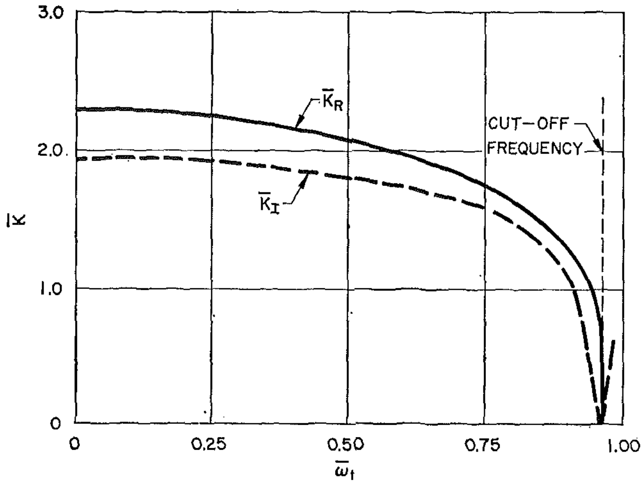

Fig. 6 Edge loading propagation constant versus frequency

Conclusion

The frequency dependence of the zeroth mode phase velocity is primarily a result of the tube constraint at high frequencies and viscosity at low frequencies. The maximum value of the zeroth mode attenuation constant occurs at a frequency near the radial natural frequency of a flexible tube; however, no peaking of the zeroth mode attenuation constant occurs when the tube is stiff.

In addition to the modes for the fluid, another mode exists, which corresponds to edge loadings on the tube. This mode is fundamentally different from the fluid modes in that it propagates with attenuation (not due to viscous effects) up to a certain frequency, beyond which there is propagation without attenuation

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APPENDIX A

Coefficients for Dispersion Equation (37)

$$C_1 = \bar{K}/\bar{\beta}$$

$$C_2 = j\bar{\lambda}\bar{\omega}J_1\{\bar{\lambda}(1 - \bar{h}/2)\}/(\bar{K}^2 - \bar{\lambda}^2)$$

$$C_3 = -J_0\{\bar{\beta}(1 - \bar{h}/2)\}/J_1\{\bar{\beta}(1 - \bar{h}/2)\}$$

$$C_4 = -j\bar{K}\bar{\omega}J_0\{\bar{\lambda}(1 - \bar{h}/2)\}/(\bar{K}^2 - \bar{\lambda}^2)$$

$$C_5 = 2\bar{\mu}\bar{K}\{J_0\{\bar{\beta}(1 - \bar{h}/2)\}/J_1\{\bar{\beta}(1 - \bar{h}/2)\} - 1/\bar{\beta}(1 - \bar{h}/2)\}$$

$$C_6 = -j2\bar{\lambda}\bar{\omega}\bar{\mu}J_1\{\bar{\lambda}(1 - \bar{h}/2)\}/(1 - \bar{h}/2)(\bar{K}^2 - \bar{\lambda}^2) - \{1 - j2\bar{\mu}\bar{\omega}/3 - j2\bar{\mu}\bar{\omega}\bar{\lambda}^2/(\bar{K}^2 - \bar{\lambda}^2)\}J_0\{\bar{\lambda}(1 - \bar{h}/2)\}$$

$$C_7 = \bar{\mu}(\bar{K}^2 + \bar{\beta}^2)/\bar{\beta}$$

$$C_8 = j2\bar{\mu}\bar{\omega}\bar{K}\bar{\lambda}J_1\{\bar{\lambda}(1 - \bar{h}/2)\}/(\bar{K}^2 - \bar{\lambda}^2)$$

$$C_9 = -j\bar{\omega}\alpha\gamma^2(1 - \bar{h}/2)/\bar{h}\phi$$

$$C_{10} = -j\bar{\omega}\alpha\gamma^2/\bar{h}\phi$$

$$C_{11} = \frac{j\bar{\omega}\alpha\gamma^2\nu\bar{K}(1 - \bar{h}/2)\{1 - \eta\xi[1 + \bar{h}/2\nu\eta\xi]\}}{\bar{h}\phi[1 - \eta\xi\{\bar{K}^2 + \gamma^2\bar{\omega}^2\}]}$$

$$C_{12} = \frac{j\bar{\omega}\alpha\gamma^2\{\nu\bar{K}\theta + \phi(1 - \bar{h}/2) - \eta\xi(\bar{K}^2 + \gamma^2\bar{\omega}^2) \times [\bar{K}\theta\nu(1 + \bar{h}/2\nu\eta\xi) + 4\phi(1 - 3\bar{h}/8)]\}}{\bar{h}\phi(\bar{K}^2 + \gamma^2\bar{\omega}^2)\{1 - \eta\xi(\bar{K}^2 + \gamma^2\bar{\omega}^2)\}}$$

DISCUSSION

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When used within the proper range of frequencies, the three-mode analysis of De Armond and Rouleau appears to be adequate for simulation of the downstream radiation field of a periodic disturbance, whose field quantities are specified at some small distance downstream of the piston face. In the instance of a rigid impermeable tube, this frequency range has been shown [5, 9, 10][4] to be in what is termed the intermediate-frequency to low high-frequency range $11.14\ \pi\ \bar{\mu} \leq \bar{\omega} \leq 3.8$. For frequencies larger than 3.8 (roughly the inviscid cut-off frequency for the first fluid mode after the De Armond and Rouleau zeroth fluid mode), mode interference can result because the first fluid mode (which in general would be excited) will attenuate spatially at the same rate as the zeroth fluid mode. For higher frequencies, even more fluid modes would be excited. For frequencies smaller than $11.14\ \pi\ \bar{\mu}$, yet a fourth set of liquid viscous modes associated with transverse shear waves (which interact with the tube wall in the form of a dilatational boundary layer) begin to come into play. If the frequency is low enough, these shear waves possess an attenuation constant which is comparable to that of the De Armond and Rouleau zeroth mode. These latter modes are a manifestation of the requirement that the specified radial velocity at or near the radiating source must also be satisfied.[5] With some modification it seems clear that these rigid impermeable tube results will allow the correct elastic tube work of De Armond and Rouleau to carry over to the high- and low-frequency range by inclusion of the required additional modes.

Additional References

9 Scarton, H. A., and Rouleau, W. T., "Axisymmetric Waves in Compressible Newtonian Liquids Contained in Rigid Tubes. Part 1. Steady-Periodic Mode Shapes and Dispersion by the Method of Eigenvalleys," *Journal of Fluid Mechanics* (to be published), 1972.

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[4] Numbers [9–14] in brackets designate Additional References at end of discussion.

[5] All of these complex modes were found for the rigid impermeable tube using a contour [11] technique called the Method of Eigenvalleys [12] to solve the complex dispersion relation involving complex Bessel functions [13] which proved the dispersion curves of Gerlach and Parker [14] to be incorrect.

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Authors' Closure

The authors appreciate Professor Scarton's discussion. His comment on low frequency behavior is especially valuable since it points out that our three-mode analysis would lose physical significance if extended to the zero-frequency limit.

In the legend on Fig. 3, the viscosity for Case study 3 should read $\bar{\mu} = 0.0000076$.