Further, if we obtain the unextrapolated convergence rate \( \beta_L \) by substituting \( \omega = 1 \) in equation (11),
\[
\beta_L = 0.042 \pm 0.002
\]
and compare it with \( \rho_L \) (equation (8)), which for our network parameters, \( p = 24 - 109 \) and \( q = 6 \), varies between,
\[
\rho_L < 0.085 \quad \text{and} \quad \rho_L < 0.076,
\]
it is clear that the convergence rate of our special line iterative method is about half Parter's estimated rate for the two-line Liebmann method for the Biharmonic equation under his boundary conditions.

Equation (11), however, unlike Young's Formula cannot be used to predict the optimum relaxation parameter. It is found that the optimum and maximum relaxation parameters of our convergences are determined by the non-linearity of the equations. In certain regions of the network, as the Reynolds Number increases, the coefficients of the side nodes of the stencil equation (3) become progressively greater than the central coefficient. This causes instability of the single line process at lower and lower relaxation parameters (Bye, 1962).

7. Conclusion

The convergence rate \( \beta \) of this set of solutions of the Navier–Stokes equation by a special line iteration process varied with \( \omega \) in approximately the same manner as the convergence rate of S.O.R. for consistently ordered matrices, provided that \( \beta < 2 - \omega \). The maximum relaxation parameter \( \omega^* (Re) \), however, decreased steadily as the Reynolds Number increased. By Reynolds Number 200 the efficiency of the single line process had been seriously impaired.

It appears that if solutions are to be obtained effectively at higher Reynolds Numbers a more elaborate iteration scheme is required. This should not depend on the constant element iteration matrix (5), but take into account the local changes which occur in the coefficients of the nodes of the complete finite-difference stencil (3) as the convergence proceeds.

The development of such methods for the Navier–Stokes and other equations, although almost certainly demanding on computer time, may well make possible the numerical solution of important non-linear problems.

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References


Correspondence

To the Editor,
The Computer Journal.

“Irascible Genius”

Dear Sir,

Mr. Ord-Smith, in briefly reviewing Miss Moseley's book upon Charles Babbage, might have called attention to two astonishing misstatements regarding that mathematical genius the late Dr. A. M. Turing, F.R.S. On page 258 she says:

Had he [Babbage] come back within seventy years, in 1936, he would have found another Englishman of genius, Alan Turing, wresting the touch from him and passing it on to others.

The reference is presumably to a paper entitled "On Computable Numbers" which Dr. Turing submitted to the London Mathematical Society on 28 May 1936, in which he discussed in terms of pure mathematics the computational limitations of such a machine as an electronic binary scale development of Babbage's analytical engine which had been discussed at the Institute of Actuaries four months earlier, namely on 27 January 1936.

Miss Moseley is not even consistent; in her Prologue, on page 17, she quotes the view that "Charles Babbage's ideas had begun to be properly appreciated only after the Second World War;" on the same page she refers to Turing as "the originator of the pilot ACE." Team work on the Pilot ACE commenced, to be pedantically precise, at 10.30 a.m. on Friday 15 January 1943. Turing did not join the team until the autumn of 1945.

Yours truly,

WILLIAM PHILLIPS.

Lamb Building,
4 February 1965.