Gravity as a Gauge Theory with Spontaneously Broken Internal Super-Poincaré Symmetry

Hirofumi SAITO

Department of Physics, Osaka City University, Osaka 558

(Received November 6, 1989)

An effective gauge theory of gravity is constructed with the aid of the method of nonlinear realizations. The internal super-Poincaré gauge symmetry breaks down spontaneously to the Lorentz one. The \( e^\mu \) is constructed out of gauge fields, Nambu-Goldstone fields and a spectator field so that it leads to a super-Poincaré gauge invariant metric. This \( e^\mu \) remains invariant under super-transformations, and thus its super-partner is not needed, in contrast to the case of conventional supergravity. Further, the internal supersymmetry does not require photino, Wino, Zino, \( \cdots \), in contrast to the spacetime one. Identities and conservation laws are obtained from the gauge invariance, and these identities restrict the functional dependence of Lagrangian density on field variables. The Noether currents of the Dirac field on the flat spacetime are also discussed.

§ 1. Introduction

Elegant successes in the gauge theories of electro-weak interaction and of strong interaction naturally lead to attempts to construct the gauge theory of gravity. Various gauge theories of gravity have been proposed; for example, the Lorentz gauge theory\(^1\) by Utiyama and the Poincaré gauge theory\(^2\) (PGT) by Kibble. Unfortunately, however, no answer has been established even to the fundamental questions: What is the gauge group of gravity? What is the gauge status of metric tensor? What are the Lagrangians of gravitational gauge fields?

Among the theories, Poincaré gauge theory\(^3\)\textsuperscript{--9} (PGT) by Kawai is formulated on the basis of the principal fiber bundle having the covering group of the proper orthochronous Poincaré group as the structure group. The Poincaré\(^*\) symmetry is introduced as an internal gauge symmetry. Nevertheless, the generators of internal translations and Lorentz rotations give correct expressions for energy-momentum and total angular momentum, respectively.\(^5,8\) It may be, therefore, that the internal translation plays a key role in description of gravity. The introduction of the internal translation, however, permits the existence of unobserved particles\(^9\) which carry nonvanishing "intrinsic" energy-momentum (= the quantum number associated with the internal translation). Thus there is a possibility that the internal translation symmetry breaks down spontaneously in the actual world.\(^*\)

Local translations are identified with general coordinate transformations (GCT's) in conventional supergravity (and also in PGT). This identification, however, destroys the super-Poincaré algebra: (i) The index mismatch\(***\) occurs on the r.h.s.

\(^*\) In Refs. 3\textsuperscript{--9}, the word Poincaré is used for the covering group of the Poincaré group.

\(^**\) This symmetry breaking has been mentioned in Ref. 3.

\(^***\) This mismatch has been discussed in quantum supergravity.\(^10\)
of the anti-commutation relation \( \{ Q, \bar{Q} \} = -2i\gamma^k P_k \), since \(^*) P_k \) (or rather \( P_\mu \)) have a world index while the \( \gamma \)-matrix carries a local (internal) index \( k \). (ii) GCT’s commute with local (internal) Lorentz rotations.

The supersymmetric extension\(^)** \) of the internal Poincaré symmetry necessarily leads to the concept of the internal super-Poincaré symmetry, which is irrelevant to GCT. In contrast to conventional supergravity, the super-Poincaré algebra holds\(^**** \) as local version. The internal supersymmetry is expected, like the spacetime supersymmetry,\(^**** \) to lead to unified description of fermions and bosons and to soften ultraviolet divergences. This internal symmetry, however, must break down spontaneously, because we have no direct experimental evidence for the supersymmetry.

The purpose of the present paper is to construct an effective gauge theory of gravity in which the internal super-Poincaré\(^***** \) gauge symmetry breaks down spontaneously to the Lorentz one.\(^***** \) We employ the method of nonlinear realizations,\(^13 \) which is useful in constructing an effective theory when a certain symmetry breaks down spontaneously. For simplicity, we confine ourselves to the \( N=1 \) super-Poincaré symmetry, but extension to the case with \( N\geq2 \) will easily be done.

In § 2, the internal super-Poincaré group is discussed briefly, and a nonlinear realization of this group is given in § 3. In § 4, the vierbein field is constructed. In §§5 and 6, we discuss the invariance of an action integral under gauge transformations and GCT’s. In § 7, we give comments on the choice of the set of independent field variables. In § 8, conserved currents are examined for the Dirac field on the flat spacetime. A summary and remarks are given in § 9, and some notations, conventions and formulae are enumerated in the Appendix.

§ 2. Internal super-Poincaré group

We introduce the super-Poincaré symmetry as an internal gauge symmetry, which is irrelevant to GCT. The algebra of this symmetry group is given by

\[
\begin{align*}
[M_{kl}, M_{mn}] &= i(\eta_{km}M_{ln} + \eta_{ln}M_{km} - \eta_{km}M_{lm} - \eta_{ln}M_{mn}) , \\
[M_{kl}, P_m] &= i(\eta_{km}P_l - \eta_{lm}P_k ) , \\
[P_k, P_l] &= 0 , \\
[M_{kl}, Q] &= \frac{i}{2} \sigma_{kl}Q ,
\end{align*}
\]

\(^*) \) The Latin indices \( k, l, \ldots \) are for internal ones, while the Greek indices \( \mu, \nu, \ldots \) are for world (external) ones.

\(^**) \) Without introducing (internal) translation, Nakanishi has proposed a new local supersymmetry by extending the local Lorentz symmetry alone.\(^11 \) His theory is quite different from that in the present paper.

\(^**** \) In PGT, the Poincaré algebra holds as local version, while it does not in PGT.

\(^**** \) By spacetime supersymmetry, we mean conventional supersymmetry, for which super-transformations are realized as certain coordinate transformations on superspace \( (x, \theta) \).\(^12 \)

\(^***** \) Since we take the Dirac field into account, the Lorentz symmetry should be the \( \text{SL}(2, \mathbb{C}) \)-symmetry, and accordingly the (super-)Poincaré symmetry also should be the symmetry represented by “covering group” of the (super-)Poincaré group. We shall refer to them as the Lorentz symmetry and the (super-)Poincaré symmetry, however, since we are not concerned with mathematical rigor in the present paper.
\[ [P_k, Q] = 0, \quad (2.1e) \]
\[ (Q, \bar{Q}) = -2i\gamma^k P_k, \quad (2.1f) \]

where \( M_{kl} \) is the generator of internal Lorentz rotations, \( P_k \) is of internal translations and \( Q \) is of internal super-transformations. In contrast to the conventional formulation of supergravity, this algebra holds as local version.

Additional internal symmetry with gauge group \( G_0 \) can be incorporated by taking the whole gauge group \( G \) to be the direct product of \( G_0 \) and the super-Poincaré group. Then the algebra of \( G \) reads as

\[ [M_{kl}, X_a] = 0, \quad (2.1g) \]
\[ [P_k, X_a] = 0, \quad (2.1h) \]
\[ [Q, X_a] = 0, \quad (2.1i) \]
\[ [X_a, X_b] = i f_{ab}^c X_c, \quad (2.1j) \]
in addition to Eqs. (2.1a)~(2.1f), where \( X_a \) is the generator of \( G_0 \) and \( f_{ab}^c \) is the structure constant.

We here introduce the gauge fields \( \phi_\mu, A^k_\mu, A^k_{\mu} \) and \( A^a_\mu \) for super-transformations, translations, Lorentz rotations and \( G_0 \)-transformations, respectively.

We can determine the infinitesimal gauge transformations of these gauge fields in a usual manner:

\[ \delta \phi_\mu = -\hat{\nabla}_k \epsilon + \frac{i}{2} \omega^{hl}(\frac{i}{2} \sigma_{kl}) \phi_\mu, \quad (2.2a) \]
\[ \delta A^k_\mu = -\hat{\nabla}_k t_\alpha + \omega^k_\mu A^m_\mu + 2 \epsilon \gamma^k \phi_\mu, \quad (2.2b) \]
\[ \delta A^k_{\mu} = -\hat{\nabla}_\mu \omega^k, \quad (2.2c) \]
\[ \delta A^a_\mu = -(\partial_\mu \alpha^a + A^b_\mu \alpha^b \alpha^c) \quad (2.2d) \]

with\(^*)

\[ \hat{\nabla}_k \epsilon := \partial_k \epsilon + \frac{i}{2} A^{hl}(\frac{i}{2} \sigma_{hl}) \epsilon, \quad (2.3a) \]
\[ \hat{\nabla}_k t_\alpha := \partial_k t_\alpha + A^k_{\mu} t_\alpha^m, \quad (2.3b) \]
\[ \hat{\nabla}_k \omega^{hl} := \partial_k \omega^{hl} + A^k_{\mu} \omega^{ml} + A^l_{\nu} \omega^{km}, \quad (2.3c) \]

where the Majorana spinor \( \epsilon(x) \) is an infinitesimal parameter of a super-transformation, and functions \( t^h(x), \omega^{hl}(x), \alpha^a(x) \) are the parameters of a translation, a Lorentz rotation and a \( G_0 \)-transformation, respectively.

Equation (2.2d) shows that \( A^a_\mu \) remains invariant under super-transformations, and thus its super-partner is not needed. Putting \( G_0 = U_{em}(1) \), for example, we can incorporate the electromagnetic interaction without introducing the photino, in contrast to the case of the spacetime supersymmetry.

\(^*) \text{We use the symbols} : = \text{and} = \text{for definitions and identities, respectively.}
\[ \text{Latin indices are raised (lowered) using} \ y^{hl}(\eta_{kl}). \]
Suppressing the $G_\psi$-symmetry for simplicity, we shall put from now on $G=$the super-Poincaré group. The $G_\psi$-symmetry can easily be restored, if necessary.

For later convenience, we here define the field strengths $\mathcal{R}_{\mu \nu}$, $\mathcal{R}^k_{\mu \nu}$ and $\mathcal{R}^{kl}_{\mu \nu}$ by*)

\[
\mathcal{R}_{\mu \nu} := 2\left( \partial_{[\mu} \psi_{\nu]} + i \frac{1}{2} A^{kl}_{[\mu} \left( \sigma_{kl} \right) \psi_{\nu]} \right),
\]

\[
\mathcal{R}^k_{\mu \nu} := 2\left( \partial_{[\mu} A^k_{\nu]} + A^k_{[\mu} \left( \partial_{\nu]} \right) A^l + \bar{\psi}_{[\mu} \gamma^k \psi_{\nu]} \right),
\]

\[
\mathcal{R}^{kl}_{\mu \nu} := 2\left( \partial_{[\mu} A^{kl}_{\nu]} + A^k_{[\mu} A^m_{\nu]} \right),
\]

which transform according as

\[
\delta \mathcal{R}_{\mu \nu} = -i \frac{1}{2} \mathcal{R}^{kl}_{\mu \nu} \left( \sigma_{kl} \right) \varepsilon + i \frac{1}{2} \omega^{kl} \left( \sigma_{kl} \right) \mathcal{R}_{\mu \nu},
\]

\[
\delta \mathcal{R}^k_{\mu \nu} = 2 \varepsilon \gamma^k R_{\mu \nu} - \omega^m R^k_{\mu \nu} + \omega^k R^m_{\mu \nu},
\]

\[
\delta \mathcal{R}^{kl}_{\mu \nu} = \omega^m R^{kl}_{\mu \nu} + \omega^k R^{lm}_{\mu \nu}.
\]

For the matter field $f(x)$ belonging to a linear representation $\tau$ of $G$, the infinitesimal transformation rule is given by**

\[
\delta f(x) = i \varepsilon(x) \tau_\ast(Q) f(x) + i t^k(x) \tau_\ast(P_k) f(x) + i \frac{1}{2} \omega^{kl}(x) \tau_\ast(M_{kl}) f(x).
\]

We find generally that $\tau_\ast(Q) \neq 0$ and $\tau_\ast(P_k) \neq 0$, namely, a matter field carries nonvanishing quantum numbers associated with the internal super-transformations and translations.

We can assign the quarks and leptons to fields for which $\tau_\ast(Q) = 0$, $\tau_\ast(P_k) = 0$ and $\tau_\ast(M_{kl}) = (-i/2) \sigma_{kl}$. When we assign so, the super-partners of the quarks and leptons need not be introduced. (There remains the possibility that the quarks and leptons form the supermultiplets for which $\tau_\ast(Q) \neq 0$ and $\tau_\ast(P_k) \neq 0$ with unobserved their partners.)

**§3. Nonlinear realization of internal super-Poincaré group**

In this section, we apply***) the method of nonlinear realizations\textsuperscript{[13]} to the spontaneous breakdown of the internal super-Poincaré symmetry to the internal Lorentz one.

In some neighborhood of the identity of $G$ (=the super-Poincaré group), every group element $g \in G$ is represented in the form

\[
g = e^{ie_0} e^{i t^P \varepsilon_{(i/2)} \omega \cdot M},
\]

\textsuperscript{(*)} We denote the anti-symmetrization of indices by a square bracket $[\cdot\cdot\cdot]$, for example, $\partial_{\mu} \phi_{\nu} := (1/2)(\partial_{\mu} \phi_{\nu} - \partial_{\nu} \phi_{\mu})$.

\textsuperscript{(**)} The representation $\tau_\ast$ of the algebra is the "differential" of $\tau$.

\textsuperscript{(***)} Since we consider the internal symmetry, we need not give the proper care which has to be taken in applying the method to spacetime symmetry.\textsuperscript{[14]} For a nonlinear realization of the spacetime supersymmetry, see Refs. 15) and 16).
Gravity as a Gauge Theory

where

\[ t \cdot P = \sum_k t^k P_k , \quad \omega \cdot M = \sum_{k,l} \omega^{kl} M_{kl} \]  \hfill (3.2)

with \( \epsilon \) being anticommuting constant Majorana spinor and \( t^k, \omega^{kl} \) being real constants.

In order to have a nonlinear realization of the internal super-Poincaré group, we introduce the Nambu-Goldstone fermion \( \lambda(x) \) and boson \( \xi^k(x) \) which correspond to the broken generators \( Q \) and \( P_k \), respectively. Each element of the left coset space \( G/H \), where \( H \) is the Lorentz group, can be characterized by the representative

\[ L(x) = e^{i\lambda(x)} e^{it(x) \cdot P} , \]  \hfill (3.3)

and the group action of the element \( g \) on this representative is defined by

\[ g \cdot L(x) = L'(x) \cdot h \]  \hfill (3.4)

with

\[ L'(x) = e^{i\lambda'(x)} e^{it'(x) \cdot P} , \quad h \in H . \]  \hfill (3.5)

From Eqs. (3.4) and (3.5), we determine the super-Poincaré transformations of the Nambu-Goldstone fields for \( g \):

\[ g: \lambda(x) \to \lambda'(x) , \quad \xi^k(x) \to \xi'^k(x) . \]  \hfill (3.6)

Further, we introduce the spectator field \( \Phi(x) \) which is "orthogonal" to the Nambu-Goldstone fields. (This field \( \Phi \) corresponds to the field \( \varphi_1 \) in Goldstone's model.\(^8\)) The transformation of \( \Phi \) for \( g \) is given by

\[ g: \Phi(x) \to \Phi'(x) = D(h) \Phi(x) , \]  \hfill (3.7)

where \( h \) is that in Eq. (3.4) and \( D \) is a linear representation of \( H \). The above transformations (3.6) and (3.7) give a nonlinear realization of the internal super-Poincaré group. The explicit expressions for infinitesimal transformations can easily be obtained as

\[ \delta \lambda(x) = \epsilon + \frac{i}{2} \omega^{kl} \left( -\frac{i}{2} \sigma_{kl} \right) \lambda(x) , \]  \hfill (3.8a)

\[ \delta \xi^k(x) = \bar{\epsilon} \gamma^k \lambda(x) + t^k + \omega^{kl} \xi^l(x) , \]  \hfill (3.8b)

\[ \delta \Phi(x) = i \omega^{kl} D_k (M_{kl}) \Phi(x) , \]  \hfill (3.8c)

using the super-Poincaré algebra (2.1a)\( \sim \) (2.1f) together with the formula (A.10). Equations (3.8a) and (3.8b) show that the Nambu-Goldstone fields transform nonlinearly in general.

To construct an invariant Lagrangian density out of the basic fields and their first

\(^8\) The model is that of a complex boson \( \varphi = (\varphi_1 + i \varphi_2)/\sqrt{2} : L = \delta \varphi \delta \varphi - (1/4) \lambda_0 (\varphi^* \varphi)^2 + (1/2) \mu_0 (\varphi^* \varphi) \) with \( \lambda_0 \) and \( \mu_0 \) being positive real constants, and the vacuum \( |0 \rangle \) is fixed as \( \langle 0 | \varphi | 0 \rangle = \mu_0 / \lambda_0 . \)
derivatives, we define the covariant derivatives $D_{\mu} \lambda$, $D_{\mu} \xi^k$ and $D_{\mu} \Phi$.

Consider the Cartan form $L(x)^{-1}dL(x)$ and decompose it into the following form:

$$L(x)^{-1}dL(x) = i\Delta \lambda(x)Q + i\Delta \xi^k(x)P^i + \frac{i}{2} \Delta u^{kl}(x)M_{kl}, \quad (3.9)$$

where $L(x)$ is of Eq. (3.3) and $\Delta \lambda$, $\Delta \xi^k$, $\Delta u^{kl}$ are the coefficients in this decomposition. Using these coefficients, we define the covariant derivatives:

$$D_{\mu} \lambda(x) := \frac{d\lambda(x)}{dx^\mu} = \partial_{\mu} \lambda(x), \quad (3.10a)$$

$$D_{\mu} \xi^k(x) := \frac{d\xi^k(x)}{dx^\mu} = \partial_{\mu} \xi^k(x) - \lambda(x) \gamma^k \partial_{\mu} \lambda(x), \quad (3.10b)$$

$$D_{\mu} \Phi(x) := \partial_{\mu} \Phi(x) + \frac{i}{2} \Delta u^{kl}(x)D_{\Phi}(M_{kl})\Phi(x) = \partial_{\mu} \Phi(x), \quad (3.10c)$$

where we have used the formulae (A.10) and (A.11) to obtain the explicit expressions.

Since these covariant derivatives remain invariant under super-transformations and translations, we can construct a global super-Poincaré invariant Lagrangian density $L_{sp}$ as an internal Lorentz-scalar function of $D_{\mu} \lambda$, $D_{\mu} \xi^k$, $D_{\mu} \Phi$ and $\Phi$:

$$L = L_{sp}(D_{\mu} \lambda, D_{\mu} \xi^k, D_{\mu} \Phi, \Phi). \quad (3.11)$$

To construct a local super-Poincaré invariant theory, we introduce the complete covariant derivatives $\mathcal{D}_{\mu} \lambda$, $\mathcal{D}_{\mu} \xi^k$ and $\mathcal{D}_{\mu} \Phi$ which transform according as

$$\delta(\mathcal{D}_{\mu} \lambda) = \frac{i}{2} \omega^{kl}(x)(-\frac{i}{2} \sigma_{kl})(\mathcal{D}_{\mu} \lambda), \quad (3.12a)$$

$$\delta(\mathcal{D}_{\mu} \xi^k) = \omega^k(x)(\mathcal{D}_{\mu} \xi^i), \quad (3.12b)$$

$$\delta(\mathcal{D}_{\mu} \Phi) = \frac{i}{2} \omega^{kl}(x)D_{\Phi}(M_{kl})(\mathcal{D}_{\mu} \Phi), \quad (3.12c)$$

under full super-Poincaré gauge transformations with $x$-dependent parameters $\epsilon(x)$, $t^k(x)$ and $\omega^{kl}(x)$. (Only under global transformations, $D_{\mu} \lambda$, $D_{\mu} \xi^k$ and $D_{\mu} \Phi$ transform in the same way as Eqs. (3.12a)~(3.12c).) We define these covariant derivatives by

$$\mathcal{D}_{\mu} \lambda(x) := \nabla_{\mu} \lambda(x), \quad (3.13a)$$

$$\mathcal{D}_{\mu} \xi^k(x) := \nabla_{\mu} \xi^k(x) - \lambda(x) \gamma^k \nabla_{\mu} \lambda(x), \quad (3.13b)$$

$$\mathcal{D}_{\mu} \Phi(x) := \nabla_{\mu} \Phi(x), \quad (3.13c)$$

by making the following replacements in $D_{\mu} \lambda$, $D_{\mu} \xi^k$ and $D_{\mu} \Phi$:

$$\partial_{\mu} \lambda \rightarrow \nabla_{\mu} \lambda := \partial_{\mu} + \frac{i}{2} A^{kl}(x)(-\frac{i}{2} \sigma_{kl}) \lambda + \phi_{kl} \lambda, \quad (3.14a)$$

$$\partial_{\mu} \xi^k \rightarrow \nabla_{\mu} \xi^k := \partial_{\mu} \xi^k + A^{kl}(x)\xi^l + A^{kl}(x) + \phi_{kl} \gamma^k \lambda, \quad (3.14b)$$

$$\partial_{\mu} \Phi \rightarrow \nabla_{\mu} \Phi := \partial_{\mu} \Phi + \frac{i}{2} A^{kl}(x)D_{\Phi}(M_{kl}) \Phi. \quad (3.14c)$$
Gravity as a Gauge Theory

Now, we can construct a super-Poincaré gauge invariant Lagrangian density $L_{\text{SP}}$:

$$L = L_{\text{SP}}(\partial_\mu \bar{\lambda}, \partial_\mu \bar{\xi}^k, \partial_\mu \Phi, \bar{\phi}_{\mu, \nu}, A^{\mu}, A^{\mu, \nu}, A^{h\mu}, A^{h\mu, \nu})$$

(3.15)

by replacing $D_\mu$ by $\partial_\mu$ in $L_{\text{SP}}$ of Eq. (3.11) and further by taking account of the contributions of the gauge fields.

Here we redefine the Nambu-Goldstone fields $\lambda(x)$ and $\xi^k(x)$ so that they have the correct dimensions of fermion and of boson, respectively.

Introducing constants $c_s$ and $c_T$ which are of dimension $(\text{mass})^3$, we put

$$\lambda'(x) = c_s^2 \lambda(x), \quad \xi^k'(x) = c_T^2 \xi^k(x),$$

(3.16)

and will drop the primes from now on. The redefined fields thus have the correct dimensions:

$$[\lambda(x)] = (\text{mass})^{3/2}, \quad [\xi^k(x)] = (\text{mass})^1.$$  

(3.17)

The constants will give the breaking scale of the internal supersymmetry and the internal translation symmetry, $A_{\text{Susy}}$ and $A_{\text{Trans}}$, respectively:

$$c_s \sim A_{\text{Susy}}, \quad c_T \sim A_{\text{Trans}}.$$  

(3.18)

The transformation laws (3.8a) and (3.8b) now become

$$\delta \lambda(x) = c_s^2 \varepsilon + \frac{i}{2} \omega^k \left( -\frac{i}{2} \sigma_{k\ell} \right) \lambda(x),$$

(3.19a)

$$\delta \xi^k(x) = c_s^2 \bar{\varepsilon} \gamma^k \lambda(x) + c_T^2 t^k + \omega^k \xi^l(x).$$

(3.19b)

The complete covariant derivatives are also redefined, but the transformation laws (3.12a)–(3.12c) still hold.

We will use the redefined fields in the rest of this paper.

§ 4. Vierbein field

We construct the co-frame field** $e^k_\mu$ out of the elements of $L_{\text{SP}}$ so that the metric***

$$g_{\mu\nu} := e^k_\mu \eta_{k\ell} e^\ell_\nu,$$

(4.1)

is invariant under nonlinear super-Poincaré gauge transformations.

This invariance prevents us from identifying the translation gauge field $A^k_\mu$ with $e^k_\mu$. If we identify $\partial_\mu e^k$ with $e^k_\mu$ (up to a constant factor), the metric is invariant as is seen from the transformations (3.12b). However, this identification seems to be

---

* For a field $f(x)$, $f_\mu$ stands for $\partial_\mu f$.
** Composite vierbeins have been introduced also in Refs. 18 and 19) with the aid of the method of nonlinear realizations. In Ref. 19), composite achtbein superfields have further been introduced in a model of composite supergravity.
*** Greek indices are raised [lowered] using $g^{\mu\nu}$, where $(g^{\mu\nu}) := (g_{\mu\nu})^{-1}$. 
unnatural in the point that the Nambu-Goldstone fields and/or the gauge fields must have nonvanishing "vacuum expectation value (VEV)" (to assure $e^k_\mu \neq 0$ in "vacuum"). To avoid this unnaturalness, we assume that the field $\Phi(x)$ contains the Lorentz vector $\phi^k$ which will have nonvanishing "VEV":

$$\Phi = \begin{pmatrix} \phi^k \\ \phi^\sigma \end{pmatrix}, \quad (4.2)$$

where $\phi^\sigma$ stands for other fields collectively, and we identify the co-frame with the linear combination of $\mathcal{D}_\mu \phi^k$ and $\mathcal{D}_\mu \xi^k$:

$$e^k_\mu = \frac{1}{c_T}(c_\varphi \mathcal{D}_\mu \phi^k + c_\varepsilon \mathcal{D}_\mu \xi^k) \quad (4.3)$$

with $c_\varphi$, $c_\varepsilon$ being dimensionless constants and

$$\mathcal{D}_\mu \phi^k = \partial_\mu \phi^k + A^k_\mu \phi^i. \quad (4.4)$$

The correspondence of $\Phi$ to the $\varphi_1$-field (mentioned in § 3) will make the nonvanishing "VEV" of $\phi^k$ acceptable. (See the expansion (8.2c) of $\phi^k$ in § 8.)

Further we define the vierbein field\footnote{Using $e^k_\mu$, $e^\sigma_\mu$, we convert Latin (Greek) indices to Greek (Latin) ones.} $e^\sigma_\mu$ through

$$(e^\sigma_\mu)^-1.$$

The identification (4.3) and the definition (4.5) are based on the assumption $\det(e^k_\mu) \neq 0$.

Now we find that $e^k_\mu$ transforms according as

$$\delta e^k_\mu = \omega^k_\mu e^\sigma_\mu, \quad (4.6)$$

under full super-Poincaré gauge transformations. The internal Lorentz symmetry is a symmetry on tangent space\footnote{The geometrical treatment of the internal translation has been given in Ref. 9). There, it has been shown that the internal Poincaré transformation induces a Poincaré transformation on the tangent affine space at each point of spacetime.} as is seen from Eq. (4.6). This equation also shows that $e^k_\mu$ is invariant under super-transformations (and translations), and consequently the introduction of gravitino (=the super-partner of $e^k_\mu$) is not needed. Correspondingly, the super gauge field $\phi_\mu$ does not play the role of gravitino, in contrast to conventional supergravity, in which theory the gauge field for the spacetime supersymmetry is the super-partner of $e^k_\mu$.

The covariant derivative $\mathcal{D}_\mu \phi^k$ is written as

$$\mathcal{D}_\mu \phi^k = \partial_\mu \phi^k + \omega^k_\mu D^k_\sigma(M_{kl}) \phi^\sigma. \quad (4.7)$$

with $D^\sigma$ being the linear representation to which $\phi^\sigma$ belongs. The transformation laws of $\phi^\sigma$ and $\mathcal{D}_\mu \phi^\sigma$ are

$$\delta \phi^\sigma = -\frac{i}{2} \omega^k_\mu D^k_\sigma(M_{kl}) \phi^\sigma, \quad \delta(\mathcal{D}_\mu \phi^\sigma) = -\frac{i}{2} \omega^k_\mu D^k_\sigma(M_{kl})(\mathcal{D}_\mu \phi^\sigma). \quad (4.8)$$
§ 5. Super-Poincaré gauge invariance of action integral

In this and next sections, we give identities and restrict the functional dependence of the Lagrangian density by following Noether’s method. (See Ref. 20 for PGT.)

Consider the action integral

\[ I = \int_\mathcal{D} L d^4x \]  \hspace{1cm} (5.1a)

with

\[ L = L(\bar{\lambda}, \bar{\sigma}, \xi^k, \xi^{k\nu}, \phi^k, \phi^{k\nu}, \bar{\phi}_\mu, \bar{\phi}_{\mu\nu}, A^k, A^{k\mu}, A^{k\mu\nu}, A^{k\mu\nu\sigma}, \phi^\sigma, \phi^{\sigma\mu}, q, q_{\nu}) \]  \hspace{1cm} (5.1b)

where \( \mathcal{D} \) is a compact region in spacetime and \( q(x) \) denotes a matter field. The field \( q(x) \) is assumed to belong to a linear representation \( \rho \) of the Lorentz group, because we are constructing an effective theory in which the gauge symmetry breaks down spontaneously to the Lorentz symmetry. Thus, we have

\[ \delta q = i \frac{\omega_{\mu} \rho_s(M_{kl})}{2} q \]  \hspace{1cm} (5.2)

for infinitesimal gauge transformations.

5.1. Identities and the Noether currents

We require that the action integral \( I \) remains invariant under (nonlinear) super-Poincaré gauge transformations.

For an infinitesimal transformation with the parameters \( \varepsilon(x), t^k(x), \omega^{kl}(x) \), the requirement of the gauge invariance gives

\[ \delta L = \varepsilon^q I + t^k M_{kl} I_{kl} + \partial_\mu D^\mu = 0 \]  \hspace{1cm} (5.3)

with

\[ q_I = c_s^2 \frac{\partial L}{\partial \lambda} + c_s^2 \gamma^k \lambda \frac{\partial L}{\partial \xi^k} + \frac{i}{2} A^k \left( \frac{1}{2} \sigma_{kl} \frac{\partial L}{\partial \phi} + 2 \gamma^k \phi^\mu \frac{\partial L}{\partial A^k_{\mu}} + \partial_\mu \left( \frac{\delta L}{\delta A^k_{\mu}} \right) \right) \]  \hspace{1cm} (5.4a)

\[ t_k I = c_t^2 \frac{\partial L}{\partial \xi^k} + \frac{\delta L}{\delta A^k_{\mu}} + \partial_\mu \left( \frac{\delta L}{\delta A^k_{\mu}} \right) \]  \hspace{1cm} (5.4b)

\[ M_{kl} = \frac{i}{2} \left( \frac{1}{2} \sigma_{kl} \right) \frac{\delta L}{\delta \lambda} + \frac{\delta L}{\delta \xi^{[k}} \xi^{l]} + \frac{\delta L}{\delta \phi_{[k}} \phi_{l]} + \frac{i}{2} \frac{\delta L}{\delta A^k_{\mu}} \left( \frac{1}{2} \sigma_{kl} \right) \frac{\delta L}{\delta \phi_{l]}}, \]  \hspace{1cm} (5.4c)

and
Here the quantities defined by
\[ Q^\mu = c_s^2 \frac{\partial L}{\partial \Lambda_{,\mu}} + c_T^2 \gamma^\mu \frac{\partial L}{\partial \xi_{,\mu}} + \frac{i}{2} A^{h,\mu} \left( \frac{i}{2} \delta_{h,\mu} \right) \frac{\partial L}{\partial \psi_{,\mu}} + 2 \gamma^\mu \psi_{,\mu} \frac{\partial L}{\partial A_{A,\mu}}, \quad (5.6a) \]
\[ P^h_{h,\mu} = c_T^2 \frac{\partial L}{\partial \xi_{,\mu}} + A_{,\mu} \frac{\partial L}{\partial A_{,\mu}}, \quad (5.6b) \]
\[ ^M J_{h,\mu} = 2 \left( \frac{i}{2} \Lambda \left( \frac{i}{2} \sigma_{h,\mu} \right) \frac{\partial L}{\partial \Lambda_{,\mu}} + \frac{\partial L}{\partial \xi_{,\mu}} \xi_{,\mu} + \frac{\partial L}{\partial \phi_{,\mu}} \phi_{,\mu} \right) \]
\[ + \frac{i}{2} \psi_{,\mu} \left( \frac{i}{2} \sigma_{h,\mu} \right) \frac{\partial L}{\partial \psi_{,\mu}} + \frac{\partial L}{\partial A_{,\mu}} A_{,\mu} + 2 \frac{\partial L}{\partial A_{,\mu}} A_{,\mu} \]
\[ + \frac{i}{2} D^2_{\mu}(M_{h,\mu}) \phi_{,\mu} \frac{\partial L}{\partial \phi_{,\mu}} + \frac{i}{2} \rho_{,\mu}(M_{h,\mu}) q \frac{\partial L}{\partial q_{,\mu}}, \quad (5.6c) \]
are the Noether currents associated with super-transformations, translations and Lorentz rotations, respectively.

Furthermore, requiring that the parameters \( s(x), t^h(x), \omega^{h,\mu}(x) \) together with their first derivatives vanish on the boundary surface of \( \mathcal{D} \), we obtain
\[ \int_{\mathcal{D}} (\varepsilon^0 I + t^h P^h I^h + \omega^{h,\mu} M^h I^h) d^4 x = 0, \quad (5.7) \]
from the gauge invariance of the action integral. Since the transformation parameters and their derivatives are arbitrary within the integration domain, we deduce
\[ Q^0 I = 0, \quad P^0 I^h = 0, \quad (5.8a) \]
\[ ^M I_{h,\mu} = 0, \quad (5.8b) \]
from the identity (5.7), and hence
\[ \partial_\mu D^\mu = 0, \quad (5.9) \]
from the identity (5.3).

The identities (5.8a) show that the field equations of the Nambu-Goldstone fields are automatically satisfied if the field equations of \( \psi_{,\mu} \) and \( A_{,\mu} \) are satisfied. Thus, the Nambu-Goldstone fields are not independent dynamical variables. The gauge freedoms for super-transformations and translations enable us to eliminate these redundant freedoms, i.e., we can take the gauge in which the Nambu-Goldstone fields "disappear":
\[ \lambda = 0, \quad \xi^h = 0. \quad (5.10) \]
This is what we call "(super-)Higgs mechanism".
Now, from the identity (5·9), we obtain the set of identities,*

\[ \varepsilon_{\nu \mu} : \frac{\partial L}{\partial \phi_{(\nu, \mu)}} = 0, \]  
(5·11a)

\[ t^k_{\nu \mu} : \frac{\partial L}{\partial A^k_{(\nu, \mu)}} = 0, \]  
(5·11b)

\[ \omega^{kl}_{\nu \mu} : \frac{\partial L}{\partial A^{kl}_{(\nu, \mu)}} = 0, \]  
(5·11c)

\[ \varepsilon : q J^\mu = \partial_{(\nu} \left( \frac{\partial L}{\partial \phi_{\nu \mu}} \right) + \frac{\partial L}{\partial \phi_{\mu}}, \]  
(5·12a)

\[ t^k : p J^\mu = \partial_{(\nu} \left( \frac{\partial L}{\partial A^k_{\nu \mu}} \right) + \frac{\partial L}{\partial A^k_{\mu}}, \]  
(5·12b)

\[ \omega^{kl} : m J^\mu_{kl} = \partial_{(\nu} \left( 2 \frac{\partial L}{\partial A^{kl}_{\nu \mu}} \right) + 2 \frac{\partial L}{\partial A^{kl}_{\mu}}, \]  
(5·12c)

\[ \varepsilon : \partial_\mu q J^\mu \equiv \partial_\mu \left( \frac{\partial L}{\partial \phi_{\mu}} \right), \]  
(5·13a)

\[ t^k : \partial_\mu p J^\mu_k \equiv \partial_\mu \left( \frac{\partial L}{\partial A^k_{\mu}} \right), \]  
(5·13b)

\[ \omega^{kl} : \partial_\mu m J^\mu_{kl} \equiv \partial_\mu \left( 2 \frac{\partial L}{\partial A^{kl}_{\mu}} \right). \]  
(5·13c)

All of these identities are not independent conditions.

The differential conservation laws of the Noether currents:

\[ \partial_\mu q J^\mu = 0, \quad \partial_\mu p J^\mu = 0, \quad \partial_\mu m J^\mu_{kl} = 0, \]  
(5·14)

follow from the identities (5·13a)~(5·13c).

5.2. Restrictions on functional dependence of the Lagrangian density

We restrict the functional dependence of the Lagrangian density \( L \) using the identities given in the preceding sub-section.

First, we express \( L \) as

\[ L = L_1(\mathcal{D}_{\mu} \bar{\lambda}, \mathcal{D}_{\mu} \phi^k, \phi^k, \mathcal{D}_{\mu} \phi^k, \bar{\phi}_{\mu}, \bar{\phi}_{\mu}, A^k_{\mu}, A^k_{\nu}, A^{kl}_{\mu}, A^{kl}_{\nu}, \phi^\sigma, \mathcal{D}_{\mu} \phi^\sigma, q, \bar{\nu}_\mu q), \]  
(5·15)

with the covariant derivative

\[ \bar{\nu}_\mu q := \partial_\mu q + \frac{i}{2} A^{kl}_{\mu} \rho^* (M_{kl}) q. \]  
(5·16)

From the identities (5·11a)~(5·11c) and Eqs. (3·13a), (4·4) and (4·7), it follows that \( L \) is expressed as

---

* We denote the symmetrization of indices by a round bracket ( ), for example, \( \bar{\phi}_{(\nu, \mu)} := (1/2)(\bar{\phi}_{\nu \mu} + \bar{\phi}_{\mu \nu}). \)
1014  H. Saitoh

\[ L = L_0(\bar{\lambda}, \nabla \mu \bar{\lambda}, \xi^k, \mathcal{D}_\mu \xi^k, \phi^k, \bar{\mathcal{D}}_\mu \phi^k, \overline{\mathcal{V}}_\mu, A^\mu, A^{\mu \nu}, \overline{\mathcal{V}}_{\mu \nu}, T^{\nu}_{\rho \mu}, \phi^\sigma, \bar{\mathcal{V}}_\mu \phi^\sigma, q, \bar{\mathcal{V}}_\mu q). \]  

(5.17)

Here we have defined the "super-torsion" \( \Psi_{\mu \nu} \) and the torsion \( T^{\nu}_{\rho \mu} \) by

\[ \Psi_{\mu \nu} := c_s^{\pm \sigma} R_{\mu \nu} + \frac{i}{2} R^{\mu \nu} \bigg( -\frac{i}{2} \sigma_{\mu \nu} \bigg), \]

(5.18a)

\[ T^{\nu}_{\rho \mu} := 2(\partial_{[\nu e^k \nu]} + A^\mu_{[\nu e^k \nu]}), \]

(5.18b)

which transform according as

\[ \delta \Psi_{\mu \nu} = \frac{i}{2} \omega^{\mu \nu} \bigg( -\frac{i}{2} \sigma_{\mu \nu} \bigg) \Psi_{\mu \nu}, \quad \delta T^{\nu}_{\rho \mu} = \omega^{\nu}_{\rho} T^{\tau}_{\nu \mu}, \]

(5.19)

under full super-Poincaré gauge transformations. The identities (5.12a)–(5.12c) now read as

\[ \frac{\partial L_2}{\partial \phi_\mu} = 0, \quad \frac{\partial L_2}{\partial A^\mu} = 0, \quad \frac{\partial L_2}{\partial A^{\mu \nu}} = 0. \]

(5.20)

For \( L_2 \) satisfying these identities, the identities (5.8a) are equivalent to

\[ \frac{\partial L_3}{\partial \lambda} = 0, \quad \frac{\partial L_3}{\partial \xi^k} = 0. \]

(5.21)

We find, from the identities (5.20) and (5.21),

\[ L = L_0(\nabla \nu \lambda, \mathcal{D}_\nu \xi^k, \phi^k, e^k_\mu, \overline{\mathcal{V}}_\nu, T^{k l m}, R^{k l m}, \phi^\sigma, \bar{\mathcal{V}}_\nu \phi^\sigma, q, \bar{\mathcal{V}}_\nu q), \]

(5.22)

where \( \nabla \nu \lambda := e^\nu_\mu \nabla \nu \lambda, \mathcal{D}_\nu \xi^k := e^\nu_\mu \mathcal{D}_\nu \xi^k, \ldots \). Then the identity (5.8b) becomes

\[
\begin{align*}
&i \frac{\partial L_3}{\partial \nabla \nu \lambda} \bigg( \frac{i}{2} \sigma_{\nu \lambda} \bigg) \frac{\partial L_3}{\partial (\nabla \nu \lambda)} + (\nabla \nu \lambda) \eta_{\nu \lambda} \frac{\partial L_3}{\partial (\nabla \nu \lambda)} + \frac{\partial L_3}{\partial (\mathcal{D}_\nu \xi^k)} \frac{\partial L_3}{\partial (\mathcal{D}_\nu \xi^k)} \\
&\quad+ \frac{\partial L_3}{\partial (\mathcal{D}_\nu \xi^k)} \eta_{\nu \lambda} \frac{\partial L_3}{\partial (\mathcal{D}_\nu \xi^k)} + \frac{\partial L_3}{\partial \phi^\sigma} (\nabla \nu \phi^\sigma) \frac{\partial L_3}{\partial (\nabla \nu \phi^\sigma)} + \frac{\partial L_3}{\partial (\mathcal{D}_\nu \xi^k)} \frac{\partial L_3}{\partial (\mathcal{D}_\nu \xi^k)} \\
&\quad+ 2 \bar{\mathcal{V}}_{(\nu \lambda)} \frac{\partial L_3}{\partial (\mathcal{D}_\nu \xi^k)} + \frac{\partial L_3}{\partial T^{(k l m)}} + 2 \frac{\partial L_3}{\partial T^{(k l m)}} + 2 \frac{\partial L_3}{\partial R^{(k l m)}} \frac{\partial L_3}{\partial R^{(k l m)}} + 2 \frac{\partial L_3}{\partial R^{(k l m)}} \frac{\partial L_3}{\partial R^{(k l m)}} \\
&\quad+ \frac{i}{2} D^\omega_\nu (M_{\nu \lambda}) \frac{\partial L_3}{\partial \phi^\omega} + \frac{i}{2} e^\nu_\mu D^\omega_\nu (M_{\nu \lambda}) (\nabla \nu \phi^\omega) \frac{\partial L_3}{\partial (\nabla \nu \phi^\omega)} + \frac{\partial L_3}{\partial (\mathcal{D}_\nu \xi^k)} \frac{\partial L_3}{\partial (\mathcal{D}_\nu \xi^k)} \\
&\quad+ \frac{i}{2} \rho (M_{\nu \lambda}) q \frac{\partial L_3}{\partial q} + \frac{i}{2} e^\nu_\mu \rho (M_{\nu \lambda}) (\nabla \nu q) \frac{\partial L_3}{\partial (\nabla \nu q)} + (\nabla \nu q) \frac{\partial L_3}{\partial (\nabla \nu q)} = 0. \quad \text{(5.23)}
\end{align*}
\]

\section{6. Invariance under general coordinate transformations}

\subsection{6.1. Identities and the Noether currents}

We also require that the action integral \( I \) is invariant under an arbitrary general coordinate transformation (GCT)
Gravity as a Gauge Theory

$$x'' - x'' = x'' + \delta x''$$  \hspace{1cm} (6.1)

with $\delta x''$ being an arbitrary function. This requirement is equivalent to $\delta L + \partial_{\mu}(\delta x'')L = 0$, which reads as

$$(-\delta x'')_{\mu\nu}S_{\rho}^{\mu\nu} - (-\delta x'')_{\rho\mu} \left( \bar{\Theta}_{\rho}^{\mu} - \bar{\Phi}_{\rho}^{\mu} \frac{\delta L}{\delta \Phi_{\nu}} A_{\nu}^{h} \right) \frac{\delta L}{\delta A_{\mu\nu}} A^{kl}_{\mu\nu} - \partial_{\rho}S_{\rho}^{\mu\nu} = 0,$$  \hspace{1cm} (6.2)

where

$$S_{\rho}^{\mu\nu} = -\bar{\Phi}_{\rho}^{\mu} \frac{\delta L}{\delta \Phi_{\nu}} A_{\nu}^{h} + \frac{\partial L}{\partial A^{h}_{\mu\nu}} A^{kl}_{\mu\nu},$$  \hspace{1cm} (6.3)

$$\bar{\Theta}_{\mu}^{\nu} = \delta_{\mu}^{\nu} L \left( \frac{\partial L}{\partial \Phi_{\mu}} \Phi_{\nu}^{h} + \frac{\partial L}{\partial \Phi_{\nu}} \Phi_{\mu}^{h} + \frac{\partial L}{\partial A^{\mu\nu}} A^{kl}_{\mu\nu} + \frac{\partial L}{\partial A^{\nu\mu}} A^{kl}_{\mu\nu} \right).$$  \hspace{1cm} (6.4)

Since $\delta x''$ is arbitrary, we obtain, from the identity (6.2),

$$S_{\rho}^{\mu\nu} = 0,$$  \hspace{1cm} (6.5)

$$\bar{\Theta}_{\mu}^{\nu} = \partial_{\rho} S_{\rho}^{\mu\nu} + \bar{\Phi}_{\rho}^{\mu} \frac{\partial L}{\partial \Phi_{\nu}} A_{\nu}^{h} + \frac{\partial L}{\partial A^{h}_{\mu\nu}} A^{kl}_{\mu\nu}.$$  \hspace{1cm} (6.6)

These identities lead to the differential conservation laws of the canonical energy-momentum density $\bar{\Theta}_{\mu}^{\nu}$ and the extended orbital angular momentum density, defined by $M_{\rho}^{\alpha\nu}$

$$M_{\rho}^{\alpha\nu} = -2(x^\rho \bar{\Theta}_{\rho}^{\mu} - S_{\rho}^{\mu\nu}).$$  \hspace{1cm} (6.7)

### 6.2. Restriction on functional dependence of the Lagrangian density

We further restrict the functional dependence of $L$ using the identities (6.5) and (6.6).

The Lagrangian density $L_3$ satisfies the identity (6.5), because $L_3$ satisfies the identities (5.11a)~(5.11c). From the identity (6.6), we obtain that the Lagrangian density which leads to a gauge and GCT invariant action is of the form

$$L = L_4 - eL_4(\nabla_{\mu}^{\lambda}, \nabla_{\rho}^{\nu}, \phi^{h}, \bar{\Phi}^{h}, T^{km}, R^{km}, \phi^{e}, \phi^{s}, g, \bar{\nu}^{k})$$  \hspace{1cm} (6.8)

with

$$e = \det(e^{h}_{\mu}).$$  \hspace{1cm} (6.9)

This $L_4$ should satisfy the identity obtained from Eq. (5.23) by excluding the term $(\partial L_4/\partial e^{h}_{\mu})e_{1\nu}$ after the replacement $L_3 \rightarrow L_4.$

---

*The antisymmetric part $M_{\rho}^{\alpha\nu} := \eta_{\alpha\eta}M_{\rho}^{\mu\nu}$ is the orbital angular momentum density: For a Lorentz coordinate transformation $\delta x'' = \Omega_{\mu}^{\nu} x''$ with $\Omega_{\mu}^{\nu}$ being an infinitesimal constant parameter which satisfies $\Omega_{\mu}^{\mu} := \eta_{\mu\nu} \Omega_{\nu}^{\mu} = -2\Omega_{\mu}^{\mu}$, we have the relation $L + (\delta x'')_{\mu} L = \partial_{\nu}(1/2)(\Omega_{\mu}^{\nu}(\eta_{\alpha\eta}M_{\rho}^{\mu\nu}))$ provided that the field equations of all the gauge fields are satisfied.*
6.3. Invariant Lagrangian densities

We give the typical examples of $L_4$ in the following:

1) The Lagrangian density of PGT:

$$L = e(aR) + L_T + L_R$$

with

$$L_T = e(\alpha t_{kim} t_{km} + \beta \nu^k \nu_k + \gamma a^k a_k),$$

$$L_R = e(a_1 A_{kimn} A_{klmn} + a_2 B_{kimn} B_{klmn} + a_3 C_{kimn} C_{klmn}$$

$$+ a_4 E^{kl} E_{kl} + a_5 I^{kl} I_{kl} + a_6 R^2),$$

satisfies our requirements. In the above, $a$, $\alpha$, $\beta$, $\gamma$, $a_i (i=1 \sim 6)$ are all real constants, and $t_{kim}$, $\nu^k$, $a^k$ are the irreducible components of $T_{kim}$, and $A_{kimn}$, $B_{kimn}$, $C_{kimn}$, $E^{kl}$, $I^{kl}$, $R$ are those of $R^{klmn}$. The Einstein-Hilbert Lagrangian density is a special case of Eq. (6·10).

2) The Lagrangian density

$$L = e \xi^k \xi_k,$$  \hspace{1cm} (6·12)

which consists of the kinetic term of $\xi^k$, also satisfies our requirements. This reduces to the mass term of the boson field $c_r A_{\mu}^r$, in the gauge in which Eq. (5·10) holds.

3) Finally we present

$$L = \frac{1}{cs^2} \left( \frac{i}{2} \xi_{kimn} \nabla \xi_{klmn} \nabla \lambda \gamma_{\lambda_{\mu}} \frac{1}{2} \Psi_{mn} \right) + \frac{1}{2} c_s \frac{1}{2} \xi_{k} \xi_{l} \xi_{m} \xi_{n},$$  \hspace{1cm} (6·13)

which reduces to the Lagrangian density of the Rarita-Schwinger field $c_s \xi_{\mu}$ with the mass $c_s$, in the gauge in which Eq. (5·10) holds.

§ 7. Comments on choice of set of independent field variables

Besides the natural set $\{ X, \xi^k, \phi^k, \bar{\phi}_\mu, A^k_{\mu}, A^{kl}_{\mu, \nu}, \phi^\sigma, q \}$ (for short, $\{ \cdots, A^k_{\mu}, \cdots \}$) of independent field variables, we can employ an alternative set $\{ \lambda, \bar{\lambda}, \xi^k, \phi^k, \bar{\phi}_\mu, e^k_{\mu}, A^k_{\mu}, \phi^\sigma, q \}$ (for short, $\{ \cdots, e^k_{\mu}, \cdots \}$). Then we can express $L$ which satisfies Eq. (5·11b) as

$$L(\lambda, \bar{\lambda}, \xi^k, \phi^k, \bar{\phi}_\mu, e^k_{\mu}, A^k_{\mu}, A^{kl}_{\mu, \nu}, \phi^\sigma, \phi^\sigma_{\mu}, q, q_{\mu})$$

$$= \bar{L}(\lambda, \bar{\lambda}, c^k, \phi^k, \bar{\phi}_\mu, e^k_{\mu}, A^k_{\mu}, A^{kl}_{\mu, \nu}, \phi^\sigma, \phi^\sigma_{\mu}, q, q_{\mu}).$$  \hspace{1cm} (7·1)

Accordingly, we can define alternative field equations and the Noether currents using $\bar{L}$ and $\{ \cdots, e^k_{\mu}, \cdots \}$. We examine the relations of these alternatives to the corresponding ones defined with $L$ and $\{ \cdots, A^k_{\mu}, \cdots \}$.\footnote{In Refs. 3) and 7), the choice of the set of independent field variables has been discussed within the framework of PGT.}
(i) Field equations

We find the following relations between the Euler derivatives:

\[
\frac{\delta L}{\delta \lambda} = \frac{\delta \tilde{L}}{\delta \lambda} - 2 \frac{c_s}{c_s^2} \gamma^\lambda \gamma^\nu \frac{\delta \tilde{L}}{\delta e_{i k}^\mu} \frac{\partial}{\partial e_{i k}^\mu} + c_s \gamma^\lambda \left[ \partial_{\mu} \left( \frac{\delta \tilde{L}}{\delta e_{i k}^\mu} \right) + A_{i k}^\mu \left( \frac{\delta \tilde{L}}{\delta e_{i k}^\mu} \right) \right], \tag{7.2a}
\]

\[
\frac{\delta L}{\delta \xi_{i k}} = \frac{\delta \tilde{L}}{\delta \xi_{i k}} - \frac{c_s}{c_s^2} \left\{ \partial_{\mu} \left( \frac{\delta \tilde{L}}{\delta e_{i k}^\mu} \right) + A_{i k}^\mu \left( \frac{\delta \tilde{L}}{\delta e_{i k}^\mu} \right) \right\}, \tag{7.2b}
\]

\[
\frac{\delta L}{\delta \phi^i} = \frac{\delta \tilde{L}}{\delta \phi^i} - \frac{c_s}{c_s^2} \left\{ \partial_{\mu} \left( \frac{\delta \tilde{L}}{\delta e_{i k}^\mu} \right) + A_{i k}^\mu \left( \frac{\delta \tilde{L}}{\delta e_{i k}^\mu} \right) \right\}, \tag{7.2c}
\]

\[
\frac{\delta L}{\delta \phi^\nu} = \frac{\delta \tilde{L}}{\delta \phi^\nu} + \frac{c_s}{c_s^2} \gamma^\lambda \gamma^\nu \frac{\delta \tilde{L}}{\delta e_{i k}^\mu}, \tag{7.2d}
\]

\[
\frac{\delta L}{\delta A_{i k}^\mu} = c_s \frac{\delta \tilde{L}}{\delta e_{i k}^\mu}, \tag{7.2e}
\]

\[
\frac{\delta L}{\delta A_{i k}^\mu} = \frac{\delta \tilde{L}}{\delta A_{i k}^\mu} + \frac{1}{c_s^2} \frac{\partial}{\partial c_s^2} \left( c_s^2 (\phi_i^1 + c_s \phi_i^2) \frac{\delta \tilde{L}}{\delta e_{i k}^\mu} \right) + \frac{c_s^2}{c_s^4} \gamma^\nu \frac{i}{2} \sigma_{\mu k} \frac{\delta \tilde{L}}{\delta e_{i k}^\mu}, \tag{7.2f}
\]

\[
\frac{\delta L}{\delta \phi^\nu} = \frac{\delta \tilde{L}}{\delta \phi^\nu}, \tag{7.2g}
\]

\[
\frac{\delta L}{\delta q} = \frac{\delta \tilde{L}}{\delta q}. \tag{7.2h}
\]

Hence, the set of the hatted\footnote{We refer to the field equations (currents, \ldots) defined with $\tilde{L}$ and \{\ldots, $e_{i k}^\mu$, \ldots\} as the hatted field equations (currents, \ldots), and those defined with $L$ and \{\ldots, $A_{i k}^\mu$, \ldots\} as the unhatted ones.} field equations is equivalent to that of the unhatted\footnote{We refer to the field equations (currents, \ldots) defined with $\tilde{L}$ and \{\ldots, $e_{i k}^\mu$, \ldots\} as the hatted field equations (currents, \ldots), and those defined with $L$ and \{\ldots, $A_{i k}^\mu$, \ldots\} as the unhatted ones.} ones:

\[
\frac{\delta L}{\delta \lambda} = 0, \ldots, \frac{\delta L}{\delta A_{i k}^\mu} = 0, \ldots, \frac{\delta L}{\delta q} = 0 \iff \frac{\delta \tilde{L}}{\delta \lambda} = 0, \ldots, \frac{\delta \tilde{L}}{\delta A_{i k}^\mu} = 0, \ldots, \frac{\delta \tilde{L}}{\delta q} = 0. \tag{7.3}
\]

(ii) The Noether currents associated with gauge transformations

The hatted Noether currents are defined by

\[
q^\mu J_\mu := \frac{c_s^4}{c_s^2} \frac{\delta \tilde{L}}{\delta A_{i k}^\mu} + \frac{c_s^2}{c_s^2} \gamma^\lambda \gamma^\nu \frac{\delta \tilde{L}}{\delta e_{i k}^\mu} + \frac{i}{2} A_{i k}^\mu \left( \delta \frac{\tilde{L}}{\delta \phi_{\nu, \mu}} \right), \tag{7.4a}
\]

\[
p^\mu J_\mu := c_s \frac{\delta \tilde{L}}{\delta \xi_{i k}}, \tag{7.4b}
\]

\[
m^\mu J^\nu_{\kappa \lambda} := \frac{c_s^4}{c_s^2} \frac{\delta \tilde{L}}{\delta \phi_{\nu, \mu}} + \frac{c_s^2}{c_s^2} \gamma^\lambda \frac{\delta \tilde{L}}{\delta e_{i k}^\mu} + \frac{i}{2} \gamma^\nu \frac{\delta \tilde{L}}{\delta \phi_{\nu, \mu}}, \tag{7.4c}
\]

\[
+ \frac{\delta \tilde{L}}{\delta e_{i k}^\mu} \frac{\delta \tilde{L}}{\delta e_{i k}^\mu} + \frac{c_s^2}{c_s^4} \gamma^\nu \frac{i}{2} \sigma_{\mu k} \frac{\delta \tilde{L}}{\delta e_{i k}^\mu} + \frac{i}{2} \gamma^\nu \frac{\delta \tilde{L}}{\delta \phi_{\nu, \mu}} + \frac{i}{2} \gamma^\nu \frac{\delta \tilde{L}}{\delta \phi_{\nu, \mu}}.
\]
and these currents are related to the unhatted ones through

\[ qJ^\mu = q\tilde{J}^\mu + \partial_\nu \left( 2\frac{c_5}{c_s^2} \gamma^\lambda \frac{\partial \tilde{L}}{\partial \phi_{\mu,\nu}^h} \right) + 2\frac{c_5}{c_s^2} \gamma^\lambda \frac{\partial \tilde{L}}{\partial e_{\mu,\nu}^s}, \tag{7.5a} \]

\[ pJ^\mu = p\tilde{J}^\mu + \partial_\nu \left( c_5 \frac{\partial \tilde{L}}{\partial \phi_{\mu,\nu}^h} \right) + c_5 \frac{\partial \tilde{L}}{\partial e_{\mu}^h}, \tag{7.5b} \]

\[ M_{J_{\mu}} = M_{\tilde{J}_{\mu}} + \partial_\nu \left( \frac{2}{c_t^2} \frac{\partial \tilde{L}}{\partial e_{\mu,\nu}^h} \left( c_3 \phi_{\mu} + c_4 \xi_{\mu} \right) + 2\frac{c_5}{c_s^4} i \left( \frac{i}{2} \bar{\sigma}_{\mu} \right) \gamma^\lambda \frac{\partial \tilde{L}}{\partial e_{\mu,\nu}^s} \right) \]

\[ + \frac{2}{c_t^2} \frac{\partial \tilde{L}}{\partial e_{\mu}^h} \left( c_3 \phi_{\mu} + c_4 \xi_{\mu} \right) + 2\frac{c_5}{c_s^4} i \left( \frac{i}{2} \bar{\sigma}_{\mu} \right) \gamma^\lambda \frac{\partial \tilde{L}}{\partial e_{\mu,\nu}^s}. \tag{7.5c} \]

We obtain the notable result that the hatted current \( p\tilde{J}_{\mu} \) vanishes identically:

\[ p\tilde{J}_{\mu} = 0, \tag{7.6} \]

from the identity (5.12b) and the relations (7.2e), (7.5b) and \( \partial \tilde{L}/\partial e_{\mu,\nu}^h = (1/c_5) \partial L/\partial A_{\mu,\nu}^h \).

The differential conservation laws of \( q\tilde{J}^\mu \) and \( M_{\tilde{J}_{\mu}} \) are satisfied when the field equations of \( \phi_{\mu} \) and of \( A_{\mu}^h \) are satisfied, respectively.

(iii) The Noether currents associated with GCT

We define the hatted currents \( \tilde{\Theta}_{\mu} \) and \( \tilde{M}_{\mu} \) by

\[ \tilde{\Theta}_\mu = \phi_{\mu} \left[ \frac{1}{c_t^2} \left( c_3 \phi_{\mu} + c_4 \xi_{\mu} \right) + c_5 \frac{\partial L}{\partial \phi_{\mu}} \right] + \frac{1}{c_s^2} \left( c_3 \phi_{\mu} + c_4 \xi_{\mu} \right) \gamma^\lambda \frac{\partial L}{\partial e_{\mu}^s}. \tag{7.7} \]

\[ \tilde{M}_\mu = -2 \left( x^\sigma \tilde{\Theta}_\mu - S_\mu \right). \tag{7.8} \]

with

\[ S_\mu = \frac{1}{c_t^2} \left( c_3 \phi_{\mu} + c_4 \xi_{\mu} \right) + c_5 \frac{\partial L}{\partial e_{\mu}^s} A_{\mu}^h. \tag{7.9} \]

The differential conservation laws of these currents are satisfied when the field equations of \( \phi_{\mu}, e_{\mu}^h \) and \( A_{\mu}^h \) are all satisfied. From the definitions of \( \tilde{\Theta}_{\mu}, \tilde{M}_{\mu} \) and of \( M_{\mu}^\sigma, M_{\mu}^s \), we obtain the following relations:

\[ \tilde{\Theta}_\mu = \tilde{\Theta}_\mu - \partial_\alpha \left[ \frac{1}{c_t^2} \left( c_3 \phi_{\mu} + c_4 \xi_{\mu} \right) \gamma^\lambda \frac{\partial L}{\partial e_{\mu,\nu}^h} \right] + \frac{1}{c_s^2} \left( c_3 \phi_{\mu} + c_4 \xi_{\mu} \right) \gamma^\lambda \frac{\partial L}{\partial e_{\mu,\nu}^s}. \tag{7.10} \]

\[ M_\mu^\sigma = M_\mu^\sigma + \partial_\alpha \left[ 2x_\sigma \gamma^\lambda \frac{\partial L}{\partial e_{\mu,\nu}^h} \right] + \frac{1}{c_t^2} \left( c_3 \phi_{\mu} + c_4 \xi_{\mu} \right) \gamma^\lambda \frac{\partial L}{\partial e_{\mu,\nu}^s} + \frac{c_5}{c_s^4} \left( c_3 \phi_{\mu} + c_4 \xi_{\mu} \right) \gamma^\lambda \frac{\partial L}{\partial e_{\mu,\nu}^s}. \tag{7.11} \]
§ 8. Dirac field on flat spacetime

We have defined five kinds of currents: $\mathbf{J}^\mu$, $\mathbf{P}^\mu$, $\mathbf{M}^\mu_{kl}$, $\mathbf{\tilde{\Theta}}_\nu^\mu$, and $\mathbf{M}_\rho^{0\mu}$ and the corresponding hatted ones. To understand a part of the physical contents of these currents, we give their explicit expressions for a typical case: the Dirac field on the flat spacetime.) For this purpose, we first give the currents of the Dirac field on curved spacetime, and next reduce these currents to their special relativistic limits.**

In the present section, we restrict ourselves, for simplicity, to a coordinate system which tends to a Cartesian one in the special relativistic limit.

8.1. Special relativistic limit

We will consider a limit \( \varepsilon \to 0 \) in which the torsion and curvature tensors tend to zero:

\[
T^k_{\mu\nu} \to 0, \quad R^k_{\mu\rho\nu} \to 0,
\]

in the following way. We first introduce the constant \( \kappa \) which represents the coupling strength between the gauge fields and the Dirac spinor \( \psi \), and then we expand the fields as

\[
\lambda = \sqrt{\kappa} \lambda^{(1/2)} + \kappa \lambda^{(1)} + \cdots, \quad (8.2a)
\]

\[
\xi^k = \sqrt{\kappa} \xi^{(1/2)k} + \kappa \xi^{(1)k} + \cdots, \quad (8.2b)
\]

\[
\phi^k = \frac{1}{c_\phi} e^k_{\mu} e^{(0)k\mu} + \phi_c^{(0)k} + \sqrt{\kappa} \phi^{(1/2)k} + \kappa \phi^{(1)k} + \cdots, \quad (8.2c)
\]

\[
\phi_\mu = \sqrt{\kappa} \phi_\mu^{(1/2)} + \kappa \phi_\mu^{(1)} + \cdots, \quad (8.2d)
\]

\[
A^k_\mu = \sqrt{\kappa} A^{(1/2)k}_\mu + \kappa A^{(1)k}_\mu + \cdots, \quad (8.2e)
\]

\[
A^k_{\rho\mu} = \sqrt{\kappa} A^{(1/2)k}_{\rho\mu} + \kappa A^{(1)k}_{\rho\mu} + \cdots, \quad (8.2f)
\]

\[
\phi = \phi^{(0)} + \sqrt{\kappa} \phi^{(1/2)} + \kappa \phi^{(1)} + \cdots \quad (8.2g)
\]

with \( e^{(0)k}_\mu \) and \( \phi_c^{(0)k} \) being constants, where all the coefficients of \( \sqrt{\kappa}, \kappa, \cdots \) on the r.h.s.'s are independent of \( \kappa \). From these expansions, \( e^k_\mu \) is expanded as

\[
e^k_\mu = e^{(0)k}_\mu + \sqrt{\kappa} e^{(1/2)k}_\mu + \cdots, \quad (8.3)
\]

where the coefficients \( e^{(i)k}_\mu (i=1/2, 1, \cdots) \) are expressed in terms of the coefficients of the other fields through the "definition" (4.3) of \( e^k_\mu \). Then \( e^{(0)k}_\mu \) satisfies

\[
e^{(0)k}_\mu \eta_{kl} e^{(0)l}_\nu = \eta_{\mu\nu}, \quad e^{(0)} := \det(e^{(0)k}_\mu) = 1. \quad (8.4)
\]

*) The Dirac field on the flat spacetime has been discussed in PGT.\(^6\)

**) The procedure employed here is different from that in Ref. 8), where we start from the special relativistic Dirac Lagrangian density.
We find that Eq. (8·1) follows in the limit $\kappa \to 0$, which will be called the special relativistic limit (SRL).\(^*)\)

### 8.2. The Noether currents in special relativistic limit

We start from the Dirac Lagrangian density $L_D$ on curved spacetime:

$$L_D = e \left\{ -\frac{1}{2} (\bar{\psi} e^\kappa \sigma^\mu \overleftarrow{\nabla}_\mu \psi - \overrightarrow{\nabla}_\mu \bar{\psi} e^\kappa \sigma^\mu \psi) - m \bar{\psi} \psi \right\}$$  \hspace{1cm} (8·5)

with $\bar{\psi} : = -i\psi^\dagger \gamma^\rho$. This $L_D$ is manifestly super-Poincaré gauge invariant, and leads to the Dirac equation on the curved spacetime:

$$\left\{ e^\kappa \gamma^\mu \left( \overrightarrow{\nabla}_\mu - \frac{1}{2} v_\mu \right) + m \right\} \psi = 0$$  \hspace{1cm} (8·6)

with

$$v_\mu : = -\frac{1}{e} e^\kappa (ee^\kappa)_\mu - e_{\kappa \rho} e_{\omega} A^{\kappa \mu \omega}.$$  \hspace{1cm} (8·7)

In SRL, $L_D$ tends to the special relativistic Dirac Lagrangian density $L_{D}^{(0)}$:

$$\lim_{\kappa \to 0} L_D = L_{D}^{(0)} : = -\frac{1}{2} (\bar{\psi}^{(0)} e^{(0) \mu} \gamma^\rho \partial_\mu \psi^{(0)} - \partial_\mu \bar{\psi}^{(0)} e^{(0) \mu} \gamma^\rho \psi^{(0)}) - m \bar{\psi}^{(0)} \psi^{(0)},$$  \hspace{1cm} (8·8)

and Eq. (8·6) also tends to the special relativistic Dirac equation:

$$(e^{(0) \mu} \gamma^\rho \partial_\mu + m) \psi^{(0)} = 0.$$  \hspace{1cm} (8·9)

The Noether currents associated with super-Poincaré gauge transformations reduce to the following in the SRL:

$$^{\lambda}J^{(0) \mu} : = \lim_{\kappa \to 0} ^{\lambda}J^{\mu} = 0,$$  \hspace{1cm} (8·10a)

$$^{\nu}J^{(0) \mu} : = \lim_{\kappa \to 0} ^{\nu}J^{\mu}$$

$$= c e e^{(0) \kappa}\left\{ \delta_\mu^{\nu} L_{D}^{(0)} + \frac{1}{2} (\bar{\psi}^{(0)} e^{(0) \mu} \gamma^\rho \partial_\mu \psi^{(0)} - \partial_\mu \bar{\psi}^{(0)} e^{(0) \mu} \gamma^\rho \psi^{(0)}) - m \bar{\psi}^{(0)} \psi^{(0)} \right\},$$  \hspace{1cm} (8·10b)

$$^{M}J^{(k) \mu} : = \lim_{\kappa \to 0} ^{M}J^{k \mu}$$

$$= -\frac{i}{2} e^{(0) \mu \nu} e_{k \lambda \mu \nu} \bar{\psi}^{(0)} \gamma_\lambda \gamma_\mu \psi^{(0)} - 2 e^{(0) \mu \nu} \left\{ \delta_\mu^{\nu} L_{D}^{(0)} + \frac{1}{2} (\bar{\psi}^{(0)} e^{(0) \mu} \gamma^\rho \partial_\mu \psi^{(0)} - \partial_\mu \bar{\psi}^{(0)} e^{(0) \mu} \gamma^\rho \psi^{(0)}) - m \bar{\psi}^{(0)} \psi^{(0)} \right\}.,$$  \hspace{1cm} (8·10c)

The first is consistent with the fact that the Dirac field $\psi^{(0)}$ carries no super (fermionic) charge. The rest show that the Noether currents $^{\lambda}J^{\mu}$ and $^{M}J^{k \mu}$ give, up to constant factors, the correct expressions for the energy-momentum density and the total

\(^*) We employ the expansions (8·2a)~(8·2g) for which the Nambu-Goldstone fields and the gauge fields vanish in the limit, although some other expansions can realize Eq. (8·1).
Gravity as a Gauge Theory

\( (=\text{spin+orbital}) \) angular momentum density, respectively. The differential conservation laws of them follow:

\[ \partial_{\mu} J^{(0)\mu} = 0, \quad \partial_{\mu} J^{(0)\kappa}_{\kappa} = 0, \quad \partial_{\mu} J^{(0)\mu}_{\kappa l} = 0. \tag{8.11} \]

The first is trivial from Eq. (8.10a), and the rest are satisfied when Eq. (8.9) is satisfied.

On the other hand, the currents associated with GCT are easily shown to vanish in the SRL:

\[ \lim_{\kappa \to 0} \bar{\Theta}^{\mu}_{\kappa} = 0, \quad \lim_{\kappa \to 0} M_{\kappa}^{\rho \rho} = 0, \tag{8.12} \]

thus they carry no dynamical information.

Incidentally, we have also defined the hatted currents with an alternative set \( \{ \cdots, e_{\kappa}^\mu, \cdots \} \). Among them, we find that \( \hat{J}^\mu \) and \( \hat{J}^\kappa_{\kappa} \) vanish identically for the Dirac field \( \phi \), and easily obtain the limits of the other currents as

\[ M\hat{J}^{(0)}_{\kappa l} = \lim_{k \to 0} \hat{J}^{(0)}_{\kappa l} = -\frac{i}{2} e^{(0)\mu\nu} e_{\kappa l\kappa} \bar{\phi}^{(0)} \gamma_5 \gamma^\mu \phi^{(0)}, \tag{8.13} \]

\[ \bar{\Theta}^{(0)}_{\nu} = \lim_{k \to 0} \bar{\Theta}^{(0)}_{\nu} = \partial^\kappa L^D_{\kappa} + \frac{1}{2} (\bar{\phi}^{(0)} e^{(0)\kappa \mu} \gamma^\mu \partial_{\nu} \phi^{(0)} - \partial_{\nu} \bar{\phi}^{(0)} e^{(0)\kappa \mu} \gamma^\mu \phi^{(0)}), \tag{8.14} \]

\[ M^{(0)}_{\rho} = \lim_{k \to 0} M^{(0)}_{\rho} = -2 x^\mu \bar{\Theta}^{(0)\rho \mu}. \tag{8.15} \]

The differential conservation of the canonical energy-momentum density \( \bar{\Theta}^{(0)\rho \mu} \) follows when Eq. (8.9) is satisfied. On the other hand, the differential conservation laws of \( M\hat{J}^{(0)}_{\kappa l} \) and \( M^{(0)}_{\rho} \) do not follow even when the Dirac equation is satisfied.\(^*\) As is widely known, however, the differential conservation of the total angular momentum density

\[ \partial_{\nu} (e^{(0)\kappa \rho} \eta_{\nu\mu} (\tilde{M}^{(0)}_{\kappa} - M(0)_{\kappa l})_{\mu} - M\hat{J}^{(0)}_{\rho})_{\kappa} = 0, \tag{8.16} \]

follows when the Dirac equation (8.9) is satisfied, while each of the spin part \( - M\hat{J}^{(0)}_{\rho} \) and orbital part is not conserved by itself.

\[ \S 9. \quad \text{Summary and remarks} \]

We have constructed an effective gauge theory of gravity by applying the method of nonlinear realizations to the broken internal super-Poincaré symmetry. Now, we summarize the results and make a few remarks.

a) The \textit{internal} super-Poincaré symmetry does not require the super-partners of gauge bosons even when additional gauge interaction is incorporated besides the super-Poincaré one. Hence, our theory is consistent with the fact that photino, Wino, Zino, \cdots have never been observed. This is very different from the situation in spacetime supersymmetric field theories, where these particles are suppressed by the supersymmetry breaking.

\(^*\) This is consistent with the conservation laws of \( \hat{J}^{\rho l}_{\kappa} \) and \( M^{\rho \rho} \), because we should not use now (the limits of) the field equations of \( \bar{\phi}, e^{\rho l}_{\kappa} \) and \( A^{\kappa l}_{\rho} \).
b) The introduction of the internal super-Poincaré symmetry permits the existence of the particles which carry nonvanishing quantum numbers associated with the internal super-transformations (and translations). These quantum numbers, however, will not be observed in low energy experiments because of the symmetry breaking. When we assign the quarks and leptons to fields which carry no such quantum numbers, the super-partner of the quarks and leptons need not be introduced.

c) The Nambu-Goldstone fields $\lambda$ and $\xi^k$ "disappear" by virtue of the "(super-) Higgs mechanism". The fields $\phi^\sigma$ and $\phi_\mu$ are expected to have acquired heavy masses by the supersymmetry breaking (and/or not to have sufficiently strong interactions with observed fields), because $\phi^\sigma$ and $\phi_\mu$ have never been observed.

d) The co-frame $e^k_\mu$ has been constructed out of fundamental fields so that it leads to a gauge invariant metric.

e) In contrast to conventional supergravity, gravitino (=super-partner of $e^k_\mu$) is not needed because $e^k_\mu$ remains invariant under super-transformations. Correspondingly, the super gauge field $\psi_\mu$ in our scheme does not play the role of the super-partner.

f) The Lagrangian density $L$ has been restricted to be of the form given by Eq. (6.8) from the gauge and GCT invariance of the action integral $I$.

g) We have five kinds of the Noether currents: $^\alpha J^\mu (^\alpha \tilde{J}^\mu)$, $^P J^\kappa (^P \tilde{J}^\kappa)$, $^M J^k (^M \tilde{J}^k)$ associated with super-Poincaré transformations, and $^\Theta_\nu (^\Theta_\nu)$, $^M \rho (^M \rho)$ associated with GCT.

h) Using the natural set $\{\cdots, A^k_\mu, \cdots\}$, we have obtained the following for the Dirac field on the flat spacetime: (i) The Noether currents $^P J^\kappa$ and $^M J^k$ give (up to constant factors) the correct expressions for the energy-momentum density and the total angular momentum density, respectively. Note that these currents are associated with internal gauge transformations. (ii) $^\Theta_\nu = 0$, which is consistent with the fact that the Dirac field carries no super (fermionic) charge. (iii) $^M \rho$, $^M \rho$ are also vanishing, and thus the currents associated with GCT carry no dynamical information.

Acknowledgements

The author is grateful to Professor T. Kawai for valuable discussions and a careful reading of the manuscript.

Appendix

(a) Minkowski metric:

$$(\eta_{kl}) := \text{diag}(-+++), \quad (\eta^{kl}) := (\eta_{kl})^{-1}. \quad (A.1)$$
(b) $\gamma$-matrices:
- $\{\gamma^k, \gamma^l\} = 2\eta^{kl}$, \hspace{1cm} (A\cdot2)
- $\gamma^5 = \gamma^5 : = i\gamma_0 \gamma_1 \gamma_2 \gamma_3 = \frac{i}{4!} \epsilon^{klmn} \gamma_k \gamma_l \gamma_m \gamma_n$, \hspace{0.5cm} (A\cdot3)
- $\sigma_{kl} = \frac{1}{2} [\gamma_k, \gamma_l]$, \hspace{1cm} (A\cdot4)
- $\gamma^{\dagger} = \gamma^0 \gamma^k \gamma^l \gamma^5$, \hspace{0.5cm} (A\cdot5)
- $\gamma^5 = -\gamma^0$, \hspace{0.5cm} (A\cdot6)
- $\gamma^{\dagger} = \gamma^0$, \hspace{0.5cm} (A\cdot7)

(c) 4-component Majorana spinor
- Majorana conjugate:
  $\bar{\theta} = -\theta^T C^{-1}$. \hspace{1cm} (A\cdot7)
  ($\theta$: Majorana spinor, $C$: charge conjugation matrix)

  - Filip property:
    $\bar{\theta}_1 \gamma^k \gamma^l \cdots \gamma^m \theta_2 = (-1)^k \bar{\theta}_2 \gamma^m \cdots \gamma^l \gamma^k \theta_1$. \hspace{1cm} (A\cdot8)
    ($\theta_1, \theta_2$: Anti-commuting Majorana spinor)

Examples: $\bar{\theta}_1 \gamma^k \theta_2 = -\bar{\theta}_2 \gamma^k \theta_1$, $\bar{\theta}_1 \gamma^k \sigma_{lm} \theta_2 = \bar{\theta}_2 \sigma_{lm} \gamma^k \theta_1$. \hspace{1cm} (A\cdot9)

(d) Formulae\textsuperscript{16} for generators $Y, Z$
- $e^Y Z e^{-Y} = e^Y \wedge Z$, \hspace{1cm} (A\cdot10)
- $e^Y \delta e^{-Y} = \frac{1 - e^Y}{Y} \wedge \delta Y$. \hspace{1cm} (A\cdot11)

  ($Y \wedge Z := [Y, Z]$, $Y^2 \wedge Z := [Y, [Y, Z]]$, \ldots)

(e) Derivative with respect to an anti-symmetric field $A^{k \mu} = -A^{\mu k}$:
- $\frac{\partial A^i_j}{\partial A^{k \nu}} = \frac{1}{2} (\delta_k^i \delta^i_j - \delta^i_k \delta^\nu_j) \delta^\mu_\nu$. \hspace{1cm} (A\cdot12)

(f) Euler derivative
- $\frac{\delta L}{\delta f} = \frac{\delta L}{\delta f} - \partial_\mu \left( \frac{\delta L}{\delta f^\mu} \right)$. \hspace{1cm} (A\cdot13)

References

   For further developments, see M. Abe and N. Nakanishi, Prog. Theor. Phys. 80 (1988), 913, and references therein.
12) A. Salam and J. Strathdee, Phys. Rev. D11 (1975), 1521.
14) A. Salam and J. Strathdee, Phys. Rev. 184 (1969), 1760.
20) K. Hayashi and A. Bregman, Ann. of Phys. 75 (1973), 562.