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Discussion

Discussion: "Kinematics of the Translational 3-URC Mechanism" [Di Gregorio, R., 2004, ASME J. Mech. Des., 126, pp. 1113–1117]

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The author of [1] proposed a 3-URC translational parallel mechanism (TPM) and presented a comprehensive study on the kinematics of the 3-URC TPM. He concluded that "only one solution exists both for the direct and for the inverse position analyses." However, we do not agree with his result on the inverse position analysis and his statement that Ref. [2] "presented a class of TPMs with linear input-output equations that contain some translational 3-URC mechanisms."

In this discussion, we will show that the 3-URC TPM does not belong to the class of TPMs with linear input-output equations [2–6] by investigating the inverse position analysis of the 3-URC TPM.

Leg i of a 3-URC TPM is shown in Fig. 1. In addition to the notations used in [1], $\mathbf{h}_i$ is used to denote a unit vector directed from A_i to C_i. For leg i, the following holds: $\mathbf{w}_{1i}\cdot\mathbf{v}_i=0$, $\mathbf{w}_{1i}\cdot\mathbf{w}_{2i}=0$, and $\mathbf{w}_{2i}\cdot\mathbf{h}_i=0$.

In the coordinate system $O-XYZ$, we have

$$\mathbf{A}_i + h_i\mathbf{h}_i - d_i\mathbf{w}_{2i} + s_i\mathbf{w}_{1i} = \mathbf{B}_{i0} \tag{1}$$

where

$$\mathbf{w}_{2i} = \cos\theta_{1i}\mathbf{v}_i + \sin\theta_{1i}\mathbf{w}_{1i}\times\mathbf{v}_i$$

$$\mathbf{w}_{2i}\times\mathbf{w}_{1i} = -\cos\theta_{1i}\mathbf{w}_{1i}\times\mathbf{v}_i + \sin\theta_{1i}\mathbf{v}_i$$

$$\mathbf{h}_i = \cos\theta_{2i}\mathbf{w}_{1i} + \sin\theta_{2i}\mathbf{w}_{2i}\times\mathbf{w}_{1i}$$

$$\mathbf{B}_{i0} = \mathbf{P} + \mathbf{R}_{bp}{}^{P}(\mathbf{B}_{i0} - \mathbf{P})$$

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The inverse position analysis of the 3-URC TPM can be performed by solving Eq. (1) for θ_{1i}, θ_{2i}, and s_i in sequence.

Solution for θ_{1i}

Taking the inner product of Eq. (1) with $\mathbf{w}_{2i}$, we obtain

$$\mathbf{w}_{2i}\cdot(\mathbf{A}_i - \mathbf{B}_{i0}) - d_i = 0 \tag{2}$$

i.e.,

$$(\mathbf{B}_{i0} - \mathbf{A}_i)\cdot(\mathbf{w}_{1i}\times\mathbf{v}_i)\sin\theta_{1i} + (\mathbf{B}_{i0} - \mathbf{A}_i)\cdot\mathbf{v}_i\cos\theta_{1i} + d_i = 0 \tag{3}$$

Define an angle α_i by

$$\cos\alpha_i = [(\mathbf{B}_{i0} - \mathbf{A}_i)\cdot(\mathbf{w}_{1i}\times\mathbf{v}_i)]/a_i$$
$$\sin\alpha_i = [(\mathbf{B}_{i0} - \mathbf{A}_i)\cdot\mathbf{v}_i]/a_i \tag{4}$$

where $a_i=[(\mathbf{B}_{i0}-\mathbf{A}_i)\cdot\mathbf{v}_i]^2+[(\mathbf{B}_{i0}-\mathbf{A}_i)\cdot(\mathbf{w}_{1i}\times\mathbf{v}_i)]^2$.

Equation (3) can be rewritten as

$$\sin(\theta_{1i} + \alpha_i) = -d_i/a_i \tag{5}$$

Solving $\sin^2(\theta_{1i}+\alpha_i)+\cos^2(\theta_{1i}+\alpha_i)=1$, we obtain two solutions for $\cos(\theta_{1i}+\alpha_i)$ as

$$\cos(\theta_{1i} + \alpha_i) = \pm[1 - \sin^2(\theta_{1i} + \alpha_i)]^{1/2} \tag{6}$$

Equations (5) and (6) show that there are two solutions for $(\theta_{1i}+\alpha_i)$. For each $(\theta_{1i}+\alpha_i)$, one solution for θ_{1i} can be obtained as

$$\sin\theta_{1i} = \sin(\theta_{1i} + \alpha_i)\cos\alpha_i - \cos(\theta_{1i} + \alpha_i)\sin\alpha_i$$
$$\cos\theta_{1i} = \cos(\theta_{1i} + \alpha_i)\cos\alpha_i + \sin(\theta_{1i} + \alpha_i)\sin\alpha_i \tag{7}$$

From Eqs. (4)–(7), we learn that there are usually two solutions for θ_{1i}.

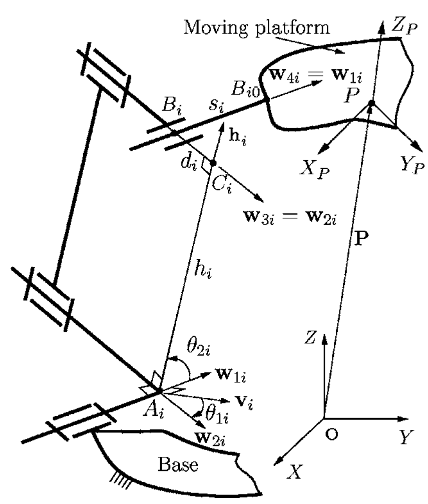

Fig. 1 URC leg

Solution for θ_{2i}

For each θ_{1i} obtained using Eq. (7), $\sin\theta_{2i}$ can be obtained by taking the inner product for Eq. (1) with $\mathbf{w}_{2i}\times\mathbf{w}_{1i}$ as

$$\sin\theta_{2i}=\{\sin\theta_{1i}[(\mathbf{B}_{i0}-\mathbf{A}_i)\cdot\mathbf{v}_i]-\cos\theta_{1i}[(\mathbf{B}_{i0}-\mathbf{A}_i)\cdot(\mathbf{w}_{1i}\times\mathbf{v}_i)]\}/h_i$$

Substituting Eq. (4) into the above equation, we obtain

$$\sin\theta_{2i}=-a_i\cos(\theta_{1i}+\alpha_i)/h_i \tag{8}$$

Solving $\sin^2\theta_{2i}+\cos^2\theta_{2i}=1$, we obtain two solutions for $\cos\theta_{2i}$ as

$$\cos\theta_{2i}=\pm(1-\sin^2\theta_{2i})^{1/2} \tag{9}$$

Equations (8) and (9) show that there exist two solutions for θ_{2i} for a given θ_{1i}.

Solution for s_i

Once θ_{1i} and θ_{2i} have been determined, s_i can be obtained by taking the inner product of Eq. (1) with $\mathbf{w}_{1i}$ as

$$s_i=[(\mathbf{B}_{i0}-\mathbf{A}_i)\cdot\mathbf{w}_{1i}]-h_i\cos\theta_{2i} \tag{10}$$

Number of Solutions to the Inverse Displacement Analysis

The above analysis shows that for a given position of the moving platform, there are usually two solutions (Eqs. (4)–(7)) for the input θ_{1i} for each leg i and four sets of solutions (Eqs. (4)–(10))[2] for the joint variables in each leg i. Thus, for a given position of the moving platform, there are usually eight ($=2^3$) sets of solutions for the inputs θ_{11}, θ_{12}, and θ_{13} and 64 ($=4^3$) sets of solutions for all the joints variables in the 3-URC TPM.

In summary, it has been shown that for a given position of the moving platform, there are usually two solutions for each input and eight sets of solutions for all the inputs in the 3-URC TPM. Thus, we have proved that the 3-URC TPM does not belong to the class of TPMs with linear input-output equations [2–6]. In fact, it belongs to the class of linear TPMs, whose forward displacement analysis can be performed by solving a set of linear equations, dealt with systematically in [5,6]. The work reported in [5,6] is an extension of the work reported in [3,4].

[2]From Eqs. (5), (6), and (8)–(10) in this paper or Eq. (11*b*) in [1], it is learned that there are usually two solutions for s_i for a given position of the moving platform.

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