$$\Omega=\sqrt{\frac{m_{02}}{\pi}}\,E''=\sqrt{\frac{m_{02}}{\pi}}\left(\frac{E'}{2}\right) \quad \text{(B13}c\text{)}$$

$$k'=\frac{0.4777\lambda}{1-1.3211\lambda} \quad \text{(B13}d\text{)}$$

$$k=\sqrt{1-k'^2} \quad \text{(B13}e\text{)}$$

$$K(k)=\int_0^{\frac{\pi}{2}}\frac{d\psi}{\sqrt{1-k^2\sin^2(\psi)}} \quad \text{(B13}f\text{)}$$

m_{20} and m_{02} are the second spectral moments corresponding to surface roughness directions along the grain and across the grain, respectively. These values are calculated for $\gamma = 9$ surfaces:

$$\sqrt{m_{20}}=0.5205\sigma\pi/\lambda_x^*$$
$$\sqrt{m_{02}}=1.548\sigma\pi/\lambda_x^*$$

The above values are substituted into (B13*b*–*f*) giving:

$\lambda=0.3363$

$\Omega=0.8732\pi\sigma E'/(2\lambda_x^*)$

$k'=0.2892$

$k=0.9573$

$K(k)=2.663$

Equation (B13*a*) now becomes

$$\frac{\overline{p}}{A_c/A_{nom}}=\frac{0.5586\pi\sigma E'}{2\lambda_x^*} \quad \text{(B14)}$$

or

$$\overline{P}=0.5586A_c/A_{nom} \quad \text{(B15)}$$

DISCUSSION

I. Etsion[1]

The authors are dealing with the very important subject of contacting rough surfaces. Unfortunately, by ignoring a vast amount of prior work they are not doing justice to the subject. The reader of the paper is mislead to believe that since the work of Bush et al. in 1979 nothing was done on the problem of contacting rough surfaces until 1992. The state of the art is of course completely different, and a number of other papers can be found in the literature e.g., Yamada et al. (1978), McCool (1986), Webster and Sayles (1986), Chang et al. (1987), Seabra and Berthe (1987), and Bhushan and Doerner (1989) to name just a few. In some of these previous works e.g., Change et al. (1987) the unrealistic assumption that all the asperities remain elastic was relaxed. In these cases it was shown that unless the surfaces are extremely smooth or very hard the contact is actually more plastic than elastic. A very important parameter in the problem of contacting rough surfaces is the plasticity index ψ suggested by Greenwood and Williamson (1966). The plasticity index is a measure of the level of plastic contact. For $\psi<1$ the contact is mainly elastic for $\psi>3$ the contact is mainly plastic. A low value of ψ is associated with low roughness and hard materials. Bhushan and Doerner (1989) and Etsion and Amit (1993) found ψ values near or below unity for extremely smooth surfaces of magnetic rigid disks. Plasticity index values above $\psi=3$ were reported by Etsion and Front (1992) on lapped faces of mechanical seals. Most engineering surfaces exhibit roughness values higher than those obtained by lapping and hence, completely elastic contact is limited to a very narrow range of application. Even in this narrow range of magnetic rigid disks the ratio A_c/A_{nom} is of the order of 10^{-4} to 10^{-5}, that is several orders of magnitude less than the values shown by the authors in Figs. 5, 7, and 9. Values of real area of contact as shown by the authors require atomically smooth surfaces, a condition that is determined to tribological applications.

In light of all these facts it is not clear what does the present work contribute to the state of the art and how can its results be useful in solving the real life problems of friction wear and lubrication.

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Authors' Closure

The authors certainly acknowledge the significance of recent works on the subject of contact simulations of rough surfaces. Indeed there are many in the literature. In our previous paper (Ren and Lee, 1993), the predecessor to the present work, the state-of-the-art works of many authors were discussed including some of those cited by the discusser. The noteworthy recent works which should be mentioned again are: Francis (1983), Lai and Cheng (1985), West and Sayles (1987), Bailey and Sayles (1991), and Lubrecht and Ioannides (1991).

The present work is applicable for surfaces with plasticity indexes, ψ, less than 1.0. Under these conditions, the asperities remain predominantly elastic during contacts. While it is generally accepted that many engineering surfaces exhibit roughness values resulting in much higher than a unity value of ψ, this is limited mainly to virgin surfaces. Under well lubricated conditions, the plasticity index drops sharply as the highest peaks of the asperities are run-in. This was demonstrated in a well known work done by Greenwood and Williamson in 1966 where surface-ground mild steel specimens were slid against stainless steel woven grids under lubricated conditions. The changes in the plasticity indexes of the specimens were measured as a function of time. Figure 11 shows their test results of specimens which had the high initial plasticity indexes of 3.0 and 9.0, but dropped quickly to reach steady-state ψ-values of less than 1.0—the elastic regime.

The discusser questions the merits of using the elastic model for contact analyses of rough surfaces. His arguments are based primarily on past studies of the magnetic head/disk contact interfaces. However there are numerous mechanical compo-

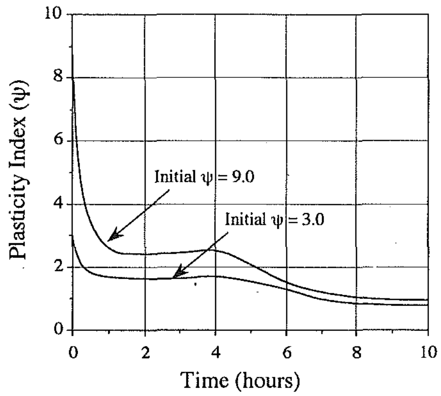

Fig. 11 Effect of continued sliding on the plasticity index. The behavior of mild steel specimens slid against stainless-steel woven grids at 100 cm/s under loads of 1.5 kg.

nents including bearings, gears, cams, tappets, and piston rings which will operate in the near elastic regime after running-in. It was pointed out in this paper that although the plastic deformation assumption is valid for the first few contact cycles, it would be unreasonable to assume this for machine components which would make millions of contact cycles during their lifetime. For general rough surface contact analyses, including those of magnetic disk applications, the authors have extended the present work to incorporate the effects of the elastic-plastic deformations in a soon to be published work (Lee and Ren, 1994).

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