Failure risk management of buried infrastructure using fuzzy-based techniques
Yehuda Kleiner, Balvant Rajani and Rehan Sadiq

ABSTRACT

The effective management of failure risk of buried infrastructure assets requires knowledge of their current condition, their rate of deterioration, the expected consequences of their failure and the owner’s (decision-maker) risk tolerance. Fuzzy-based techniques seem to be particularly suited to modeling the deterioration of buried infrastructure assets, for which data are scarce, cause-effect knowledge is imprecise and observations and criteria are often expressed in vague (linguistic) terms (e.g., ‘good’, ‘fair’ ‘poor’ condition, and so on). The use of fuzzy sets and fuzzy-based techniques helps to incorporate inherent imprecision, uncertainties and subjectivity of available data, as well as to propagate these attributes throughout the model, yielding more realistic results.

This paper is the second of two companion papers that describe an entire method of managing risk of large buried infrastructure assets. The first companion paper describes the deterioration modeling of buried infrastructure assets, using a fuzzy rule-based, non-homogeneous Markov process. This paper describes how the fuzzy condition rating of the asset is translated into a possibility of failure. This possibility of failure is combined with the fuzzy failure consequences to obtain fuzzy risk of failure throughout the life of the pipe. This life-risk curve can be used to make effective decisions on pipe renewal. These decisions include when to schedule the next inspection and condition assessment or alternatively, when to renew a deteriorated pipe, and what renewal alternative should be selected.

Key words | buried pipes, condition rating, deterioration, failure risk, fuzzy, infrastructure, renewal decision, risk management

NOTATION

\( b_k \) defuzzified value of \( B_k \). A fuzzy number representing the pipe fuzzy condition at time step \( t \)

\( B_k \) (\( i = 1, 2, \ldots, 5 \)) fuzzy triangular subsets (levels) in the fuzzy set \( B_k \), depicting the fuzzy deterioration rate of renewal alternative \( k \) relative to observed deterioration rate

\( C_i \) (\( i = 1, 2, \ldots, 7 \)) fuzzy triangular subsets (states) in the fuzzy set \( C \), which defines pipe condition

\( C_{ik}^k \) post-renewal condition for renewal alternative \( k \), which was applied at time step \( t^k \)

\( C_{ik}^k \) immediately before renewal implementation

\( C_{ik}^k \) immediately after renewal implementation

\( d_0 \) base deterioration unit

\( F_i \) (\( i = 1, 2, \ldots, 9 \)) fuzzy triangular subsets (levels) in the fuzzy set \( F \), which defines pipe possibility of failure
**INTRODUCTION**

To run an effective risk management program one needs to be able to predict failure risk levels throughout the life of the asset. Lawrence (1976) defined risk as a “measure of probability and severity of negative adverse effects”. In a companion paper, Kleiner et al. (2006), introduced a method to model the deterioration of large buried infrastructure assets as a rule-based, non-homogeneous fuzzy Markov deterioration process. This deterioration model yields a family of curves depicting the condition rating of the asset (in terms of membership values to defined condition states) at selected point along the life of the asset. This paper describes the following five steps in assessing and managing the risk of asset failure:

(a) How to convert the fuzzy life deterioration curve into possibility of failure.
(b) How to combine the possibility of failure with failure consequences (expressed as a fuzzy set) to obtain a life-risk curve.
(c) How to make a decision on whether to renew, rehabilitate or replace an asset or schedule its next inspection, based on the decision maker’s risk tolerance.
(d) How to use expert opinion to assess the post-renewal condition rating as well as the post-renewal deterioration rate of the asset.
(e) How to select a renewal alternative based on this post-renewal performance assessment.

The proposed fuzzy-based risk modeling and decision process is especially suited for assets for which data are scarce, and when available are often imprecise and vague. These assets are typically difficult to access and expensive to inspect. In this paper, data on buried, large-diameter water transmission mains are used to demonstrate all the aspects of the proposed method.

**FUZZY RISK OF FAILURE**

**Possibility of failure**

Kleiner et al. (2006) proposed a fuzzy rule-based Markov process to model infrastructure asset deterioration. They used a seven-grade (condition states) fuzzy set (excellent, good, adequate, fair, poor, bad, failed) to describe the condition rating of the asset. The deterioration model was formulated such that the condition of the asset at any given time could have significant memberships to no more than three contiguous condition states. It should be noted that the failed (state 7) condition state does not mean that collapse or rupture has already happened (in which case the membership would be a definite unity), rather that the asset is in such a bad condition that failure is imminent and can occur at any time as a result of the slightest perturbation. Further, since fuzzy subsets (such as triangular fuzzy numbers - TFNs) are often interpreted (Klir & Yuan 1995) as possibility distributions (in contrast to probability distribution), it follows that the membership to the failed state at any given time can be...
viewed as the possibility (not probability) of failure at that time (see brief discussion about probability versus possibility in Kleiner et al. 2006). The family of curves depicting the condition rating of the asset at every point in its life is illustrated at the top left portion of Figure 1.

A nine-grade fuzzy set $F$ (from extremely low to extremely high) defined for the possibility of failure is illustrated at the bottom right of Figure 1. The membership values to the failed state in each time step are fuzzified (or re-mapped) on to this fuzzy set $F$, as illustrated in Figure 1. This re-mapping of a membership value onto a secondary fuzzy set is similar to the concept of “type-2 fuzzy sets” (Mendel & John 2002). For example in Figure 1, membership of 0.45 to the failed state translates to 0.4 membership to moderately low and 0.6 membership to medium possibility of failure, or in a vector form (0, 0, 0.4, 0.6, 0, 0, 0, 0). The possibility of failure can be computed for each year $t$ in the life of the asset. This possibility of failure along the life of the asset is used to generate a fuzzy life-risk curve as is described later.

**Fuzzy consequences of failure**

The failure consequences of buried infrastructure assets such as large-diameter pipes can be substantial. They may comprise direct costs (emergency repair, direct damage to adjacent assets, liability), indirect costs (loss of production, accelerated deterioration) and social costs (disruption of business, loss of confidence, loss of time). Some research on how to assess these costs has been reported in the literature (e.g. Cromwell et al. 2002; PPK Consultants 1993) but it appears that much more work is required, and detailed case studies can help alleviate the lack of good data. A well-structured process of querying local experts and practitioners could be devised to interpret qualitative knowledge into fuzzy numbers. One possible approach is to develop a process based on fuzzy synthetic evaluation similar to that applied by Rajani et al. (2006) to translate distress indicators to condition ratings. However this issue is beyond the scope of the research described here.

In this paper it is assumed that the severity of asset-failure consequences can be provided through the fuzzy set $Q$, which comprises nine severity grades (triangular fuzzy numbers/subsets) that include extremely low, very low, quite low, moderately low, medium, moderately severe, quite severe, very severe and extremely severe. The utility is thus assumed to be able to provide a fuzzy set representing failure consequences. For example, the fuzzy set $(0, 0, 0.2, 0.8, 0, 0, 0, 0)$ represents an asset whose failure consequences has 20% membership to quite low and 80% membership to moderately low. The severity ratings of failure consequence can be subjective, for example, the loss of service to one thousand customers in a small municipality might be perceived as very severe, while the same loss of service in a large city would be described as quite low. It should also be noted that the fuzzy set representing failure consequences must be convex and

---

**Table:** Possibility of failure fuzzy set $F$

<table>
<thead>
<tr>
<th>Fuzzy subset ($F$)</th>
<th>Qualitative scale</th>
<th>TFN representation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F_1$ extremely low</td>
<td>(0.0, 0.125)</td>
<td>(0.0, 0.125, 0.25)</td>
</tr>
<tr>
<td>$F_2$ very low</td>
<td>(0.125, 0.25)</td>
<td>(0.125, 0.25, 0.375)</td>
</tr>
<tr>
<td>$F_3$ quite low</td>
<td>(0.25, 0.375)</td>
<td>(0.25, 0.375, 0.50)</td>
</tr>
<tr>
<td>$F_4$ moderately low</td>
<td>(0.375, 0.50)</td>
<td>(0.375, 0.50, 0.625)</td>
</tr>
<tr>
<td>$F_5$ medium</td>
<td>(0.50, 0.625)</td>
<td>(0.50, 0.625, 0.75)</td>
</tr>
<tr>
<td>$F_6$ moderately high</td>
<td>(0.625, 0.75)</td>
<td>(0.625, 0.75, 0.875)</td>
</tr>
<tr>
<td>$F_7$ quite high</td>
<td>(0.75, 0.875)</td>
<td>(0.75, 0.875, 1.0)</td>
</tr>
<tr>
<td>$F_8$ very high</td>
<td>(0.875, 1.0)</td>
<td>(0.875, 1.0, 1.0)</td>
</tr>
<tr>
<td>$F_9$ extremely high</td>
<td>(1.0)</td>
<td>(1.0, 1.0, 1.0)</td>
</tr>
</tbody>
</table>

---

**Figure 1:** Re-mapping of membership values in the “failed” state onto the “possibility of failure” fuzzy set.
must have unit cardinality, thus, sum of memberships in a fuzzy set is equal to 1.

**Fuzzy risk of failure**

As stated in the introduction, failure risk is a measure of the probability and severity of failure. Often failure risk cannot be treated with mathematical rigour during the initial or screening phase of decision-making (Lee 1996), especially when a complex system involves various contributory risk items with uncertain sources and magnitudes.

Let fuzzy set $Z$, representing failure risk, be defined by nine triangular fuzzy subsets. These subsets represent the nine failure risk levels extremely low, very low, quite low, moderately low, medium, moderately high, quite high, very high and extremely high. The rule-set $R_Z$ that governs the relationship between failure possibility (set $F$), failure consequence (set $Q$) and failure risk (set $Z$) is given in Table 1. The Mamdani (1977) algorithm, detailed in the companion paper Kleiner et al. (2006) for the multiple input single output (MISO) model, is used to compute the fuzzy risk (output) from the inputs of fuzzy possibility and fuzzy consequences of failure.

As the possibility of failure can be calculated for each year $t$ in the life of the asset, it can be combined with the fuzzy consequences of failure to obtain a fuzzy risk curve for the life of the asset, as illustrated in Figure 2. The intensity of the grey levels in Figure 2 represents the membership values to the respective risk levels. The black curve represents the defuzzified risk values, which are akin to the mean risk level. It can be seen that the defuzzified values do not always coincide with the highest membership (modal) values, which means that the fuzzy set that represents risk at any year $t$ is not necessarily symmetrical about its mode.

Kleiner et al. (2006) described the concept of a possibilistic confidence band (constructed using $\alpha$-cuts). The same concept is used to construct a confidence band for the life-risk curve, as illustrated in Figure 2 for $\alpha = 0.25$. In this example, the band defines the ranges at which membership values to any risk grade are greater or equal to 0.25. In other words, within this band are all those risk levels to which the modelled asset has membership of at least 0.25. Naturally, the confidence band will become narrower as the value of $\alpha$ increases. Conversely, for $\alpha = 0$ the confidence band will encompass the entire possibility spectrum (support of the fuzzy set).

It is clear that the upper bound of the band represents a more conservative attitude, while the lower bound represents a more optimistic attitude. It is worth reiterating

### Table 1 | Rule-set $R_Z$ for failure possibility-consequence-risk relationship

<table>
<thead>
<tr>
<th>Failure possibility, $F$</th>
<th>extremely low</th>
<th>very low</th>
<th>quite low</th>
<th>moderately low</th>
<th>medium</th>
<th>moderately severe</th>
<th>quite severe</th>
<th>very severe</th>
<th>extremely severe</th>
</tr>
</thead>
<tbody>
<tr>
<td>extremely low</td>
<td>EL</td>
<td>EL</td>
<td>VL</td>
<td>VL</td>
<td>QL</td>
<td>QL</td>
<td>ML</td>
<td>ML</td>
<td>M</td>
</tr>
<tr>
<td>very low</td>
<td>EL</td>
<td>VL</td>
<td>VL</td>
<td>QL</td>
<td>QL</td>
<td>ML</td>
<td>M</td>
<td>M</td>
<td>MH</td>
</tr>
<tr>
<td>quite low</td>
<td>VL</td>
<td>VL</td>
<td>QL</td>
<td>QL</td>
<td>ML</td>
<td>ML</td>
<td>M</td>
<td>M</td>
<td>MH</td>
</tr>
<tr>
<td>moderately low</td>
<td>QL</td>
<td>QL</td>
<td>QL</td>
<td>QL</td>
<td>ML</td>
<td>ML</td>
<td>M</td>
<td>M</td>
<td>MH</td>
</tr>
<tr>
<td>medium</td>
<td>QL</td>
<td>ML</td>
<td>ML</td>
<td>ML</td>
<td>ML</td>
<td>M</td>
<td>MH</td>
<td>MH</td>
<td>QH</td>
</tr>
<tr>
<td>moderately high</td>
<td>ML</td>
<td>ML</td>
<td>ML</td>
<td>M</td>
<td>M</td>
<td>MH</td>
<td>MH</td>
<td>QH</td>
<td>QH</td>
</tr>
<tr>
<td>quite high</td>
<td>ML</td>
<td>M</td>
<td>M</td>
<td>M</td>
<td>MH</td>
<td>MH</td>
<td>QH</td>
<td>QH</td>
<td>VH</td>
</tr>
<tr>
<td>very high</td>
<td>M</td>
<td>M</td>
<td>MH</td>
<td>MH</td>
<td>QH</td>
<td>VH</td>
<td>VH</td>
<td>EH</td>
<td></td>
</tr>
<tr>
<td>extremely high</td>
<td>M</td>
<td>MH</td>
<td>MH</td>
<td>QH</td>
<td>QH</td>
<td>VH</td>
<td>VH</td>
<td>EH</td>
<td></td>
</tr>
</tbody>
</table>

EL = extremely low, VL = very low, QL = quite low, ML = moderately low, M = medium, MH = moderately high, QH = quite high, VH = very high, EH = extremely high.
here that the possibilistic confidence band cannot be interpreted in the same way as a probabilistic confidence band, as it provides a notional rather than a frequentist-based quantitative idea about the likelihood of the ‘best estimate’ prediction to be in the corresponding interval. Further, there generally is no ‘standard’ α value used for this type of analysis (analogous to, say, 90% or 95% confidence level in probabilistic analysis. This possibilistic confidence band is used in the decision process as discussed later.

**Decision-making process**

Ideally the optimal strategy of renew/repair/inspect will be the one, which minimizes the present value of the total life-cycle costs (including direct, indirect and social costs of failure) that are associated with the asset. This strategy requires accurate forecasting of the asset deterioration and its probability of failure over its life cycle, as well as of the expected consequences of failure. Additionally, it requires the forecasting of asset deterioration and probability of failure after rehabilitation or renewal, which may change the characteristics of the renewed asset and its behaviour.

Several decision-making strategies for various infrastructure assets have been described in the literature. Examples that include WRe (1995, 1994), Edmonton (1996) and Zhao and McDonald (2000), provide guidance for inspection frequency of sewers. These guidelines are largely qualitative and prescriptive, for example, “condition x requires that the asset be inspected every y years”, and as such tend to provide a rather broad and general range of actions with only an implicit consideration of deterioration rates. However, these guidelines have gained significant popularity because of their simplicity and their need for few data. Other, researchers have proposed more elaborate and quantitative methods, for example, Madanat and Ben Akiva (1994), Li et al. (1995, 1997), Pandey (1998), Hong (1998), Jiang et al. (2000), Kleiner (2001), and Guillaumot et al. (2003). These, however, do not seem to have gained much popularity, possibly because underlying models are either too complex or are data hungry, or both.

Traditional quantitative approaches to decision-making present some limitations in the data-poor realm of buried infrastructure. The first limitation is the requirement for sufficient data to train the underlying deterioration models. The second is that, in order to evaluate a renewal alternative, post-renewal deterioration (for which typically no data exist) has to be evaluated. The third is that, the subjectivity and vagueness in the determination of the asset condition, as well as the consequences of failure, are not considered.

In the proposed fuzzy rule-based approach described in this paper, calculations of life cycle costs are not crisp, and ‘true probabilistic’ values for failure occurrences are not determined explicitly. Consequently, the traditional concept of discounting of future costs (present value of stream of costs) cannot be explicitly applied and life-cycle costs of various renewal alternatives cannot be directly compared.
**Criterion for renewal or scheduling next inspection/condition assessment**

A maximum risk tolerance (acceptable risk) value is proposed as a decision criterion, since the life-cycle costs of various alternatives cannot be directly compared. A water utility, through a consensus-building process like Delphi (Linstone & Turoff 2002) or other, will define its maximum risk tolerance (MRT) $z_{max}$ for its infrastructure asset. In the present context, maximum risk tolerance $z_{max}$ will be one of nine risk-level of the fuzzy set $Z$. Since the term risk is a composition of both the possibility of failure and the failure consequences, it is possible that one $z_{max}$ will suffice for the entire inventory of large-diameter transmission mains. At the same time, special consideration(s), which might not be readily integrated into the set of factors that determine failure consequences, may render more than one $z_{max}$ necessary.

It is assumed that any decision about renewal of the asset will always be preceded by an inspection and condition assessment. Thus, if the deterioration model of an asset predicts that maximum risk tolerance $z_{max}$ is going to be reached at year $t_z$, it follows that at year $t_z$ (or thereabouts) an inspection/condition assessment will be scheduled. This inspection/condition assessment can have one of two outcomes: either the observed condition of the asset is better than predicted, that is, the deterioration model overestimated the deterioration rate, or the observed condition of the asset is the same or worse than the model predicted, that is, the deterioration model was accurate or underestimated the deterioration rate. In the case of the former outcome, the deterioration model needs to be re-calibrated to include the newly acquired data, and then re-applied to obtain a new $t_z$. If it is the latter outcome, renewal work has to be planned immediately and implemented as soon as possible.

The decision maker’s tolerance (attitude) for risk can be expressed in two independent manners. First, as was described earlier in this section, the decision maker provides an explicit measure of maximum risk tolerance $z_{max}$. Second, the decision maker can select the $\alpha$-level of the possibilistic confidence limit. As discussed earlier, the confidence band will become wider (and the higher the confidence) as the $\alpha$-cut value decreases. Consequently, a decision maker with a conservative attitude increasingly makes more conservative decisions as the selected $\alpha$-cut is lowered. Figure 3 illustrates an example in which the MRT ($z_{max} = \text{moderately high}$) is forecasted to be met between 109 and 138 years of age at an $\alpha$-value of 0.6. The conservative decision maker will schedule the next inspection/condition assessment to occur at the asset age of 109 years.

**Examination and comparison of renewal alternatives**

As described earlier, the comparison of renewal alternatives requires the knowledge (or assumptions) about the expected performance of the asset after renewal. It is assumed that this post-renewal expected performance can be determined by (a) the post-renewal condition of the asset (i.e., the degree of improvement), and (b) the post-renewal deterioration rate of the asset.

![Figure 3](https://iwaponline.com/aqua/article-pdf/55/2/81/402747/81.pdf)
Evaluation of post-renewal condition

Renewal of an asset should invariably lead to the improvement of its condition rating. Typically, the shift in the condition rating from before to after renewal will depend on factors such as technology, material, process, and so on, of the specific renewal alternative. Quantitative information on how a specific renewal alternative improves the condition rating is most often unavailable because the renewal alternatives are based on technologies that are relatively new and conditions under which they are applied can vary. A raw condition improvement matrix, $P_k$, is introduced to formalize a process for making educated judgments about the shift of condition state from before to after the application of renewal alternative $k$.

In the absence of sufficient field data to assign deterministic or even probabilistic values, the raw condition improvement matrix, $P_k$, is constructed based on expert opinion, where linguistic terms, e.g., highest, medium, and lowest are used to capture the belief of experts about a shift from one condition state to another, as is illustrated in Table 2. In turn, each of these linguistic terms can be assigned relative weights, say, 0.7, 0.4 and 0.1, respectively (every row in $P_k$ must be convex). Once the linguistic terms are substituted by the corresponding weights, the raw condition improvement matrix, $P_k$, is normalized so that the sum of weights in each row equals unity, to obtain the normalized condition improvement matrix, $P_k$:

$$p_{ij} = \frac{p'_{ij}}{\sum_{j=1}^{7} p'_{ij}}$$

Table 2

<table>
<thead>
<tr>
<th>Confidence to get condition shift</th>
<th>To condition</th>
</tr>
</thead>
<tbody>
<tr>
<td>From condition</td>
<td>excellent</td>
</tr>
<tr>
<td>excellent</td>
<td>highest</td>
</tr>
<tr>
<td>good</td>
<td>highest</td>
</tr>
<tr>
<td>adequate</td>
<td>medium</td>
</tr>
<tr>
<td>fair</td>
<td>medium</td>
</tr>
<tr>
<td>poor</td>
<td>lowest</td>
</tr>
<tr>
<td>bad</td>
<td>medium</td>
</tr>
<tr>
<td>failed</td>
<td>lowest</td>
</tr>
</tbody>
</table>

lowest = 0.1, medium = 0.4, highest = 0.7, ‘blank’ = 0.
where the left matrix is the raw condition improvement matrix, \( P_k \). The post-renewal condition rating of the asset is obtained from equation (3) as follows:

\[
\tilde{C}_k^b = C_t \cdot P_k
\]  

(3)

where the operator \( \cdot \) indicates a simple matrix multiplication and \( t^k \) denotes the time of renewal alternative \( k \). The post-renewal condition \( \tilde{C}_k^b \) has a cardinality of unity, that is, the sum of all its members, which are membership values, equals unity. The renewal process itself is assumed to be instantaneous (with respect to the length of a time step which is typically one year), where \( \tilde{C}_t \) denotes the asset condition immediately after renewal, and \( \tilde{C}_k \) the asset condition immediately before renewal. For example, suppose that at age 60 the condition rating of an asset is \( C_{t=60} = (0, 0, 0, 0.3, 0.5, 0.2, 0, 0) \). If condition improvement matrix \( P_k \) from Table 2 is applied, the resulting post-renewal condition rating becomes \( \tilde{C}_{k=60} = (0.12, 0.50, 0.36, 0.02, 0, 0, 0) \).

Let the term post-renewal equivalent age, \( \tau^k \), be defined as the age of the asset, prior to renewal, at which its condition was equal (or very close to) to the post-renewal condition \( \tilde{C}_k^b \). Post-renewal equivalent age \( \tau^k \) is found by minimizing the sum of square deviations between \( C_{t=\tau} \) and \( \tilde{C}_k^b \) as depicted in (4):

\[
\tau^k \text{ such that } C_{t=\tau} \equiv \tilde{C}_k^b; (\tau^k < t^k);
\]

by finding:

\[
\min \sum_{i=1}^{7} \left( \mu_{\tilde{C}_k^b}^i - \mu_{C_t^i} \right)^2
\]

(4)

where \( \mu_{\tilde{C}_k^b}^i \) is the membership value in condition state \( i \) \((i = 1, \ldots, 7)\) of the post-renewal condition rating \( \tilde{C}_k^b \) (when renewal alternative \( k \) was implemented at time \( t^k \)) and \( \mu_{C_t^i} \) is the membership value in condition state \( i \) \((i = 1, \ldots, 7)\) of the pre-renewal fuzzy condition rating \( C_t^k \) \((\tau^k < t^k)\). This means that renewal option \( k \) made the asset functionally ‘younger’ by \((t^k - \tau^k)\) years.

In the example above, the post-renewal condition rating of the asset is \( \tilde{C}_{k=60} = (0.12, 0.50, 0.36, 0.02, 0, 0, 0) \). Suppose that searching through all the pre-renewal condition ratings of the asset \( C_t \) \((t = 1, \ldots, 60)\) it is found that \( C_{t=24} = (0.093, 0.533, 0.374, 0, 0, 0, 0) \) has the closest match of membership values to \( \tilde{C}_{k=60}^b \), that is, the minimum sum of square deviations between corresponding membership values. Consequently, \( \tau^k = 24 \) years (the asset is as good as it was at age 24) and renewal alternative \( k \) made the asset functionally ‘younger’ by \((t^k - \tau^k) = 36 \) years.

**Evaluation of post-renewal deterioration**

Once an asset segment has been renewed, it will undergo deterioration at a rate that may be the same or different from the asset segment before it was renewed. How slow or fast the renewed asset segment will deteriorate will largely depend on the characteristics of the selected renewal alternative. For a fair comparison between candidate renewal alternatives, the post-renewal deterioration rate must also be considered. However, for lack of available field data this evaluation must also be based on expert opinion.

Similar to the condition improvement matrix, expert opinion is solicited to provide input on how the expected post-renewal deterioration rate will be relative to the deterioration rate observed prior to renewal. This expert opinion is expressed in the same linguistic terms used for the raw condition improvement matrix. Let \( B_k \) denote a 5-tuple fuzzy set \((\text{much lower, lower, same, higher, much higher})\) as shown in Figure 4. The fuzzy number \( B_k \) is used to evaluate the post-renewal deterioration rate of renewal alternative \( k \) relative to the observed (or historical) deterioration rate (Table 3), similar to the manner in which \( P_k \) is used to evaluate post-renewal condition rating.

For convenience, the confidence in each of the elements of the \( B_k \) is expressed using the same linguistic terms (and relative weights) as those used for \( P_k \). Once the expert expresses belief on anticipated post-renewal deterioration rates, these linguistically expressed beliefs are substituted for their respective weights, which are mapped on fuzzy set \( B \), as is shown in Table 3. Subsequently, \( B_k \) is defuzzified (using the centre of area method) to obtain the post-renewal relative deterioration factor \( b_k \). In the example illustrated in Table 3 and Figure 4, the linguistic input in Table 3 is substituted by the respective weights to obtain \( B_k = (0, 0.4, 0.7, 0.1, 0) \), which is then mapped onto \( B \) (Figure 4) and defuzzified to obtain the post-renewal relative deterioration factor \( b_k \) \((= 0.944)\). Note that the solid line in Figure 4 is a graphical representation of the fuzzy number obtained from Table 3.
The post-renewal deterioration is modelled as if the asset continues to deteriorate from the post-renewal equivalent age, $t_k$, using the pre-renewal deterioration rate, $D_t = t_k$, modified by the post-renewal relative deterioration factor $b_k$, i.e., the asset continues to deteriorate through the same deterioration process described in the companion paper Kleiner et al. (2006). The only difference is that all the defuzzified deterioration factors $D_t^i$ are multiplied by the scalar $b_k$. In the above example, applying $b_k = 0.944$ will yield a post-renewal deterioration rate that is somewhat lower than the pre-renewal rate.

Quantification of post-renewal performance

Let $T_k$ be defined as the time interval it would take for the renewed asset (renewal alternative $k$) to reach condition $C_{t_k}$, which is its pre-renewal condition. It can be said colloquially that $T_k$ is the time interval which renewal alternative $k$ ‘bought’ for the asset, or in other words, it is the time interval by which the subsequent renewal can be deferred. Similar to $\tau_k$, $T_k$ is found by minimizing the sum of square deviations between $C_{t_k+T_k}$ and $C_{t_k}$ as depicted in equation (5):

$$T_k \text{ such that } C_{t_k+T_k} \equiv C_{t_k}; (\tau_k < \tau^k);$$

by finding:

$$\min \sum_{i=1}^{7} \left( \mu_{C_{t_k}^i} - \mu_{C_{t_k+T_k}^i} \right)^2$$

where $\mu_{C_{t_k}^i}$ is the membership value in condition state $i (i = 1, \ldots, 7)$ of the pre-renewal condition rating $C_{t_k}$ (when renewal alternative $k$ was implemented at time $t_k$); $\mu_{C_{t_k+T_k}^i}$ is the membership value in condition state $i (i = 1, \ldots, 7)$ of the post-renewal fuzzy condition rating $C_{t_k+T_k}$. In the example above $T_k = 40$ years, which means that the asset in the example is expected to return to its pre-renewal condition rating 40 years after renewal alternative $k$ has been applied.

<table>
<thead>
<tr>
<th>Expression of confidence in the post-renewal deterioration rate compared to the current deterioration rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>much lower</td>
</tr>
<tr>
<td>0</td>
</tr>
</tbody>
</table>

The post-renewal deterioration is modelled as if the asset continues to deteriorate from the post-renewal equivalent age, $\tau_k$, using the pre-renewal deterioration rate, $D_{t_k}$, modified by the post-renewal relative deterioration factor $b_k$, i.e., the asset continues to deteriorate through the same deterioration process described in the companion paper Kleiner et al. (2006). The only difference is that all the defuzzified deterioration factors $D_{t_k}^i$ are multiplied by the scalar $b_k$. In the above example, applying $b_k = 0.944$ will yield a post-renewal deterioration rate that is somewhat lower than the pre-renewal rate.

Quantification of post-renewal performance

Let $T_k$ be defined as the time interval it would take for the renewed asset (renewal alternative $k$) to reach condition $C_{t_k}$, which is its pre-renewal condition. It can be said colloquially that $T_k$ is the time interval which renewal alternative $k$ ‘bought’ for the asset, or in other words, it is the time interval by which the subsequent renewal can be deferred. Similar to $\tau_k$, $T_k$ is found by minimizing the sum of square deviations between $C_{t_k+T_k}$ and $C_{t_k}$ as depicted in equation (5):

$$T_k \text{ such that } C_{t_k+T_k} \equiv C_{t_k}; (\tau_k < \tau^k);$$

by finding:

$$\min \sum_{i=1}^{7} \left( \mu_{C_{t_k}^i} - \mu_{C_{t_k+T_k}^i} \right)^2$$

where $\mu_{C_{t_k}^i}$ is the membership value in condition state $i (i = 1, \ldots, 7)$ of the pre-renewal condition rating $C_{t_k}$ (when renewal alternative $k$ was implemented at time $t_k$); $\mu_{C_{t_k+T_k}^i}$ is the membership value in condition state $i (i = 1, \ldots, 7)$ of the post-renewal fuzzy condition rating $C_{t_k+T_k}$. In the example above $T_k = 40$ years, which means that the asset in the example is expected to return to its pre-renewal condition rating 40 years after renewal alternative $k$ has been applied.
Selection of a renewal alternative

As was noted earlier, each renewal alternative $k$ is associated with a condition improvement matrix $P_k$, and a post-renewal deterioration rate. Additionally, each renewal alternative will also have an associated cost $S_k$. Since a renewal alternative $k$ essentially ‘buys’ time $T_k$ for the asset until the next expected renewal, the magnitude of $T_k$ is determined by the extent to which the asset condition is expected to improve under the expected post-renewal deterioration rate. Assuming that all renewal alternatives will restore the functionality equally well during their respective expected $T_k$ periods, the only criterion for the selection of the best renewal alternative can be cost versus time ‘bought’, or more precisely, the preferred renewal alternative will be that for which the ratio $S_k/T_k$ is the lowest.

If different renewal alternatives are anticipated to provide different levels of functionality then additional functionality criteria need to be defined and quantified for all the renewal alternatives. However, this issue is beyond the scope of this research.

EXAMPLE

The data and results presented in the companion paper (Kleiner et al. 2006) are used here to illustrate the application of concepts developed in this paper. This example referred to one of several 96” (2400 mm) diameter PCCP pipe segments that belong to Arizona Public Service Company (APS). This PCCP pipe segment was installed in 1978 and inspected in 1997, 1999 and 2002. Kleiner et al. (2006). For the example presented here, the deterioration model was trained on all three condition ratings established from observed distress indicators and the modelled condition rating of the pipe in 2002 (age 24) was obtained as $C_{t=24} = (0.08, 0.8, 0.13, 0, 0, 0)$ with SSD (sum of squared deviations) 0.017. The base deterioration rate parameter was found to be $d_0 = 0.054$. The resulting deterioration curves together with the failed curve are shown in Figure 5. Note that threshold values for condition states 3, and higher were assumed to be 0.7.

The following sub-sections are structured to correspond to each of the five steps involved in the assessment and management of failure risk that are enumerated in the introduction.

Convert fuzzy deterioration to possibility of failure

The failed curve obtained in the example discussed above was re-mapped onto the failure possibility fuzzy set (set $F$) using the process illustrated in Figure 1. This mapping was conducted for each year in the life of the pipe. It can be seen in Figure 5 that membership to the failed state is zero as long as the pipe is less than 72 years old. Consequently the model predicts its possibility of failure to be $(1, 0, 0, 0, 0, 0, 0, 0, 0)$, that is, extremely low, and increases after 72 years until it reaches extreme values at ages beyond 100 years.

Figure 5 | An example of model training on three inspections.
Combining possibility and consequences of failure to obtain risk

The fuzzy consequences of failure were arbitrarily assumed to be \( Q(0, 0, 0.2, 0.5, 0.3, 0, 0, 0) \), that is, predominantly *moderately low* with lower memberships to *medium* and *quite low*. The graphical representation of the fuzzy consequences is illustrated in Figure 6a. The fuzzy consequences (set \( Q \)) and failure possibility (set \( F \)) were then combined, using the Mamdani (1977) algorithm with the rule base provided in Table 1, to obtain the fuzzy risk for every year in the life of the pipe, as shown in Figure 6b (on the right).

**Decision: renew or schedule next inspection**

It was arbitrarily assumed that decision maker has a maximum risk tolerance \( z_{max} \, medium \), and a moderately conservative attitude which requires a confidence band of 50% (\( \alpha \) value of 0.5). Figure 6b (left) illustrates that \( z_{max} \) is anticipated to be exceeded at age of about 77 years, based on this moderately conservative attitude (if an optimistic attitude were taken, \( z_{max} \) would be anticipated to be exceeded about 15 years later). Consequently, the next inspection/condition assessment would be scheduled to that age and the resulting condition rating compared to condition rating that was predicted by the model for that age. If the newly observed condition rating indicates that the pipe is in a better condition than predicted, the model will need to be re-calibrated using the new data (the newly observed condition rating) and the subsequent inspection will then be scheduled. If, on the other hand, the observed condition rating is worse or the same as that predicted by the mode, renewal works should be implemented as soon as possible.

**Assessment of post-renewal condition rating and deterioration factor**

Suppose that the utility had contemplated the implementation of external post-stressing of the PCCP pipe at an age of, say, 60 years (earlier than the anticipated age to reach \( z_{max} \)) due to an unexpected change in attitude. This renewal action is referred to here as renewal alternative \( k = 1 \). In accordance with the trained model (Figure 5) the condition rating of the pipe at that age is predicted to be approximately \( C_{60}^{1} = (0, 0, 0.1, 0.3, 0.6, 0, 0) \), that is, predominantly poor. Suppose further that the condition improvement matrix \( P_{1} \) for renewal alternative 1 can be extracted based on expert opinion depicted in Table 2. The post-renewal condition rating compared to condition rating that was predicted by the model for that age. If the newly observed condition rating indicates that the pipe is in a better condition than predicted, the model will need to be re-calibrated using the new data (the newly observed condition rating) and the subsequent inspection will then be scheduled. If, on the other hand, the observed condition rating is worse or the same as that predicted by the mode, renewal works should be implemented as soon as possible.
The rating of the pipe is found by applying equation (3), i.e., 
\[ r_{60}^{1} = C_{60} \& P_{1} = (0.1, 0.4, 0.5, 0, 0, 0, 0). \] Applying equation (4) will reveal that the expected equivalent age of the pipe post-renewal is \( \tau^{1} = 30 \) years, that is, post-renewal condition rating is such that the pipe has been rejuvenated to an age of 30 years.

Suppose further that the post-renewal deterioration rate for renewal alternative 1 can be extracted based on expert opinion depicted in Table 3. The post-renewal deterioration factor will therefore be the same as determined in Figure 4, that is, \( b_{k=1} = b_{1} = 0.944. \)

**Selection of a renewal alternative**

Once the expected post-renewal condition rating and post-renewal deterioration rate factor are computed, the rule-base fuzzy Markov deterioration process (Kleiner et al. 2006) is applied, starting with the condition rating \( C_{60}^{1} \), which is taken at the equivalent age \( \tau^{1} \). This deterioration process is applied, however, at a slower rate of deterioration compared to the pre-renewal rate. This deterioration ‘slow down’ is achieved by multiplying the deterioration rate at every time step by the factor \( b_{1} \). The resulting pre- and post-renewal deterioration curves are illustrated in Figure 7.

Applying equation (5), it is found that \( T_{1} = 38 \) years, that is, it is expected that at age \( (60 + 38 = 98) \) the condition of the renewed pipe will be similar to its condition just before renewal. This means that renewal alternative 1 is expected to enable the deference of subsequent renewal by 38 years if none of decision criteria change. If the cost of renewal alternative 1 is say, \( S_{1} = $50,000 \), the cost per renewal year becomes \( S_{1}/T_{1} = $1316 \). Other renewal alternatives can be examined in the same manner and if all else remains unchanged, the selected renewal alternative will be the one with the lowest \( S_{k}/T_{k} \) ratio.

**DISCUSSION**

The research described in this paper and in the companion paper (Kleiner et al. 2006) can be viewed as a first attempt to formalize and standardize a consistent approach to decision-making for buried infrastructure assets such as large-diameter water transmission mains, trunk sewers, and so on. The approach consists of collecting, recording and interpreting data, using these data to model and predict deterioration, evaluating failure risk through the pipe life and making rational decisions on inspection and renewal scheduling. As noted earlier, one of the main problems observed in the course of this research and elsewhere is the severe lack of pertinent data. The adoption of this approach by water utilities will motivate practitioners to collect and record the appropriate data.

Fuzzy techniques were used for this approach because of the capability of these techniques to accommodate data that are uncertain as well as vague and imprecise. The granularities of the various fuzzy sets were taken as seven condition states for pipe condition rating, nine grades for the possibility of failure, nine levels of severity of failure consequences and nine levels of failure risk. However, it
should be noted that the mathematical framework of the approach could accommodate any granularity to suit the user’s preferences.

The proposed method can differentiate between renewal alternatives based on their cost and their longevity. Its current limitations are: (a) assumption that level of service (excluding longevity) is the same for all options, (b) assumption that the cost to implement renewal does not depend on the condition of the asset, and (c) assumption that decision makers will always want to reach maximum risk tolerance (MRT). A recent paper (Kleiner 2005) expands the proposed method, while addressing two (b and c) of these three limitations.

More research is required to formalize a consistent approach to express failure consequences as fuzzy sets. Future availability of data will enable more rigorous case studies. These may be required to better determine the predictive capabilities of the deterioration model as well as to investigate the sensitivities of the model to various assumptions.

SUMMARY

The scarcity of data about the deterioration rates of buried infrastructure assets, coupled with the imprecise and often subjective nature of assessment of pipe condition merits the usage of fuzzy techniques to model the deterioration of these assets. The deterioration process is modelled as a fuzzy rule-based non-homogeneous Markov process, in which memberships “flows” from higher to lower condition states. The model is trained using the condition rating(s) of the asset extracted from distress indicators recorded during inspection session(s). The model can be used to predict the future deterioration rate of the asset, subject to some judgment-based assumptions. The deterioration model was partially validated using available data, however, a more rigorous validation is required, with data that have more historical depth as well as consistent inspection techniques.

The prediction of the fuzzy condition rating of the asset is coupled with the fuzzy consequences of failure, using a fuzzy rule base, to obtain the fuzzy risk of failure throughout the life of the asset. This risk of failure is used to make a decision about the scheduling of the next inspection. If renewal action is required, the decision maker uses a structured process to make an educated guess about the performance of the renewal alternatives to be considered. Subsequently, the most economical alternative can be selected.

ACKNOWLEDGEMENTS

This paper is based on a research project, which was co-sponsored by the American Water Works Association Research Foundation (AWWA RF), the National Research Council of Canada (NRC) and water utilities from the United States and Canada and Australia. The project report, accompanied by a prototype computer application is available from AWWA RF. The project report number is 91087 and its title is “Risk management of large-diameter water transmission mains”.

REFERENCES

Cromwell, J. E., III, Reynolds, H., Jr., Pearson, N. & Grant, M. 2002 Cost of Infrastructure Failure. American Water Works Association Research Foundation, Denver, CO, USA.

Edmonton, City of 1996 Standard sewer condition rating system report. City of Edmonton Transportation Department, Alberta, Canada.


Lawrence, W. W. 1976 *Of acceptable risk*. William Kaufmann, Los Altos, CA, USA.


First received 28 August 2005; accepted in revised form 8 December 2005