Conclusion

It has been shown that the theory of the previous paper [1] can be modified by progressively taking account of about a dozen secondary effects, each of which can assume considerable importance under suitable conditions. Evidently an immense amount of work is necessary if a complete theory is to be developed, and the structure of such a theory may well be too awkward for it to be useful in practice. It is often simpler and cheaper to perform a suitable experiment.

The simple theory provides a starting point for further developments. It offers approximate predictions which are adequate for preliminary estimates. It provides a conceptual framework from which more detailed investigation can be seen in perspective. It is generally compatible with the results of other workers except where deviations can be traced to physical effects which are not accounted for in the theory.

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DISCUSSION

J. W. Palen[2]

The theory described in this paper depends for its simplicity on two assumptions:

1 That the gas flows in a core of diameter, D_c, surrounded by a liquid annulus of negligible velocity. The diameter of the core is related to the thickness of the liquid film, δ, and the vapor volume fraction, α, by the straightforward equation

$$\alpha = \left(\frac{D - 2\delta}{D}\right)^2$$

$$D_c = D - 2\delta$$

$$D = \text{pipe diameter}$$

2 That the friction factor for the vapor shear at the vapor-liquid interface, f_i, is linearly proportional to the ratio δ/D, as indicated by limited air, water data from 4 investigators up to $\delta/D \cong 0.04$, resulting in the empirical curve fit

$$f_i = 0.005(1 + 300\delta/D)$$

An additional slight approximation of the foregoing equation for α is also made, substituting,

$$1 - \alpha = \frac{4\delta}{D},$$

and resulting in

$$f_i = 0.005(1 + 75(1 - \alpha))$$

Since the pressure drop is based on the vapor velocity inside the core, the method breaks down at low values of α, when the cross-sectional area for vapor flow approaches zero and the pressure drop approaches infinity. This is called the limit of the "forbidden region" by the author, but actually defines one limit of applicability of the foregoing assumptions. However, since the author claims in the title applicability only to the annular regime, the foregoing limit is subtly stated, and the user has been implicitly warned not to use the method under conditions of slug and bubble flow (if he knows how to define these conditions for his system).

The author's alternate approach of defining velocity on the basis of the full tube diameter and expressing f as

$$f_{sg} = 0.005(1 + 90(1 - \alpha))$$

is essentially the "homogeneous" flow approach, but is more satisfactory for practical design, since the case of the pressure drop approaching infinity is not encountered. If this is done, however, a more general approach to the homogeneous friction factor could have been taken. As the author states, an approximate value for both the liquid and vapor friction factors in turbulent flow is 0.005, whereas, the foregoing equation gives a liquid friction factor of

$$f = (0.005)(91)$$

for the case of all liquid. A more usual way of correlation in industry is a quadratic form which gives the vapor friction factor for $\alpha = 1$, the liquid friction factor for $\alpha = 0$, and passes through a maximum in between. Such a correlation is suggested and borne out by the data of Thom.[3]

Another simplified approach similar to the author's, but which does not assume negligible liquid velocity, was recently proposed by Gloyer.[4]

In conclusion, the author is to be congratulated in recognizing that for the accuracy required by some engineering design problems, the highly sophisticated analyses of recent investigators in two-phase flow may not be necessary.

However, a better understanding of how to use the author's own method for practical design would be obtained if the percentage error in pressure-drop estimation were shown directly for all data points tested, and if the limits of applicability were more specifically stated.

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Author's Closure

Dr. Palen's extrapolation of the theory to the "all liquid" case is interesting but inappropriate. It was mentioned already at several points in the papers that there is a flow regime change for $(1 - \alpha)$ larger than about 0.2 due to bridging of the gas core by the liquid.

The physical model developed in this paper is not realistic once liquid slugs are formed. Any success in correlating data outside the annular flow regime, as appears to be the case in the comparison shown in Fig. 30 of Part I, is entirely fortuitous.

The "forbidden region" is not the result of proceeding to the limit of $\alpha = 0$. It is bounded by the envelope of the pressure gradient characteristics for vertical flow at quite moderate values of film thickness.

Regarding accuracy—the theory has been given here to one significant figure. Presumably a statistical study would show that one should replace the number 300 in equation (16) of Part I by 312, 285, 342.5, or some other value, depending on which data are used to optimize the theory. I feel that this sort of thing is misleading and inappropriate when presenting such a simple theoretical model. If an estimate of accuracy is desired I would guess that one would not have to look very far to find data which disagreed with the theory by a factor of 2, since this is about the current level of reproducibility of two-phase flow data when effects such as entrainment, surface tension variations, and entrance conditions are not adequately controlled. It was because of this well-known difficulty of obtaining a theory which will correlate a variety of data that the large number of graphical comparisons were shown in Part I to give an impression of the kind of scatter which is to be expected.