

Discussion

Calculation of Diffuser Efficiency for Two-Dimensional Flow¹

DONALD ROSS² AND J. M. ROBERTSON.³ The author presents an extension of the Buri method of diffuser analysis which should be useful in many engineering applications. His method differs from that of Buri in the use of the displacement thickness of the boundary layer where Buri used the momentum thickness. Apparently this change was made because it resulted in a more linear relation between ψ and Γ . It would be interesting to know how much of an improvement resulted from this change, and if the author tried using the total thickness of the boundary layer in the parameters.

In most previous studies of diffuser flow little attention was paid to entrance conditions, the importance of which is brought out clearly in this paper. Previous experimenters studying "two-dimensional" diffuser flow did not eliminate secondary effects caused by boundary-layer growth along the parallel walls of the conduit. In attaining truly two-dimensional flow, Hall has realized an ideal. Publication of the details of his experiments would add greatly to the fund of information on the action of a boundary layer under an adverse pressure gradient.

For the past 2 years the writers have been engaged in the design of a large high-speed water tunnel, to be constructed at the Pennsylvania State College by the U. S. Navy Bureau of Ordnance. As part of the water-tunnel design program, a study was made of flow in three-dimensional diffusers. The previous work of Donch, Nikuradse, Buri,⁴ Gruschwitz,⁵ Fediavsky,⁶ Howarth,⁷ and others was carefully reviewed, experiments were conducted on the flow in conical diffusers with a variety of entrance conditions, and analyses were attempted based on existing theories and modifications thereof. In the experiments, velocity and pressure measurements were made for three angles of diffusion, with initial turbulent boundary-layer thicknesses of 0.08 to 0.24 diam. The writers were unable to correlate the experimental results on the basis of a single parameter, as suggested by the writers mentioned and the author. In the various diffusers tested the same value of Γ did not correspond to the same shape of velocity distribution.

It seems physically unreasonable to expect that the flow in diffusers can be described by a single parameter such as Γ . The basic assumption of such a theory is that "the velocity distribution at any section depends on the velocity gradient (i.e., pres-

sure gradient) just outside the boundary layer at that section only, being affected by the state of affairs upstream only in so far as it affects the boundary-layer thickness."⁸ The objection to this is that "this property cannot hold exactly since the velocity distribution at any section depends on conditions upstream."⁹ In a turbulent boundary layer, the form of the velocity distribution is determined by the shear-stress and mixing-length distributions. The shear stress and turbulence in the flow at any point cannot differ greatly from that in the preceding flow. Thus one would expect the local velocity distribution to be controlled by its history, and only its rate of change to be governed by the local pressure gradient. The writers have found that diffuser-flow data can better be expressed as a function of two parameters: (a) a "history" parameter proportional to the pressure regain; and (b) a parameter similar to Γ . Von Doenhoff and Tetervin¹⁰ have also discarded the single parameter approach, and have presented a method of boundary-layer analysis taking into account the history effect.

The author assumes that the velocity variation in the boundary layer is a power law. The writers have found that the velocity distribution in a turbulent boundary layer under an adverse pressure gradient can better be given by an expression of the form

$$\frac{u}{U} = 1 + 2.5 (1 - \alpha) \sqrt{\frac{c_f}{2}} \ln \frac{y}{\delta} + \alpha \left(\frac{y}{\delta} - 1 \right)$$

where c_f is the wall shear-stress coefficient and α is a coefficient whose value is a function of the two parameters mentioned in the previous paragraph. It is to be noted that when α is zero, one has the von Kármán logarithmic velocity law; while, when α is unity, separation occurs. The writers have plotted the data of Donch and Nikuradse, as well as their own, on log-log paper and have not found a straight line as would be expected from a power law. A better fit was obtained using the foregoing relation.

AUTHOR'S CLOSURE

The comments and suggestions made by Mr. Ross and Mr. Robertson are appreciated. In regarding the writers' comments it is very important to distinguish between different types of diffusers. The writers' comments refer to three-dimensional diffusers, whereas the author's paper refers only to two-dimensional diffusers.

The writers were unable to correlate their experimental results on the basis of a single parameter. This was due to the fact that the diffusers were conical. The three-dimensional case may require two parameters, for it is more complicated than the two-dimensional case.

It does seem reasonable from a physical point of view to expect the flow in a two-dimensional diffuser to be described by a single parameter such as Γ . It is assumed that the diffuser is symmetrical, that the surfaces are relatively smooth, and that there is no separation. For the physical aspect involved in Buri's original proposal, it is preferable to refer to the original work of Buri who, in turn, refers to basic work done by Stodola.

The power law for the particular data was used because it provided a simple interpolation relation.

⁸ Goldstein, *ibid.*, p. 159.

⁹ Goldstein, *ibid.*, p. 374.

¹⁰ "Determination of General Relations for the Behavior of Turbulent Boundary Layers," NACA, ACR No. 3G13, July, 1943 (classification canceled).

¹ By R. C. Binder, published in the September, 1947, issue of the *JOURNAL OF APPLIED MECHANICS*, Trans. ASME, vol. 69, p. A-213.

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⁴ Buri's work is summarized in "Modern Developments in Fluid Dynamics," edited by S. Goldstein, Oxford University Press, New York, N. Y., 1938, pp. 374, 436.

⁵ "Die Turbulente Reibungsschicht in Ebener Strömung bei Druckabfall und Druchanstieg," by E. Gruschwitz, *Ingenieur-Archiv*, vol. 2, 1931, pp. 321-346. This work is summarized in Goldstein, *ibid.*, pp. 487-489.

⁶ "Turbulent Boundary Layer of an Airfoil," by K. Fediavsky, *Journal of the Aeronautical Sciences*, vol. 4, 1937, pp. 491-498.

⁷ "The Theoretical Determination of the Lift Coefficient for a Thin Elliptic Cylinder," by L. Howarth, *Proceedings of the Royal Society of London*, series A, vol. 149, 1935, pp. 558-586.