}} \frac{\rho - \rho_0}{\rho_0} (\rho - \rho_0)^2 \rho^2, \]  

(5.22)

\[ \sqrt{\frac{\sum_{45} N(\rho)}{\rho^2}} = \left\{ 1 + \frac{2}{\sqrt{3}} g_{45} \left[ \frac{\rho - \rho_0}{\rho_0} \right] \right\}^{1/2} (\rho - \rho_0), \]  

(5.23)

\[ \sum_{45} \Phi(\rho) = \left\{ \frac{1}{3} [2J_a + J_b] + g_{45} J_c - \frac{2}{\sqrt{3}} g_{45} \left[ \frac{\rho}{m} I_a - I_b \right] \right\}, \]  

(5.24)

\[ \sum_{45} M(F) = \left\{ \frac{1}{3} [2J_a + J_b] + g_{45} J_c - \frac{2}{\sqrt{3}} g_{45} \left[ \frac{\rho}{m} I_a - I_b \right] \right\}. \]  

(5.25)

Curves for (5.17), (5.18), (5.22) and (5.23) are plotted in Figs. 5 and 9. In Fig. 9 curves are drawn for rather large values of the parameters \( g_{35} \) and \( g_{45} \): namely, we have taken \( \pm (1/\sqrt{3}) \) for both \( g \)'s. If we take smaller values for both \( g \)'s, the spectra resulting from these combinations approach the pure allowed one. We see in the figures that the observed data lie between the curve for the allowed spectrum and the curves with \( g_{35} = g_{45} = + (1/\sqrt{3}) \). This means that one will be able to account for the observed data by these mixtures if one takes the mixing ratios suitably. Here we will not enter to the precise determination of the mixing ratio, but give only the following remark. In the ordinary theory about the nuclear beta-decay processes, the first term arising from the P. interaction has been classified into the first forbidden transition and usually treated as a smaller effect compared with the allowed one. However, Petshek and Marshak, who studied the \( RaE \) spectrum, found that the first term of the expansion of the P. interaction including the coupling constant is about thirteen times as large as the first forbidden one of the T. interaction. This may mean that the effect of the P. interaction is larger than usually considered, though, of course, we can not draw any definite conclusion before we have more accurate knowledge about the magnitude of the nuclear matrix elements for \( RaE \). If the effect of the P. interaction is actually not small, it may be possible that the shape of the spectrum at the low energy region is accounted for by our mixtures.

Hitherto we have considered only the linear combinations of two interactions. The linear combinations of three or more invariants are, of course, not excluded, but the general
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only the freedom of the theory. Also, it is possible to assume that the coupling constant has an imaginary value. In order to obtain further information about such possibilities, more cases of nuclear beta-decay processes must be observed and investigated, and therefore we will not discuss them now. However, there is one important case to be considered. It is a permissible linear combination of S., A. and P. interactions, which was introduced by Critchfield and Wigner according to a symmetry principle. In this case, both the cross correction terms of A. and P. with S. interaction vanish, and, as the result, the beta-spectrum obtained is the same with the case (ivb), including only the $S_P \alpha_5$ terms. We will not, therefore, discuss this possibility in detail, though the curve for the angular correlation may be different from what has been given above. Besides the Critchfield-Wigner combination, there is a special combination of V. and T. interactions, which was proposed by De Groot and Tolhoek according to their own symmetry principle. In this combination, the cross correction term is very small, so that this seems to be consistent, only when the experimental results confirm that the observed spectrum is actually in good agreement with the theoretical allowed spectrum.

Our above discussion about the linear combinations of some interactions has been founded on the assumption that below 300 Kev. the data measured by Robson are to be taken seriously. But Robson himself seems to believe that "in this region the known instrumental defects make the observed points inaccurate in their representation of true momentum spectrum" and that "above 300 Kev. their distribution is consistent with what is to be expected for an allowed transition." If the experimental Kurie plot is actually a straight line down to low energies indicating that the transition is allowed one, combinations (ii), i.e. the combinations of S. with V. and T. with A, must be excluded, while the combinations, which was not discussed above, i.e. combinations of S. with T., S. with A., V. with T. and V. with A., are all permissible.* It is because in these combinations the terms of interference either vanish or are smaller than the original (diagonal) terms, as has been indicated in Table II. If the P. coupling constant $G_5$ is of the same order of magnitude in comparison with the other, for example with $G_1$, $G_5 \alpha_5$ is smaller than the other, e.g., $G_1 \alpha_1$, and then the P. interaction does not give rise to any effect on the allowed transition so that any combinations including the P. interaction are all permissible.

Further experimental data will have to be used to decide whether the interaction is a pure one or a linear combination and whether the interference terms are necessary or not, if the combination is to be taken. In the case of the neutron decay process, more accurate information will be supplied by the experiments on the proton spectrum as well as those on the angular correlation of electrons and protons, besides the information supplied by a more accurate observation of the beta-spectrum. Feature of these combinations can be inferred by suitable consideration about the above discussed combinations of two interactions. Such combinations of many interactions increase

* According to the symmetry principle postulated by De Groot and Tolhoek, two combinations of S. with T. and V. with A. are also excluded.
The authors should like to express their sincere thanks to Prof. S. Tomonaga, Prof. S. Nakamura and Prof. T. Miyazima, Dr. Y. Fujimoto, and the members of the research group for the theory of the elementary particles in Nagoya for their kind interests and valuable advices and discussions in the course of this work. They are most grateful to Prof. J. M. Robson for having sent the numerical values of his work to Prof. S. Nakamura. Finally the authors wish to express their grateful gratitude for the financial aids from the Yukawa Yomiuri Fellowship to one of them (H. T.).

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Note added in proof: Erratum In the preceding paper (Part I) of some authors, \( n = 0 \) on page 477 (first and eighth lines) should be replaced with \( n \neq 0 \).