tual resonance frequency of any mode is distributed uniformly over an interval C centered on an average frequency given by one of the supported plate resonance frequencies. The probability of a resonance frequency occurring in any given frequency interval $\delta\omega$ is therefore increased by $(nC)!$ There is a disorder component in this model, therefore, that is very similar to equation (19), and if $B = C$, they are identical. There is another component to the disorder measure in this second model, however, since the energy distribution is of nonexponential form.

If the response frequency is ω_0, the excitation frequency is ω_s, and the damping loss factor is η, then the energy of vibration is

$$E(\omega_0/\omega_s) \doteq E_{\max}[4(\omega_0 - \omega_s)^2/\omega_s^2\eta^2 + 1]^{-1} \equiv E_{\max}(1 + \xi^2)^{-1} \quad (20)$$

where $\xi = 2(\omega_0 - \omega_s)/\omega_s\eta$. The mean value of the energy as ω_0 is chosen randomly over the interval C is

$$\langle E \rangle = \frac{E_{\max}}{\xi_{\max}} \int_0^{\xi_{\max}} \frac{d\xi}{1 + \xi^2} \simeq \frac{\pi}{2} \frac{E_{\max}}{\xi_{\max}} \quad (21)$$

where $\xi_{\max} = C/\omega_s\eta$. The probability density of the energy for this model is

$$p_2(e) = p(\xi) \frac{d\xi}{de} = \frac{1}{\xi_{\max} e^{3/2}(1 - e)^{1/2}} \quad (22)$$

defining $e \equiv E/E_{\max}$. Then $p_2(E) = p_2(e)/E_{\max}$ and the ratio of probabilities is

$$p_1(E)/p_2(E) = \exp\,(-E/\langle E \rangle)\langle E \rangle^{-1}\xi_{\max}E_{\max}e^{3/2}(1 - e)^{1/2} \quad (23)$$

Taking the log of equation (23) and then its expectation to get the contribution to the disorder function by the form of the density function:

$$\Delta D = \langle ln\, p_1/p_2 \rangle = -1 + ln\, E_{\max}/\langle E \rangle + ln\, \xi_{\max} + \frac{3}{2}\langle ln\, e \rangle + \frac{1}{2}\langle ln\,(1 - e)\rangle \quad (24)$$

The expectation of the logs in equation (24) could be obtained from the density function in equation (22) but it is simpler to average over the random variable ξ:

$$\langle ln\, e \rangle = \langle ln\,(1 + \xi^2)\rangle = -\xi_{\max}^{-1}\int_0^{\xi_{\max}} d\xi\, ln\,(1 + \xi^2) =$$

$$-\xi_{\max}^{-1}[\xi_{\max}\, ln\,(1 + \xi_{\max}^2) - 2\xi_{\max} + 2\tan^{-1}\xi_{\max}] \quad (25)$$

Similarly, $ln\,(1 - e) = ln\,\xi^2 - ln\,(1 + \xi^2)$.

$$\langle ln\,\xi^2 \rangle = \xi_{\max}^{-1}\int_0^{\xi_{\max}} d\xi\, ln\xi^2 = (\xi_{\max}\, ln\,\xi_{\max}^2 - 2\xi_{\max})/\xi_{\max} \quad (26)$$

Collecting terms,

$$\Delta D = ln\,\frac{2}{\pi}\,\frac{\xi_{\max}^2}{1 + \xi_{\max}^2} - \frac{2\tan^{-1}\xi_{\max}}{\xi_{\max}} \simeq ln\,\frac{2}{\pi}\,\xi_{\max} \quad (27)$$

for $\xi_{\max} \gg 1$.

The result in (27) represents the amount that the noise response disorder exceeds that of a randomized sinusoidal excitation. Thus, the disorder in model 2 is

$$\Delta D_2 = ln\,(nC)! - ln\,\frac{2C}{\pi\omega\eta} \quad (28)$$

This is to be compared to the result in equation (19). We see that in order for the disorder in these two models to be equal, the frequency bandwidth C must be slightly greater than the noise bandwidth B. The difference, however, will be very slight since $C!$ is a much stronger function than C.

Conclusion

We have briefly explored some properties of the disorder function $\Delta D = \langle ln\, p_1/p_2 \rangle$ where p_1 is the "maximally random" exponential energy density function for noise excited oscillators and p_2 is the energy density function for the system being evaluated. The properties of this function agree with our expectations of how disorder should be described. The definition has parallels to the H function in statistical mechanics [8] and the "negentropy" of information theory [9].

We have not demonstrated in the above, however, just how we would make use of this function in structural analysis problems. We have only illustrated its calculation in some relatively simple cases. We will leave for other contributions the discussion of the utility of the disorder measure.

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DISCUSSION

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Professor Lyon is to be congratulated for this latest attempt to render applied mechanics less precise. Anyone who has worked much in acoustics or structural dynamics is aware that present day computing techniques allow one to answer detailed questions regarding precisely formulated models of real systems. But for the complex systems which arise in practice, we neither know the plethora of required parameters very well nor do we really care much about the minute details of system response that a high order model could predict. A gross macroscopic model which could be relied upon to produce average features of systems response would often suffice and might prove to be simple to understand and to manipulate.

Unfortunately, it has not been easy to find procedures for constructing macroscopic models which yield accurate results. Lyon's disorder measure as proposed in this paper is perhaps intended to shed light on this important and vexing problem, although he has chosen to "leave for other contributions the discussion of the utility of the disorder measure." The idea of developing a thermodynamic theory of structural dynamics is an interesting one, but judging from the long and often controversial history of thermodynamics, the development may prove to be rather difficult.

Lyon's disorder measure is clearly patterned after entropy and information measures used previously. But such measures have been easier to define than to interpret properly. In an interesting paper, Jaynes,[3] for example, states that "unqualified statements that 'entropy measures randomness' are . . . totally meaningless" and that "the most difficult problem of all to learn how to state

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clearly is what is the question we are trying to answer." It would be easier to understand Lyon's disorder measure if he would try to pose the question that he is trying to answer. It is not at all clear that it is realistic to associate a disorder measure only with various models of a physical system independent of the type of information which is to be extracted from the models.

Author's Closure

The discussors have made valid points regarding my paper. I would comment, however, that what I am trying to accomplish is more of a communications theory model of structures than a thermodynamic or statistical mechanical one. Thermodynamics and statistical mechanics deal with systems in their maximally disordered state, or in perturbations very near that state. What I am trying to do with my heirarchy of models is define something like a "channel capacity" for each model of a situation. Knowing the channel capacity doesn't tell you how to build the transmission line, but it does tell you that you have to increase the capacity of the communication system (which generally means more complexity or more degrees of freedom) if the amount of information that you desire is more than the channel is capable of—or vice versa. Thus there should be a direct and derivable relation between the required complexity of a model and the amount of information that one wants from it. The real cruncher, and my cop-out in this paper, is the construction of this model sequence.