Closure to "Discussion of 'Kinematics of the Translational 3-URC Mechanism'" (2006, ASME J. Mech. Des., 128, pp. 812–813)

Raffaele Di Gregorio
Department of Engineering,
University of Ferrara,
Via Saragat, 1,
44100 Ferrara, Italy
e-mail: rdigregorio@ing.unife.it

[DOI: 10.1115/1.2205876]

Kong and Gosselin [1], with reference to what this author wrote in [2], disagree on the following two points: (i) the inverse position analysis (IPA) of the translational 3-URC has only one solution, and (ii) the translational 3-URC belongs to the class of translational parallel mechanisms (TPMs) with linear input-output equations presented in [3].

Point (i): Figure 1 of [1] shows the *i*th leg of type URC of a translational 3-URC. With reference to the notations shown in that figure, the inverse position analysis consists of calculating the values of the angle θ_{1i} $(i=1,2,3)$ compatible with an assigned position of the axis of the cylindrical pair (note that the cylindrical pair axis passes through the platform point B_{i0} and is parallel to the unit vector, $\mathbf{w}_{1i}$, of the axis of the revolute pair that is embedded in the base, whereas the unit vectors $\mathbf{w}_{2i}$ and $\mathbf{w}_{3i}$ of the axes of the two intermediate revolute pairs are parallel to each other and perpendicular to $\mathbf{w}_{1i}$). It can be shown through a simple geometric reasoning that the *i*th translational URC leg can be assembled in four different configurations (assembly modes) once the position of the cylindrical pair axis is assigned. Such configurations can be divided into two groups each of which is composed of two configurations that are symmetric with respect to the plane, the cylindrical pair axis and point A_i belong to (plane σ_i of Fig. 1), and correspond to only two values of the angle θ_{1i} (the values ${}^{(1)}\theta_{1i}$ and ${}^{(2)}\theta_{1i}$ shown in Fig. 1). Even though the axis of the cylindrical pair keeps itself parallel to $\mathbf{w}_{1i}$ during the platform translation, suitable platform translations that bring the *i*th leg into any configuration out of the four assembly modes, without dismounting and reassembling the leg, exist. Therefore, both the values of θ_{1i} that correspond to the four assembly modes are a solution of the IPA (i.e., Kong and Gosselin are right), and the formulas (16) and (17*a*), and (17*b*)reported in [2] must be changed as follows:

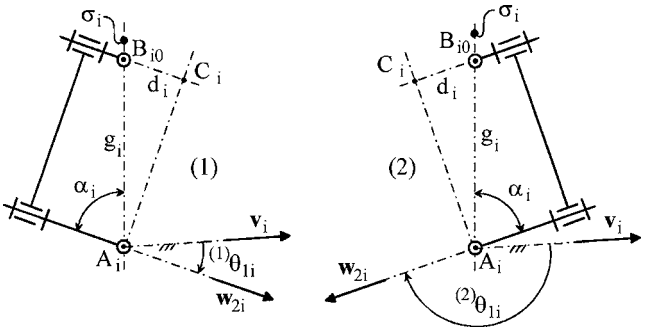

Fig. 1 Assembly modes of the *i*th translational URC leg projected onto a plane perpendicular to $\mathbf{w}_{1i}$; each of the projection ((1) or (2)) corresponds to two leg configurations with the same value of θ_{1i} and different values of s_i

$$\mathbf{w}_{2i} = \pm \sin\alpha_i \frac{\mathbf{w}_{1i}\times(\mathbf{B}_{i0}-\mathbf{A}_i)}{\|\mathbf{w}_{1i}\times(\mathbf{B}_{i0}-\mathbf{A}_i)\|} + \cos\alpha_i \frac{\mathbf{w}_{1i}\times[\mathbf{w}_{1i}\times(\mathbf{B}_{i0}-\mathbf{A}_i)]}{\|\mathbf{w}_{1i}\times(\mathbf{B}_{i0}-\mathbf{A}_i)\|} \tag{16}$$

$$\cos\theta_{1i} = \mathbf{v}_i \cdot \mathbf{w}_{2i} \tag{17a}$$

$$\sin\theta_{1i} = (\mathbf{w}_{1i}\times\mathbf{v}_i)\cdot\mathbf{w}_{2i} \tag{17b}$$

where the angle α_i is shown in Fig. 1 (it can be easily computed through the relationships: $\cos\alpha_i = d_i/g_i$ and $\sin\alpha_i = [1-(d_i/g_i)^2]^{1/2}$ with $g_i = \|(\mathbf{B}_{i0}-\mathbf{A}_i) - [(\mathbf{B}_{i0}-\mathbf{A}_i)\cdot\mathbf{w}_{1i}]\mathbf{w}_{1i}\|$).

Point (ii): Since the IPA of the translational 3-URC has two solutions per leg (i.e., 8 solutions), the translational 3-URC proposed in [2] does not belong to the class of TPMs presented in [3], and Ref. [2] must be correct on this point as Kong and Gosselin observed.

This author wishes to thank X. Kong and C. M. Gosselin for having given him the opportunity to discuss and correct his work.

References

[1] Kong, X., and Gosselin, C. M., 2006, "Discussion: 'Kinematics of the Translational 3-URC Mechanism'," ASME J. Mech. Des., **128**, pp. 812–813.
[2] Di Gregorio, R., 2004, "Kinematics of the Translational 3-URC Mechanism," ASME J. Mech. Des., **126**(6), pp. 1113–1117.
[3] Kong, X., and Gosselin, C. M., 2002, "A Class of 3-DOF Translational Parallel Manipulators With Linear Input-Output Equations," *Proceedings of the Workshop on Fundamental Issues and Future Research Directions for Parallel Mechanisms and Manipulators*, Québec City, Québec, Canada, October 3–4, pp. 25–32.

Contributed by the Mechanisms and Robotics Committee of ASME for publication in the JOURNAL OF MECHANICAL DESIGN. Manuscript received May 15, 2005; final manuscript received September 10, 2005. Review conducted by Q. Jeffrey Ge.

