can be used on complex real-world problems. It can also provide additional insight into the physical behavior of a transient system.

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DISCUSSION

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The authors have presented two applications of an optimization program involving low order linear mechanical systems. The results are of some interest although the paper is hard to follow due to misprints (Figs. 1 and 4 seem to be interchanged) and unusual notation (**s** is called a state vector while **z**, a vector of displacements and velocities, though typically called a state vector in dynamics, is not called a state vector in the paper).

The reference to "time domain optimization" in the title may be misleading. Another way to describe what the authors have done is by the phrase "parameter optimization" since the computer program apparently varied a damping parameter and a spring constant to optimize some aspects of the transient response of a particular 2 degree-of-freedom linear system model subject to inequality constraints. In related work, a group at I.I.T. investigated several possible methods of optimizing vibratory systems. Included were not only parameter optimization but also a true time domain optimization in which dynamic programming techniques were used to determine the optimal time history of forces which would achieve a minimum of a performance criterion subject to constraints, independent of the manner in which the force would actually be realized [1], [2].[3] Also, there have been attempts to compare the performance of systems optimized according to various criteria when such systems were subjected to a variety of transient and forced response situations, [3].

The authors have demonstrated their ability to achieve parameter optimization using a gradient technique, but it is not entirely clear that the method should be used on "complex real-world problems." The authors' examples are hardly complex nor do they necessarily represent the real world. Would anyone realistically construct a car bumper by trying to match it to a *linear* spring and dashpot combination? The potential of the technique might be better illustrated by using it to optimize nonlinear devices which, though suboptimal in the true time domain sense, yield responses closer to optimal than could be achieved with linear devices. Even in the vibration absorber problem, the proposed criterion of time-optimal energy dissipation is not so easy to justify. This criterion evidently yields a different optimal system for every different initial condition and indeed for every choice of percent energy remaining, ϵ. This phenomenon might be explained by realizing that the response of the system in question can at least roughly be considered the sum of the responses of two normal modes. Each modal velocity is described by an exponentially decaying sinusoid whose initial value is a function of all initial conditions. The rate of exponential decay for each mode can be different functions of the design parameters. For $\epsilon \to 1$ the tradeoff between the fast decay of a mode with high initial conditions and the slow decay of the mode with lower initial conditions is very critical. In the extreme one could hypothesize the mode with large initial value decaying very rapidly and the low initial condition mode never decaying in order for values of ϵ close to one to be achieved in optimum time. The exact nature of the trade off is a function of initial conditions and ϵ. When $\epsilon \to 0$, the responses of both modes are forced to be small as soon as possible. When $t \to T$ for this case, the difference due to initial conditions in the values of the modal responses must be small because the responses are small. Hence, initial conditions have little effect, and the main concern of the optimization process becomes making both responses small as soon as possible. This is the most reasonable criteria for optimization. Surely for most vibration absorber design techniques one desires a useful criterion which will produce a single isolation design which is optimal for a broad class of inputs or initial conditions. The authors' results for $\epsilon \to 0$ suggest the sort of result in optimal linear regulator design in which an infinite time integral square criteria yields an optimal design independent of initial conditions.

Finally, the paper illustrates the difficulties in interpretation which often arise in parameter optimization. Computed optimal parameters may be nearly useless unless supplemented by an understanding of the influence of small changes in the optimization criterion on the system parameters. In equation (13), for example, the authors allowed the absorber mass to be a parameter with a finite range to make sure that the optimum design would be "physically reasonable." On the face of it, however, it would seem that the absorber mass would always need to be as large as possible in order to optimize the system. This apparently was the case, and, at least in retrospect, one can see that the absorber mass is not usefully considered a variable parameter in the same sense as the spring constant and damping parameters. In this case, the authors could have been more specific and simply studied an absorber with, say, a mass of 10 percent of the main mass.

In another instance, the authors have taken a specific result and made a rather broad generalization from it which may not be justified. The statement that "an optimum steady state absorber will also be nearly optimum for transient conditions when ϵ is small" surely must be qualified. Though the statement is true for the authors' specific case, many other constraints and criteria might be used, and it would be amazing if the statement were universally true. Only when the systems remain entirely in the domain of linear optimum systems can one expect simple relations between optimal systems designed on the basis of transient and forced response [3].

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Authors' Closure

We wish to thank Messrs. Karnopp and Morison for their comments concerning our paper. We believe that most of the typographical errors which occurred in the preprint have been eliminated in the final manuscript.

The term "time domain optimization" is used to distinguish this work from studies done in the frequency domain. The term

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[3] Numbers in brackets designate Additional References at end of discussion.

"state variable," used in the paper, is associated with system optimization, not dynamics. Thus, the state variables describe the condition of the system at each iteration of the optimization process, not the condition of the system at a particular time.

The vibration systems discussed in this paper were chosen with a complex, real-world problem in mind—the design and evaluation of tractor-semitrailer truck systems and components.[4] The truck problem is highly nonlinear and its simulation is computationally expensive. Therefore, for an initial application of the design procedure, it was desirable to choose simpler problems whose objective functions and constraints are similar to the more complex truck problem. No attempt was made to exploit features (e.g., linearity, a priori knowledge of constraint effects, etc.) which are only characteristic of the simpler problems. In addition, no attempt was made to solve the general optimum design problem for vibration absorbers. As a consequence, the results and conclusions presented are for the objective functions and transient loadings described in the paper.

[4] A paper, "An Automated Method for Evaluating Truck Design," by A. I. Krauter and D. L. Bartel, has been submitted for presentation at the ASME Design Automation Conference; Toronto, Canada; September 1971.