The Thermal Stability of Renormalization of Open Bosonic Strings

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The thermal stability of renormalization of open bosonic strings is described with proper regard for newfashioned thermal Veneziano duality within the framework of the thermofield dynamics.

Generalization of superstring theories to nonzero temperature has gradually been regarded as a real solid subject in high energy physics. This approach has not only been materialized by Leblanc and others by the best possible use of the thermofield dynamics [TFD] but also been pursued by Witten and others within the standard framework of the Euclidean-time Matsubara formalism. Renormalizability of conventional thermal fields has long been pioneered by Umezawa and his collaborators on the basis of the Euclidean-time prescription. Their observation is as follows: Any renormalizable field theory at zero temperature remains renormalizable even at finite temperature. The thermal stability of renormalization is nowadays transparently examined in the TFD formulation based upon the thermal doublet algorithm.

Making use of the improved representation of the thermal propagator [IRTP] as well as the Neveu-Scherk regularization method, in particular, Leblanc has quite recently substantiated renormalization of bosonic thermal strings in the operational TFD framework [September, 1988]. On the other hand, one of the present authors (H. F.) has antecedently materialized the newfashioned thermal Veneziano duality by the full use of the new form of the thermal reggeon propagator in the dispersion theoretic TFD approach [August, 1988]. The typical ingredient common to Refs. 3) and 13) reads that the thermal contribution is estimated at finite temperature in regard to the physical sheet structure at zero temperature. It is then of physical urgency to clarify whether or not the possible relevance of the IRTP algorithm to the thermal stability of renormalization of string fields affords active confirmation to the newfashioned thermal Veneziano amplitude. In the present short communication, therefore, let us investigate along this line renormalizability of open bosonic thermal strings within the TFD formalism based upon the IRTP arithmetic.

Let us start with tersely describing the Neveu-Scherk renormalization procedure of open bosonic strings at zero temperature. The one-loop planar amplitude $A$ with $N$ external tachyons of momenta $k_1, \ldots, k_N$ is expressed as

$$A(k_1, \ldots, k_N) = -iG^N \int_{-\infty}^{\infty} \frac{d^2 p}{(2\pi)^2} \prod_{i=1}^{N} dx_i \int w^{-2f(w)} \prod_{s} \phi_{rs}^{2s} k_s$$

$$\times \prod_{1 \leq r < s \leq N} \phi_{rs}^{2s k_r \cdot k_s}$$ (1)
at the critical space-time dimension and the critical Regge intercept, i.e., $D=26$ and $a=1$, where

$$f(w) = \prod_{n=1}^{\infty} (1 - w^n); \quad w = x_1 \cdots x_N,$$

and $p_i = p - k_1 - \cdots - k_{i-1}$. In addition, $G, g, \alpha'$ and $\rho$ read the Chan-Paton factor of the planar loop, the coupling constant, the Regge slope parameter and the loop momentum, respectively. As can evidently be seen from Eqs. (1) and (2), $A$ diverges at $w=1$, which comes from the tachyon of zero momentum as well as the soft dilaton. As can immediately be seen from Eqs. (1) and (3), moreover, $A$ diverges at $c_{sr}=1$, which sounds like a spurious consequence of the problematic use of the integral representation of the string propagator. The loop momentum integral of Eq. (1) is legitimately performed prior to all parameter integrals, which is of principal importance in the renormalization program of open bosonic strings. The leading divergence at $w=1$ is inherent to the tachyon of zero momentum and cannot be remedied with the aid of the Neveu-Scherk renormalization prescription alone. The next to the leading divergence at $w=1$ originates from the soft dilaton and can be regularized a la Pauli and Villars through the Neveu-Scherk subtraction algorithm by the appropriate choice of a counterterm. The spurious divergence at $c_{sr}=1$ can be eliminated in the sense of the low energy theorem through the Neveu-Scherk off-shell continuation arithmetic of an additional external momentum.

Let us now turn our attention to the possible thermal stability of renormalization of open bosonic strings at finite temperature in the TFD approach based upon the IRTP arithmetic. The thermal propagator $\Delta^\beta$ of the free open bosonic string is written in the IRTP form

$$\Delta^\beta(p) = \tau_3 \int_0^1 dx \int_c x^{L_0-a-1-iaX} \sum_{n=0}^{\infty} \frac{\delta(\alpha' p^2 + n-a)}{e^{\beta |p|} - 1} \left( 1 - e^{\beta |p|} \right),$$

where $\beta = 1/kT$, $L_0 = a + \alpha'(p^2 + M^2)$ and the integration contour $c$ is taken as the unit circle around the origin. In addition, $a$ and $M^2$ read the Pauli matrix and the mass operator, respectively. The one-loop planar, thermal amplitude $\tilde{A}^\beta$ of $N$ external tachyons is then reduced to

$$\tilde{A}^\beta(k_1, \cdots, k_N) = A(k_1, \cdots, k_N) + \tilde{A}^\beta(k_1, \cdots, k_N)$$

at $D=26$ and $a=1$, where $A$ is, of course, precisely described as Eq. (1) and

$$\tilde{A}^\beta(k_1, \cdots, k_N) = -iGg^N \int_{w=0}^{\infty} d^{26} p \sum_{s_{j\neq 0}}^{N} \sum_{1 \leq i \leq N} dx_i x_i^{\sigma \rho/2} \times \int dx_s x_s^{\sigma \rho/2} w^{-2[f(w)]^{-24}}$$
\[ \times \frac{\delta(a' p_j^2 + n_j - 1)}{e^{\beta [\mathcal{H}_0] - 1}} \prod_{1 \leq r < s \leq N} (\psi'_{rs})^{2a' kr - ks} \]

\[ \vdots \]

\[ -iG g^N \int_{-\infty}^{\infty} d^{26} p \sum_{n=1}^{\infty} \sum_{n=0}^{\infty} \int \cdots \}

\[ \times \int \prod_{j=1}^{N} dx_j x_j a' p_j^2 w^{-2[f(w)]-24} \]

\[ \times \frac{\delta(a' p_j^2 + n_j - 1)}{e^{\beta [\mathcal{H}_0] - 1}} \prod_{1 \leq r < s \leq N} (\psi'_{rs})^{2a' kr - ks}. \]

By the use of Eqs. (1), (5) and (6), the thermal amplitude \( A^\beta \) can synthetically be rewritten as follows:

\[ A^\beta(k_1, \cdots, k_N) = -iG g^N \int_{-\infty}^{\infty} d^{26} p \int \prod_{i=1}^{N} dx_i x_i a' p_i^2 w^{-2[f(w)]-24} \]

\[ \times \prod_{1 \leq r < s \leq N} (\psi'_{rs})^{2a' kr - ks}, \]

where the symbolic notation \( \int^\beta \) reads either

\[ \int^\beta dx_i = \int_0^1 dx_i; \quad 1 \leq i \leq N \]

or

\[ \int^\beta dx_i = \sum_{n=0}^{\infty} \frac{\delta(a' p_i^2 + n_i - 1)}{e^{\beta [\mathcal{H}_0] - 1}} \int dx_i; \quad 1 \leq i \leq N \]

just in the same manner as in Eq. (6). The string amplitude (7) at nonzero temperature bears a salient resemblance in all respects to the one-loop planar amplitude (1) of open bosonic strings at zero temperature, which is a crucial consequence of the evaluation of the thermal effect with respect to the physical sheet structure at zero temperature. As the most typical feature common to Eqs. (1) and (7), in particular, the loop momentum integration can be carried out not only at zero but also at nonzero temperatures irrespective of the parameter integrations.\(^3\) Consequently, the Neveu-Scherk renormalization algorithm erected at zero temperature is successfully applicable at nonzero temperature to the one-loop planar, thermal amplitude of \( N \) external tachyons. We are then led to claim that the Neveu-Scherk renormalization procedure is highly effective even at finite temperature within the general framework of the TFD treatment of open bosonic strings on the basis of the IRTP algorithm of the free thermal string, and that the one-loop planar, thermal amplitude \( A^\beta \) of \( N \) tachyons is guaranteed to be not only free from the soft dilaton divergence but also free from the spurious divergence provided the temperature remains below the Hagedorn temperature,\(^2\) i.e., \( \beta \geq \beta_0 = 4\pi \sqrt{\alpha'} \) a la Hardy-Ramanujan formula.\(^1\) This observation sounds, in fact, natural in the sense that the canonical partition function of open bosonic strings cannot be well-defined beyond the Hagedorn temperature.\(^1,2\)

Let us build up the newfashioned four-tachyon tree amplitude \( V^\beta(s, t) \) for the \( s-t \)}
diagram at finite temperature within the analytic S-matrix framework based upon the dispersion theoretic TFD approach. Use is made at zero temperature of the fundamental postulates as follows: i) the Regge analyticity, ii) crossing symmetry and iii) the Veneziano duality. Since there is no preferred channel for thermal fluctuations, the s- and t-channel thermal reggeon contributions must simultaneously be taken into consideration. As has elaborately been discussed in Ref. 13) for the case of π-π scattering, then, the four-tachyon thermal amplitude $V^\#(s, t)$ turns into

$$V^\#(s, t) = \frac{i}{4\pi} \int_{L - \infty}^{L + \infty} dL \left\{ \bar{V}^\#(l, a(l)) \frac{\Gamma(-l)\Gamma(-a(s))}{\Gamma(-l-a(s))} 
+ \bar{V}^\#(l, a(s)) \frac{\Gamma(-l)\Gamma(-a(t))}{\Gamma(-l-a(t))} \right\},$$

where $a(\zeta) = a + a^\# \zeta (=-1+1/2\cdot\zeta); \zeta = s, t, a(\zeta) < L < 0$ and $\Gamma$ is the gamma function. As can be envisaged from the arguments in Ref. 13), the IRTP representation (4) of $A^\#$ in the operational TFD formalism literally corresponds to the thermal partial fraction amplitude:

$$\bar{V}^\#(l, a(\zeta)) = -g^2 \left\{ \frac{1}{l-a(\zeta)} + \frac{2\pi i}{e^{\omega t} - 1} \delta(l-a(\zeta)) \right\}$$

in the dispersion theoretic TFD approach, where $\omega t = \sqrt{\zeta}$; $\zeta = s, t$. Equations (10) and (11) result in

$$V^\#(s, t) = \frac{g^2}{2} \left( \frac{e^{\omega st}}{e^{\omega st} - 1} + \frac{e^{\omega st}}{e^{\omega st} - 1} \right) B(-a(s), -a(t)),$$

where $B$ is the Euler beta function. The newfashioned four-tachyon thermal amplitude (12) successfully satisfies all the prominent features which are inherent in the zero-temperature Veneziano amplitude

$$V(s, t) = g^2 B(-a(s), -a(t)),$$

in sharp contrast to the old-fashioned four-tachyon thermal amplitude. It is therefore possible to argue that renormalizability of open bosonic thermal strings is guaranteed to be compatible with the newfashioned thermal Veneziano amplitude as a consequence of the thermal stability of duality and renormalization.

Let us fitly conclude the present paper by emphasizing that the TFD approach based upon the IRTP formalism deserves more than passing consideration for the possible higher loop calculation of modern versions of closed thermal superstrings.

Enlightening lectures by Dr. I. Ojima on the thermal fields as well as related topics at Rikkyo University are highly appreciated.

6) See, for example, H. Umezawa, H. Matsumoto and M. Tachiki, *Thermo Field Dynamics and Condensed States* (North-Holland, Amsterdam, 1982).
14) But see, for example, S. Weinberg, Phys. Lett. B187 (1987), 278.