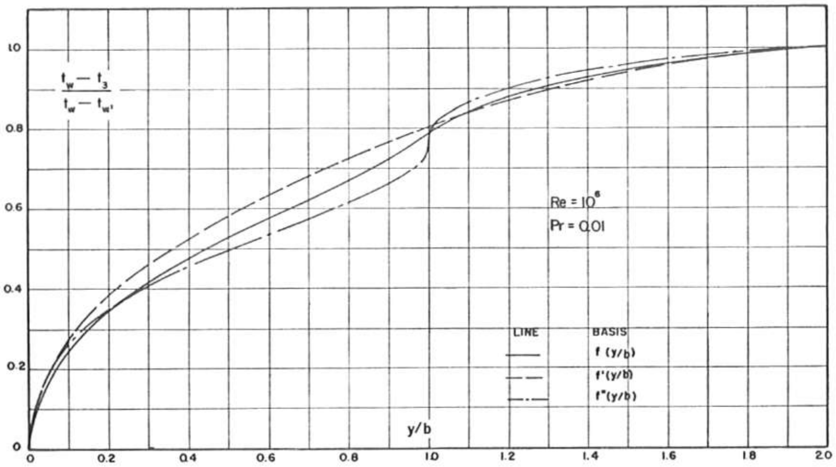

Fig. 7 Plot of Temperature Distribution Showing Effect of Eddy-Diffusivity Values for an Asymmetrical Case

CONCLUSIONS

Theoretical work by Martinelli on heat transfer to molten metals has been extended to cover the asymmetrical system having fluid moving between two plane parallel walls, only one of which permits heat transfer to the stream. At low Peclet numbers, Nusselt numbers calculated in the manner outlined are roughly 50 per cent of the values presented by Martinelli for symmetrical systems. At high Peclet numbers, asymmetrical-system calculations are about 67 per cent of the symmetrical system calculations.

Suggestions have been made regarding an experimental approach for finding the variation of eddy diffusivity across a flat channel especially in the vicinity of the center line, and also for determining heat-transfer coefficients in asymmetrical systems.

This paper is not considered to be complete in itself, but only a brief extension of the work presented earlier by Dr. R. C. Martinelli.

ACKNOWLEDGMENT

The authors wish to express their thanks to T. W. Mullikin[4] for his assistance in making the calculations for this paper, and to R. N. Lyon[4] for making many helpful suggestions. The work was sponsored by the Atomic Energy Commission under Contract W-7405-ENG-26 with the Oak Ridge National Laboratory.

BIBLIOGRAPHY

1 "Heat Transfer to Molten Metals," by R. C. Martinelli, Trans. ASME, vol. 69, 1947, pp. 947–959.

2 "Untersuchungen Über die Strömungen des Wassers in konvergenten und divergenten Kanalen," by J. Nikuradse, *Forschungs-Arbeiten auf dem Gebiete des Ingenieurwesens*, Heft 289, Berlin, Germany, 1929, 49 pp.

3 "Remarks on Turbulent Transfer Across Planes of Zero Momentum Exchange," by F. A. Brooks and W. P. Berggren, University of California, Agricultural Experiment Station, Davis, Calif.

4 "Temperature Gradients in Turbulent Gas Streams," by W. H. Corcoran, B. Roudebush, and B. H. Sage, *Chemical Engineering Progress*, vol. 43, 1947, pp. 135–142.

5 "Gesetzmassigkeiten der turbulenten Strömung in glatten Rohren," by J. Nikuradse, Forschungsheft 356, VDI Verlag, Berlin, Germany, 1932, 36 pp.

6 "A Progress Report on Theoretical Considerations of the Heat Transfer Coefficient," by R. N. Lyon, Oak Ridge National Laboratory, Oak Ridge, Tenn., April 1, 1948.

7 "Laminar-Flow Heat-Transfer Coefficients for Ducts," by R. H. Norris and D. D. Streid, Trans. ASME, vol. 62, 1940, pp. 525–533.

8 "Measurements of the Temperature and Heat Exchange in a Catalytic Reaction," by M. Jakob, Trans. AIChE, vol. 35, 1939, pp. 563–586.

Discussion

R. N. Lyon.[5] It is of interest to compare the results of these calculations with those of Martinelli in an effort to find some roughly accurate general relationship between them. If the values of N_{Nu} for symmetrically heated parallel plates, for asymmetrically heated parallel plates, and for symmetrically heated tubes are plotted versus $N_{Pe} \times N_{Re}$ or Peclet number N_{Pe}, three lines can be drawn which approximate the three conditions.

Inspection of the three lines reveals the interesting fact that the following relations hold with reasonable accuracy up to N_{Pr} about 0.5:

$N_{Nu(\text{sym})} = 3 + N_{Nu(\text{CT})}$ at the same value of N_{Pe}
$N_{Nu(\text{asym})} = 0.7\ N_{Nu(\text{CT})}$ at the same value of N_{Pe}
$N_{Nu(\text{asym})} = 0.7\ [N_{Nu(\text{sym})} - 3]$ or $[0.7\ N_{Nu(\text{sym})} - 2.1]$ at the same value of N_{Pe}

In all these cases D in $N_{Nu} = hD/k$ and $N_{Pe} = cu\gamma D/k$ is the usual $4A/P_w$ where A is the flow cross section in the channel and P_w is the wetted perimeter.

The theoretical significance of these approximations, while interesting for speculation, is probably not important. It is felt, however, that there may be a practical value in being able to convert from one situation to the other by such simple means.

R. C. Martinelli.[6] Since a paper by the writer[7] is used as a basis for the present paper, it should be pointed out that an error was made in that paper.[7] This error was first noted by Mr. R. N. Lyon.

The temperature distribution for the limiting case of $N_{Pr} = 0$ was shown in the former paper[7] as a series of broken straight lines. Actually the temperature distribution should have been parabolic. The equations in the reference are correct and give this parabolic curve, but a numerical error was made in establishing the curves, which gave the erroneous temperature distribution for $N_{Pr} = 0$. Luckily, this error seriously affects only calculations for $N_{Pr} <$ 0.003, and fluids with N_{Pr} 0.003 are rare. In order to set the record straight, however, the corrected values of $(t_w - t_m)/$

---

[4] Oak Ridge National Laboratory, Oak Ridge, Tenn.

[5] Oak Ridge National Laboratory, Oak Ridge, Tenn.

[6] Associate Professor of Mechanical Engineering, University of California, Berkeley, Calif. Mem. ASME. Deceased January 9, 1949.

[7] See authors' Bibliography (1).

$(t_w - t_c)$ and the Nusselt modulus are tabulated herewith (Tables 2–5). In these calculations the friction factors for smooth pipes presented by Moody[8] were utilized.

TABLE 2 RATIO $(t_w - t_m/t_w - t_c)$ FOR CIRCULAR PIPES

N_{Re} / N_{Pr}	2000	10^4	10^5	10^6	10^7
0	0.600	0.564	0.558	0.553	0.550
0.0001	0.610	0.568	0.560	0.565	0.617
0.001	0.616	0.570	0.572	0.627	0.728
0.01	0.621	0.589	0.639	0.738	0.813
0.10	0.669	0.692	0.761	0.823	0.864
1.00	0.806	0.865	0.877	0.897	0.912
10	0.943	0.958	0.962	0.963	0.966
100	0.991	0.992	0.993	0.993	0.994

TABLE 3 RATIO $(t_w - t_m/t_w - t_c)$ FOR FLAT PLATES

$(N_{Re})_{FP}$ / N_{Pr}	4.89×10^3	2.20×10^4	2.16×10^5	2.12×10^6	2.10×10^7
0	0.745	0.718	0.711	0.707	0.704
0.0001	0.752	0.720	0.711	0.713	0.746
0.001	0.754	0.726	0.719	0.754	0.818
0.01	0.758	0.731	0.769	0.824	0.874
0.1	0.791	0.798	0.839	0.881	0.908
1.0	0.864	0.913	0.918	0.931	0.940
10	0.966	0.972	0.974	0.976	0.977
100	9.995	0.996	0.996	0.996	0.996

TABLE 4 NUSSELT MODULUS FOR CIRCULAR TUBES

N_{Re} / N_{Pr}	2000	10^4	10^5	10^6	10^7
0	5.90	7.02	7.17	7.23	7.27
0.0001	5.82	6.98	7.29	8.11	12.5
0.001	5.78	7.08	8.06	13.3	43.3
0.003	5.85	7.15	9.18	22.9	94.7
0.007	5.93	7.60	12.5	38.2	184
0.010	6.01	7.97	14.5	48.7	244
0.030	6.34	9.66	25.7	108	605
0.070	6.75	12.2	42.6	205	1224
0.100	7.10	14.0	53.0	271	1649
0.300	8.87	21.6	107	623	3993
0.700	11.3	31.2	179	1128	7666
1.000	12.9	36.7	217	1416	9953

TABLE 5 NUSSELT MODULUS FOR FLAT PLATES

$(N_{Re})_{FP}$ / N_{Pr}	2000	10^4	10^5	10^6	10^7
0	8.2	10.2	11.2	11.3	11.3
0.0001	8.3	10.2	11.2	12.1	16.5
0.001	8.3	10.2	12.0	16.2	46.0
0.003	8.4	10.3	13.1	26.6	104
0.007	8.5	11.0	16.1	43.2	196
0.010	8.6	11.2	18.3	54.0	260
0.030	8.7	12.6	30.5	116	640
0.070	8.9	14.8	48.0	220	1300
0.100	9.2	16.0	59.0	290	1730
0.300	10.2	23.8	110	658	4200
0.700	12.1	34.0	183	1160	7800
1.000	13.9	40.0	225	1450	10200

It is evident that the values of N_{Nu} are for all practical purposes the same as those shown in the writer's paper,[7] but the values of $(t_w - t_m)/(t_w - t_c)$ differ at the lower values of N_{Pr}, where the curves in the former paper were shown dotted. None of the conclusions reached in that paper is changed by this correction.

The extension by the authors of the analogy between heat and momentum transfer to asymmetrically heated channels is very interesting and provides a badly needed basis for analyzing heat transfer in asymmetrically heated annuli.

It is evident that the greatest difference between heat transfer in symmetrically and unsymmetrically heated channels occurs with molten metals. Experimental data for heat transfer to molten metals are necessary in order to verify the analysis presented.

The data of Corcoran, Roudebush, and Sage,[9] and Woertz and Sherwood,[10] may help clarify the question of the eddy diffusivity at the center of a channel.

AUTHORS' CLOSURE

The authors wish to acknowledge with appreciation the interest shown in this paper and the discussions by Dr. Lyon and Dr. Martinelli. It is pleasing to note that the relations presented in this paper have been applied by Dr. Lyon to the analysis of experimental data on sodium-potassium alloy flowing in an annulus.[11] The data appear to confirm the predictions within the limits of experimental error and the uncertainty regarding physical properties of the alloy. Dr. Lyon has recommended approximate equations which should be very useful to workers in this field.

Further research is being conducted at the University of Tennessee on heat transfer to mercury flowing in an asymmetrically heated rectangular channel, and it is hoped that the data will be available in the near future.[12]

It is with some shock that the authors must record their inability fully to acknowledge the contributions of Dr. Martinelli. He was entitled to say, "Non omnis moriar."

---

[8] "Friction Factors for Pipe Flow," by L. F. Moody, Trans. ASME, vol. 66, 1944, pp. 671–684.

[9] "Temperature Gradients in Turbulent Gas Streams," by W. H. Corcoran, N. Roudebush, and B. H. Sage, *Chemical Engineering Progress*, vol. 43, 1947, p. 135.

[10] "The Role of Eddy-Diffusion in Mass Transfer Between Phases," by B. B. Woertz and K. T. Sherwood, Trans. AIChE, vol. 35, 1939, p. 517.

[11] "Heat Transfer at High Fluxes in Confined Spaces," by R. N. Lyon, PhD Thesis, University of Michigan, 1949.

[12] Department of Chemical Engineering, University of Tennessee, Knoxville, Tenn.