



# Discussion

## Discussion: “Kinematics of the Translational 3-URC Mechanism” [Di Gregorio, R., 2004, ASME J. Mech. Des., 126, pp. 1113–1117]

Xianwen Kong

Clément M. Gosselin<sup>1</sup>

e-mail: gosselin@gmc.ulaval.ca

Département de Génie Mécanique,  
Université Laval,  
Québec, Québec, Canada G1K 7P4

[DOI: 10.1115/1.2205875]

The author of [1] proposed a 3-URC translational parallel mechanism (TPM) and presented a comprehensive study on the kinematics of the 3-URC TPM. He concluded that “only one solution exists both for the direct and for the inverse position analyses.” However, we do not agree with his result on the inverse position analysis and his statement that Ref. [2] “presented a class of TPMs with linear input-output equations that contain some translational 3-URC mechanisms.”

In this discussion, we will show that the 3-URC TPM does not belong to the class of TPMs with linear input-output equations [2–6] by investigating the inverse position analysis of the 3-URC TPM.

Leg  $i$  of a 3-URC TPM is shown in Fig. 1. In addition to the notations used in [1],  $\mathbf{h}_i$  is used to denote a unit vector directed from  $A_i$  to  $C_i$ . For leg  $i$ , the following holds:  $\mathbf{w}_{1i} \cdot \mathbf{v}_i = 0$ ,  $\mathbf{w}_{1i} \cdot \mathbf{w}_{2i} = 0$ , and  $\mathbf{w}_{2i} \cdot \mathbf{h}_i = 0$ .

In the coordinate system  $O-XYZ$ , we have

$$\mathbf{A}_i + h_i \mathbf{h}_i - d_i \mathbf{w}_{2i} + s_i \mathbf{w}_{1i} = \mathbf{B}_{i0} \quad (1)$$

where

$$\mathbf{w}_{2i} = \cos \theta_{1i} \mathbf{v}_i + \sin \theta_{1i} \mathbf{w}_{1i} \times \mathbf{v}_i$$

$$\mathbf{w}_{2i} \times \mathbf{w}_{1i} = -\cos \theta_{1i} \mathbf{w}_{1i} \times \mathbf{v}_i + \sin \theta_{1i} \mathbf{v}_i$$

$$\mathbf{h}_i = \cos \theta_{2i} \mathbf{w}_{1i} + \sin \theta_{2i} \mathbf{w}_{2i} \times \mathbf{w}_{1i}$$

$$\mathbf{B}_{i0} = \mathbf{P} + \mathbf{R}_{bp}^P (\mathbf{B}_{i0} - \mathbf{P})$$

<sup>1</sup>Corresponding author.

Contributed by the Mechanisms and Robotics Committee of ASME for publication in the JOURNAL OF MECHANICAL DESIGN. Manuscript received April 29, 2005; final manuscript received January 19, 2006. Review conducted by Q. Jeffrey Ge.

The inverse position analysis of the 3-URC TPM can be performed by solving Eq. (1) for  $\theta_{1i}$ ,  $\theta_{2i}$ , and  $s_i$  in sequence.

### Solution for $\theta_{1i}$

Taking the inner product of Eq. (1) with  $\mathbf{w}_{2i}$ , we obtain

$$\mathbf{w}_{2i} \cdot (\mathbf{A}_i - \mathbf{B}_{i0}) - d_i = 0 \quad (2)$$

i.e.,

$$(\mathbf{B}_{i0} - \mathbf{A}_i) \cdot (\mathbf{w}_{1i} \times \mathbf{v}_i) \sin \theta_{1i} + (\mathbf{B}_{i0} - \mathbf{A}_i) \cdot \mathbf{v}_i \cos \theta_{1i} + d_i = 0 \quad (3)$$

Define an angle  $\alpha_i$  by

$$\cos \alpha_i = [(\mathbf{B}_{i0} - \mathbf{A}_i) \cdot (\mathbf{w}_{1i} \times \mathbf{v}_i)] / a_i \quad (4)$$

$$\sin \alpha_i = [(\mathbf{B}_{i0} - \mathbf{A}_i) \cdot \mathbf{v}_i] / a_i$$

where  $a_i = [(\mathbf{B}_{i0} - \mathbf{A}_i) \cdot \mathbf{v}_i]^2 + [(\mathbf{B}_{i0} - \mathbf{A}_i) \cdot (\mathbf{w}_{1i} \times \mathbf{v}_i)]^2$ .

Equation (3) can be rewritten as

$$\sin(\theta_{1i} + \alpha_i) = -d_i / a_i \quad (5)$$

Solving  $\sin^2(\theta_{1i} + \alpha_i) + \cos^2(\theta_{1i} + \alpha_i) = 1$ , we obtain two solutions for  $\cos(\theta_{1i} + \alpha_i)$  as

$$\cos(\theta_{1i} + \alpha_i) = \pm [1 - \sin^2(\theta_{1i} + \alpha_i)]^{1/2} \quad (6)$$

Equations (5) and (6) show that there are two solutions for  $(\theta_{1i} + \alpha_i)$ . For each  $(\theta_{1i} + \alpha_i)$ , one solution for  $\theta_{1i}$  can be obtained as

$$\sin \theta_{1i} = \sin(\theta_{1i} + \alpha_i) \cos \alpha_i - \cos(\theta_{1i} + \alpha_i) \sin \alpha_i \quad (7)$$

$$\cos \theta_{1i} = \cos(\theta_{1i} + \alpha_i) \cos \alpha_i + \sin(\theta_{1i} + \alpha_i) \sin \alpha_i$$

From Eqs. (4)–(7), we learn that there are usually two solutions for  $\theta_{1i}$ .

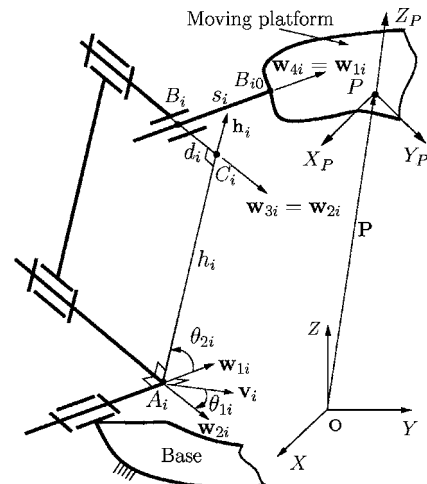


Fig. 1 URC leg

### Solution for $\theta_{2i}$

For each  $\theta_{1i}$  obtained using Eq. (7),  $\sin \theta_{2i}$  can be obtained by taking the inner product for Eq. (1) with  $\mathbf{w}_{2i} \times \mathbf{w}_{1i}$  as

$$\sin \theta_{2i} = \{\sin \theta_{1i}[(\mathbf{B}_{i0} - \mathbf{A}_i) \cdot \mathbf{v}_i] - \cos \theta_{1i}[(\mathbf{B}_{i0} - \mathbf{A}_i) \cdot (\mathbf{w}_{1i} \times \mathbf{v}_i)]\} / h_i$$

Substituting Eq. (4) into the above equation, we obtain

$$\sin \theta_{2i} = -a_i \cos(\theta_{1i} + \alpha_i) / h_i \quad (8)$$

Solving  $\sin^2 \theta_{2i} + \cos^2 \theta_{2i} = 1$ , we obtain two solutions for  $\cos \theta_{2i}$  as

$$\cos \theta_{2i} = \pm (1 - \sin^2 \theta_{2i})^{1/2} \quad (9)$$

Equations (8) and (9) show that there exist two solutions for  $\theta_{2i}$  for a given  $\theta_{1i}$ .

### Solution for $s_i$

Once  $\theta_{1i}$  and  $\theta_{2i}$  have been determined,  $s_i$  can be obtained by taking the inner product of Eq. (1) with  $\mathbf{w}_{1i}$  as

$$s_i = [(\mathbf{B}_{i0} - \mathbf{A}_i) \cdot \mathbf{w}_{1i}] - h_i \cos \theta_{2i} \quad (10)$$

### Number of Solutions to the Inverse Displacement Analysis

The above analysis shows that for a given position of the moving platform, there are usually two solutions (Eqs. (4)–(7)) for the input  $\theta_{1i}$  for each leg  $i$  and four sets of solutions (Eqs. (4)–(10))<sup>2</sup>

<sup>2</sup>From Eqs. (5), (6), and (8)–(10) in this paper or Eq. (11b) in [1], it is learned that there are usually two solutions for  $s_i$  for a given position of the moving platform.

for the joint variables in each leg  $i$ . Thus, for a given position of the moving platform, there are usually eight ( $=2^3$ ) sets of solutions for the inputs  $\theta_{11}$ ,  $\theta_{12}$ , and  $\theta_{13}$  and 64 ( $=4^3$ ) sets of solutions for all the joints variables in the 3-URC TPM.

In summary, it has been shown that for a given position of the moving platform, there are usually two solutions for each input and eight sets of solutions for all the inputs in the 3-URC TPM. Thus, we have proved that the 3-URC TPM does not belong to the class of TPMs with linear input-output equations [2–6]. In fact, it belongs to the class of linear TPMs, whose forward displacement analysis can be performed by solving a set of linear equations, dealt with systematically in [5,6]. The work reported in [5,6] is an extension of the work reported in [3,4].

### References

- [1] Di Gregorio, R., 2004, "Kinematics of the Translational 3-URC Mechanism," *ASME J. Mech. Des.*, **126**(6), pp. 1113–1117.
- [2] Kong, X., and Gosselin, C. M., 2002, "A Class of 3-DOF Translational Parallel Manipulators With Linear Input-Output Equations," *Proceedings of the Workshop on Fundamental Issues and Future Research Directions for Parallel Mechanisms and Manipulators*, Québec, Canada, October 3–4, pp. 25–32.
- [3] Gosselin, C. M., and Kong, X., 2002, "Cartesian Parallel Manipulators," International Patent No. WO 02/096605 A1.
- [4] Gosselin, C. M., and Kong, X., 2004, "Cartesian Parallel Manipulators," U.S. Patent No. 6729202.
- [5] Kong, X., and Gosselin, C. M., 2002, "Type Synthesis of Linear Translational Parallel Manipulators," *Advances in Robot Kinematics—Theory and Applications*, J. Lenarčič and F. Thomas, eds., Kluwer Academic, Dordrecht, pp. 411–420.
- [6] Kong, X., and Gosselin, C. M., 2004, "Type Synthesis of 3-DOF Translational Parallel Manipulators Based on Screw Theory," *ASME J. Mech. Des.*, **126**(1), pp. 83–92.