Analysis of Matrix Heat Exchanger Performance[1]

V. Ahuja[2] and R. K. Green[3].
Recently, a new numerical scheme for solving the equations governing matrix heat exchanger thermal performance was published in this journal. It was found that this scheme does not include the full effect of longitudinal heat transfer in the said heat exchangers. This effect is demonstrated by correcting the parameter related to longitudinal heat transfer in the approximate analytical solution for the balanced flow case and finding it to deviate considerably from the numerically calculated result.

Nomenclature

A = heat transfer surface area, m^2
A_{cr} = spacer area, m^2
A_{fr} = frontal area, m^2
b = separator width, m
C = mc_p, J/s·K
c_p = specific heat, fluid, J/kg·K
G_o = fluid mass velocity in perforation, kg/s·m^2
G = $G_o p$, fluid mass velocity in header, kg/s·m^2
H_f = fin height, m
k_{ax} = axial conductivity, W/m·K
k_p = conductivity, perforated plate, W/m·K
k_{plate} = conductivity, plate material, W/m·K
k_x = conductivity, spacer material, W/m·K
m = mass flow rate, kg/s
n = number of plates in matrix heat exchanger
Ntu = Number of transfer units
p = plate porosity
s = separator thickness, m
U = overall heat transfer coefficient, W/m^2·K
W = Plate width, m
β = surface area per unit volume, m^2/m^3
δ = plate thickness, m
λ = overall axial conduction parameter, Eq. (3)
λ_p = plate conduction parameter, Eq. (5)
ν = heat capacity rate ratio
ϕ = Ntu$_{p,i}$/Ntu$_{f,i}$

Subscripts

D = design
eff = effective
f = fluid

i = channel number (1 or 2)
o = overall
p = plate (perforated)

Review

Venkatarathnam and Sarangi (1991) derived equations governing the heat transfer in a matrix heat exchanger. These equations were reduced to two second-order ordinary differential equations and four algebraic equations describing the energy balance and heat transfer for every plate, based on the assumption that the axial temperature gradient in the plate is negligible and hence the full temperature drop takes place across the spacer. They describe a new numerical scheme to solve these equations. This analysis accounts for the discrete plate-spacer pair set structure of the matrix heat exchanger, and nonunity fin effectiveness. Details of this work and a listing of the program incorporating this numerical scheme are covered in Venkatarathnam's Ph.D. thesis (1991).

Venkatarathnam (1996) has subsequently published an approximate analytical solution for the matrix heat exchanger governing equations. The approximate analytical solution for the equations for energy balance and heat transfer, accounting for the heat transfer coefficient, axial conduction, number of plates, and fin effectiveness in the matrix heat exchangers, for balanced flow is shown in Eq. (1).

$$\text{Ntu}_{\text{eff}} = \frac{n(1 - \alpha_1)(1 - \alpha_2)}{\lambda n(1 - \alpha_1)(1 - \alpha_2) + 1 - \alpha_1\alpha_2 + (1 - \alpha_1)(1 - \alpha_2)/\text{Ntu}_{po}} \quad (1)$$

where

$$\alpha_i = e^{-\text{Ntu}_{f,i}} \quad (2)$$

$$\lambda = \frac{k_s b W}{nsC}, \quad (3)$$

and Ntu$_{po}$, the overall plate Ntu, is defined as

$$\frac{1}{\text{Ntu}_{po}} = \frac{1}{\lambda_p} + \frac{1}{3\phi_1\text{Ntu}_{f,1}} + \frac{\nu}{3\phi_2\text{Ntu}_{f,2}} \quad (4)$$

with

$$\lambda_p = \frac{k_{\text{plate}}\delta W}{bC} \quad (5)$$

$$\phi = \frac{\text{Ntu}_{p,i}}{\text{Ntu}_{f,i}} = \frac{k_p}{h\beta H_f^2} \quad (6)$$

$$k_p = k_{\text{plate}} \left(\frac{1 - p}{1 + p} \right). \quad (7)$$

[1] Venkatarathnam, G., and Sarangi, S., 1991, "Analysis of Matrix Heat Exchanger Performance," ASME JOURNAL OF HEAT TRANSFER, Vol. 113, pp. 830–836.

[2] CSIRO Manufacturing Science and Technology, Locked Bag 9 (cnr. Albert and Raglan Streets), Preston, Victoria 3072, Australia. e-mail: vikas.ahuja@mlb.dmt.csiro.au

[3] Directorate, Otago Polytechnic, Private Bag 1910, Denedin, New Zealand. e-mail: rgreen@tekotago.ac.nz
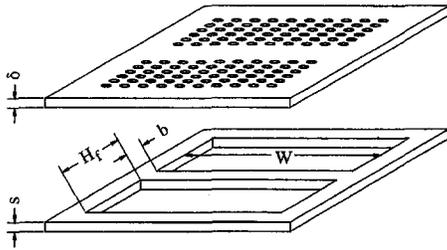
Fig. 1 Perforated plate and spacer dimensions

Plate and spacer dimensions relevant to these equations are shown in Fig. 1.

Longitudinal Conduction

The control volume for which the energy balance is considered for deriving the governing equations does not include the area over which longitudinal conduction occurs other than the separator. The axial conduction parameter defined in Eq. (3) incorporates a spacer area equivalent only to that of the separator (bW). The parameters λ_p and $\lambda_s = n\lambda$ are derived from the energy balance for the separator, for the numerical solution. Venkatarathnam has then applied λ to the approximate analytical solution as the overall axial conduction parameter to account for axial conduction as done by Kroeger (1969). However, Kroeger has used the overall axial conductivity and total area for axial conduction to define this axial conduction parameter λ as

$$\lambda = \frac{k_{ax}A_{cr}}{n(\delta + s)C} \qquad (8)$$

where

$$k_{ax} = \frac{\delta + s}{\dfrac{\delta}{k_{plate}} + \dfrac{s}{k_s}} \cdot \qquad (9)$$

Venkatarathnam's use of λ thus represents a small proportion of the area over which axial conduction occurs, and the net axial conductivity. The effect of taking λ from (3) or (8) is shown in Fig. 2.

Venkatarathnam's results were presented as Ntu_{eff} versus Ntu_D graphs for constant ϕ and λ. A matrix heat exchanger of given geometry at one flow condition would represent a single point on such a graph. Figure 2 shows Ntu_{eff} versus Ntu_D plotted for given constant geometric parameters. Venkatarathnam's ap-
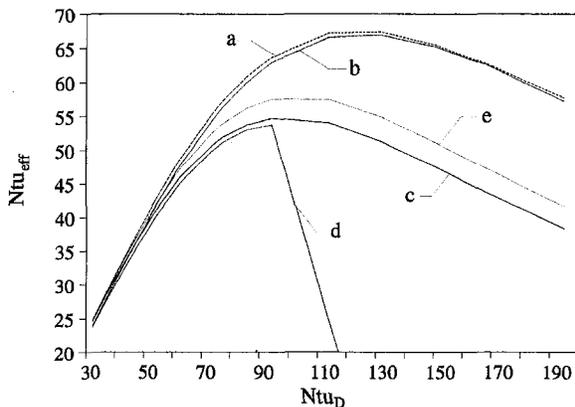

Fig. 2 Ntu_{eff} versus Ntu_D for a: Venkatarathnam, analytical; b: Venkatarathnam, numerical; c: analytical, using λ from (8); d: numerical, using λ from (8); e: analytical solution using Fleming's (1969) equation for fin effectiveness and Kroeger's method for longitudinal conduction
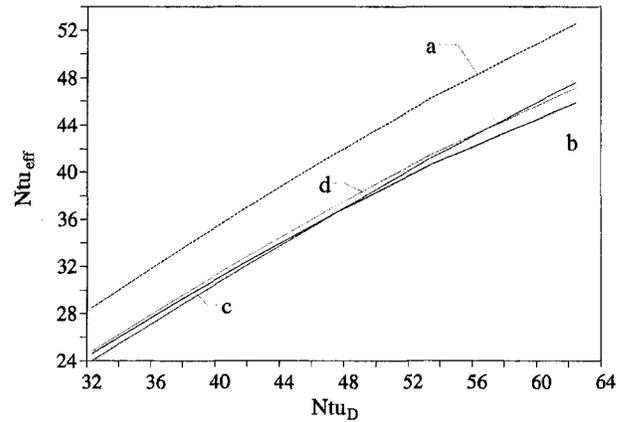

Fig. 3 Ntu_{eff} versus Ntu_D for a: Venkatarathnam, analytical with $\lambda_p = \infty$; b: this work, with finite values for λ_p; c: Venkatarathnam, numerical; d: analytical solution using Fleming's equation for fin effectiveness and Kroeger's method for longitudinal conduction

proximate analytical and numerical solutions only show agreement provided λ is calculated for the separator alone. If λ is taken as the overall axial conduction parameter, the approximate analytical solution deviates considerably from the numerical solution. As to whether the numerical solution can take into account axial conduction in the heat exchanger body other than in the separator is uncertain.

The source code of the program incorporating Venkatarathnam's numerical solution for the energy balance and heat transfer equations, as presented in his thesis, does not run. A running version of the program in his thesis was obtained through correspondence with the author (1995). He stated that the program included in his thesis is a pared down version of a larger program and suggested changes to the listed source code. The modified version gives results which concur with the graphs presented in his various papers and thesis. However, the modified version still produces absurd results (negative values of Ntu_{eff}) if the axial conduction parameter is proportionately higher than some undefined limit. An example of this is shown in Fig. 2, by taking a value of λ comparable to that from (8) for the numerical solution. The efficacy of this numerical solution and associated program is thus questionable.

Lateral Conduction

Venkatarathnam has used the simplification of neglecting transverse resistance in the separator by assuming the plate conduction parameter $\lambda_p = \infty$. Thus for $\alpha_1 = \alpha_2$

$$Ntu_{eff} = \frac{n(1 - \alpha_i)}{n\lambda(1 - \alpha_i) + (1 + \alpha_i) + \dfrac{(1 - \alpha_i)}{Ntu_{po}}}, \qquad (10)$$

and from

$$Ntu_D = n\left(\frac{1}{Ntu_{f,1}} + \frac{1}{Ntu_{f,2}} + \frac{1}{\lambda_p}\right)^{-1} \qquad (11)$$

$$Ntu_{f,i} = \frac{2Ntu_D}{n} \qquad (12)$$

$$Ntu_{po} = \frac{3}{2} Ntu_{p,i} = \frac{3k_p\delta}{2G_o pc_p H_f^2} \cdot \qquad (13)$$

The effect of taking $\lambda_p = \infty$ is shown in Fig. 3. The convection heat transfer coefficients for the matrix heat exchanger are much higher than those normally associated with gaseous heat transfer media. It is apparent that the simplification of $\lambda_p = \infty$ may not

be justifiable for the low Reynolds number flows which are characteristic of matrix heat exchangers.

Sizing Equation

The analytical solution for the balanced flow case lends itself to further simplification. For design purposes a sizing equation can be derived from Eq. (1) by re-arranging the terms and solving by substitution. The sizing equation gives the surface area required for any known desired Ntu. From (1), (2), (8), and (13), neglecting conduction resistance

$$Ntu_{eff} = \frac{n}{\dfrac{k_{ax}A_{cr}}{(\delta + s)C} + \dfrac{1 + e^{-Ntu_{f,i}}}{1 - e^{-Ntu_{f,i}}} + \dfrac{2G_o p c_p H_f^2}{3k_p \delta}} . \quad (14)$$

Taking the number of plates as

$$n = \frac{A}{\beta A_{fr} \delta} , \quad (15)$$

Eq. (14) can be solved as

$$A = \left[e^{2\tanh^{-1}(-h\beta\delta/Gc_p)} + \frac{k_{ax}A_{cr}}{(\delta + s)C} \right.$$

$$\left. + \frac{2Gc_p H_f^2}{3k_p \delta} \right] \beta W H_f \delta Ntu_{eff} . \quad (16)$$

If conduction resistance is included then

$$Ntu_{po} = \frac{3k_p \delta W}{C\left(2H_f + 3\left(\dfrac{1-p}{1+p}\right)b\right)} \quad (17)$$

giving

$$A = \left[e^{2\tanh^{-1}(-h\beta\delta/Gc_p)} + \frac{k_{ax}A_{cr}}{(\delta + s)C} \right.$$

$$\left. + \frac{GH_f c_p\left(2H_f + 3\left(\dfrac{1-p}{1+p}\right)b\right)}{3k_p \delta} \right] \beta W H_f \delta Ntu_{eff} . \quad (18)$$

Conclusion

The new numerical scheme developed by Venkatarathnam and Sarangi (1991) for solving the equations governing heat transfer in matrix heat exchangers has been shown to neglect the full effect of longitudinal heat transfer. The efficacy of the program listing available in open literature, which incorporates this numerical scheme, is questionable. It is recommended that for matrix heat exchanger performance analysis, the analytical solution be used with the corrected axial conduction parameter.

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Authors' Closure

Vikas Ahuja has discussed the need to include the full cross-sectional area of the walls that separate the streams from the environment as well as that which separates the streams from one another in calculating the longitudinal conduction parameter originally presented by the authors. He has also raised some points on the accuracy of the work. The salient features of the methods proposed by us are reviewed and the points raised by Ahuja have been answered in this closure.

Introduction

Ahuja has compared the effective number of transfer units (effectiveness) estimated using a numerical scheme presented by the authors in the *Journal of Heat Transfer* (Venkatarathnam and Sarangi, 1991) and an analytical solution presented by Venkatarathnam (Venkatarathnam, 1996) recently in *Cryogenics*. The main point raised by Ahuja is that the definition of the longitudinal heat conduction parameter should include the total foot print area of the low conductivity spacer that separates the copper perforated plates, and not the area of the wall that separates the two streams alone, as defined in our paper. He has also compared the performance when the lateral resistance of the wall is neglected, and has stressed the need for accounting for the same. The points raised by Ahuja are answered in this closure.

Longitudinal Heat Conduction

The walls of the perforated plate matrix heat exchanger are formed by bonding a stack of alternating low thermal conductivity (plastic/stainless steel) spacers and copper perforated plates to form a monolithic block (Fig. 1). The walls that separate the streams from the environment and that which separate the two streams are thus interconnected. In deriving the governing equations, it was assumed that the longitudinal heat conduction occurs only through the walls that separate the streams, to simplify the analysis. This assumption has been made by many workers (Kroeger, 1967) who have studied the performance degradation due to longitudinal heat conduction through the walls of heat exchangers. These assumptions are meant only to simplify the analysis and make the problem tractable. While analyzing the performance of any real heat exchanger it is necessary to take the longitudinal heat conduction through all the walls. In most cases, the performance of the heat exchanger will be slightly underpredicted if the longitudinal heat conduction through the outer walls is accounted for by adding the cross-sectional areas of the outer and inner walls, and assuming that all the longitudinal heat conduction occurs only through the inner wall alone (Narayanan, 1997).

It is well known that the longitudinal heat conduction parameter should be estimated using the area of all the walls of any exchanger, including perforated plate matrix heat exchangers. To demonstrate the basic principles of heat exchangers, Mills (1996) used the total foot print area to evaluate the longitudinal heat conduction parameter and predict the performance degradation due to longitudinal heat conduction in perforated plate heat exchangers. Thus, the suggestion made by Ahuja is not new.

Figure 2 shows the temperature profile in the wall separating the streams. Because of the small thickness of the copper perforated plates (typically 0.2 to 1.0 mm thick), the temperature of any copper perforated plate will essentially be uniform in the longitudinal direction (Sarangi and Barclay, 1984, Venkatarathnam and Sarangi, 1991). The total temperature drop across the wall is almost totally sustained by the low conductivity (plastic or stainless steel) spacers separating the copper perforated

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[1] G. Venkatarathnam, Department of Mechanical Engineering, Indian Institute of Technology, Chennai 600 036, India and S. Sarangi, Cryogenic Engineering Centre, Indian Institute of Technology, Kharagpur 721 302, India.