All-Order No-Renormalization of the Mass and Interaction Lagrangians for Anti-de Sitter Supersymmetry

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We consider the interacting model of chiral and real gauge superfields in Anti-de Sitter space. The superconformal invariance of the massless model allows us to implement an expansion in the curvature effects in terms of the interaction vertices of the quantum model. In a previous application of the superfield formalism, the renormalization of the linear superfield term in the superpotential was derived. In the present work, it is argued that simple power counting arguments suffice to prove the no-renormalization of the mass and chiral self-interaction actions to all orders in perturbation theory.

§ 1. Introduction

The motivations of the present work originate from the results obtained in a series of papers which recently have been dealing with the quantum features of supersymmetry realized in four-dimensional Anti-de Sitter (AdS4) background space. The exploratory analysis carried in Ref. 1) shows that a one-loop renormalization of the mass lagrangian of the Wess-Zumino model, allowed for the theory in AdS4, in fact does not take place. There it is also verified that the interaction lagrangian does not get one-loop renormalized, as it is expected on purely dimensional grounds.

An anomaly with respect to the flat behaviour emerges from the work of Refs. 2) and 3): Part of the no-renormalization theorem holding in flat background4) suffers a breakdown in AdS4, due to the non-vanishing space time curvature. What happens is that the insertion of a linear superfield term in the action of the model, which is possible at the classical level, turns out to be necessary for the purpose of maintaining the renormalizability of the model when one-loop quantum corrections are included. The insertion of this counterterm does not produce the spontaneous breaking of the supersymmetry invariance of the model, as it is shown in Ref. 6) where boundary condition effects are not taken into account (for the treatment of the general case see Refs. 10) and 16) below). The above linear insertion introduces a violation of the no-renormalization theorem in what concerns the possibility of generating counterterms which, being allowed by the symmetries of the theory, yet were not included in the classical action.

Here we study the possibility of further breakdown of the theorem in AdS4 and examine the eventuality of a renormalization of the mass lagrangian, the only option allowed by dimensional analysis. Anyhow, our analysis turns out to be applicable immediately to the no-renormalization of the cubic interaction lagrangian as well.

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We consider the superfield formulation of the model with interacting chiral and real gauge superfields in AdS$_4$ following the line suggested in Ref. 5). We are able to implement a perturbative treatment of the effects of the background space by exploiting a partial superconformal invariance of the model. This perturbative analysis already allowed us to carry explicitly the one-loop renormalization of the chiral self-interacting model. The above expansion enables us now to analyze, to any order in the loop expansion, the generation of a mass or interaction counterterm. We argue that there is no renormalization of the mass nor of the cubic interaction action to all orders in perturbation theory and that the theorem holding in flat space-time should also be valid in AdS$_4$ background, except for the above mentioned linear superfield renormalization already present at the one-loop level. Our argument is easily extended to the model with gauge interactions.

In the previous literature it was erroneously claimed that the choice of boundary conditions for the field propagators influences the ultraviolet divergent counterterms of the theory. Under this misconception the same authors considered they had no other choice but to abandon this investigation. We were thus stimulated to carefully study the role of the so-called reflective boundary conditions in the renormalization of the model. We first showed that a mass term—which is boundary condition dependent—is radiatively generated in AdS$_4$. Then we carried out an exact (i.e., non-perturbative) computation of the one-loop effective action, including finite terms, proving that the choice of boundary conditions does not produce the breakdown of supersymmetry invariance.

The possibility of using different regularization prescriptions in order to preserve in the quantum calculations the supersymmetry invariance of the classical action was also explored. Alternatively, we introduced finite counterterms which violate supersymmetry and legitimate the use in the effective action of a supersymmetry-violating subtraction of divergences. Then, the proof that the boundary conditions needed for supersymmetry invariance to hold in AdS$_4$ cannot play any role in the renormalization of the effective action, irrespective of the order in the loop expansion, was given. This result holds more in general. In fact, using a technique of power series expansion, we proved that the trace anomaly is not affected by the choice of boundary conditions or by the classical vacuum around which the theory is quantized. After the publication of this result Dusedau et al. agreed with it. The universality of the renormalization coefficients for the purely geometrical tensors is shown with complete generality in Ref. 17).

From a previous investigation of the mass and self-interaction one-loop counterterms we obtain that the analysis of possible divergent renormalizations to higher loops may depart from the behavior of the theory in flat space time only because of the presence of internal lines in our formulation of the massless model and/or the boundary of the conformal mapping of AdS$_4$ into Minkowski space time. Those are features that result from taking into account the boundary conditions demanded by supersymmetry in AdS$_4$. Since we showed in our previous work that they cannot play any role in the renormalization of the theory irrespective of the order in the loop expansion, in carrying out, in the following, the analysis of the possible counterterms to all orders in perturbation theory we may neglect the above-mentioned features.
of the model depending on the choice of boundary conditions. There is a clear indication that the effect of taking these features into account will be restricted to finite terms in the higher-loop effective action. The generation of a finite mass term by one-loop corrections to the classical action of the massless model is an example of such an effect to the lowest order. This mass shift is related to the nonconservation of the axial charge implied by the boundary conditions. The order parameter of chiral symmetry breaking is calculated in Ref. 9).

This paper is organized as follows. Section 2 is devoted to giving the superfield formulation of the theory and to introduce the expansion in the background effects. In § 3 we describe our argument for the no-renormalization of the mass and interaction lagrangians. Finally, the conclusions of this work are drawn.

§ 2. The superfield formulation

In the functional integral approach to quantization, the Wess-Zumino model in AdS$_4$ is described by the generating functional

$$Z(J) = \int \mathcal{D} \eta \mathcal{D} \bar{\eta} e^{S(\eta, \bar{\eta}) + i \int d^4x d^4\theta \phi \bar{\eta} \eta + h.c.},$$

$$S(\eta, \bar{\eta}) = \int d^4xd^4\theta \phi \bar{\eta} \eta + \left( \frac{m}{2} \int d^4xd^2\theta \phi^3 \eta^2 + \frac{\lambda}{3!} \int d^4xd^2\theta \phi^3 \eta^3 + h.c. \right).$$

(1)

(2)

The chiral compensator $\phi$ is introduced in the action (2) to make it covariant with respect to the background. The equation of motion for the $\phi$ field in the chiral representation is

$$\bar{D}^2 \phi = a \phi^2$$

(3)

and has the regular solution

$$\phi = \frac{1}{1 - a \bar{\alpha} x^2} \frac{\bar{\alpha} \theta^2}{(1 - a \bar{\alpha} x^2)^2}.$$

(4)

The model possesses a partial superconformal invariance, what makes useful the introduction of a superconformal transformation according to the canonical weights of the fields

$$\bar{\eta} = \varphi \eta, \quad \bar{J} = \varphi^2 J.$$

(5)

This removes the explicit presence of the background in the description of the quantum model in terms of the transformed fields, except for a term proportional to $m$ which represents the deviation from an exact superconformal invariant theory.7) Defining $\xi = \phi - 1$ and splitting the action in a free-field part

$$S_0 = \int d^4xd^4\theta \bar{\eta} \eta + \left( \frac{m}{2} \int d^4xd^2\theta \eta^2 + h.c. \right).$$

(6)
and an interaction part
\[ S_{\text{int}} = \frac{m}{2} \int d^4 x d^2 \theta \xi \eta^2 + \frac{\lambda}{3!} \int d^4 x d^2 \theta \eta^3 + \text{h.c.}, \] (7)

we can derive a consistent description of the quantum model from the generating functional
\[ \tilde{Z}(\bar{f}) = Z(f) = \exp \left[ \frac{m}{2} \int d^4 x d^2 \theta \phi \bar{\phi} \left( \frac{\delta}{\delta f} \right)^2 + \frac{\lambda}{3!} \int d^4 x d^2 \theta \bar{\phi} \left( \frac{\delta}{\delta f} \right)^3 + \text{h.c.} \right] \times \int \mathcal{D} \bar{\eta} \mathcal{D} \eta \exp \left[ \int d^4 x d^2 \theta \bar{\eta} \eta + \left( \frac{m}{2} \int d^4 x d^2 \theta \bar{\eta}^2 + \int d^4 x d^2 \theta \bar{f} \eta + \text{h.c.} \right) \right]. \] (8)

The calculation of the free-field propagators can be carried out following the flat space-time analogue to obtain the momentum-space expressions
\[ \langle \bar{\eta} \eta \rangle_0 = -\frac{mD^2}{p^2(p^2 + m^2)} \delta^4(\theta - \theta'), \]
\[ \langle \bar{\eta} \eta \rangle_0 = \frac{1}{p^2 + m^2} \delta^4(\theta - \theta'). \] (9)

After taking into account the covariant functional derivative
\[ \frac{\delta \tilde{f}(z)}{\delta \tilde{f}(z')} = \frac{\phi(z)}{\phi(z')} \left( \frac{\delta}{\delta f} \right) \delta f(z), \]
\[ \frac{\delta \tilde{f}(z)}{\delta \tilde{f}(z')} = \frac{\phi(z)}{\phi(z')} \left( \frac{\delta}{\delta f} \right) \delta f(z'), \]
\[ = \frac{\phi(z)}{\phi(z')} \left( \frac{\delta}{\delta f} \right) \delta f(z), \] (10)
the naive contributions from Eq. (8) to the quadratic vertex
\[ \frac{m}{2} \phi^2 \bar{\xi}, \] (11)
and to the cubic vertex
\[ \frac{\lambda}{3!} \phi^3 \xi, \] (12)
in computing a generic derivative of the generating functional \( \tilde{Z}(\bar{f}) \) reduce to \((m/2)\bar{\xi}\) and to \(\lambda/3!\), respectively. We notice that the algebra of the \(D\)'s remains the flat space-time one and we conclude the equivalence of the model in AdS_4 background to the flat space time massive Wess-Zumino model with the addition of the quadratic vertex \((m/2)\bar{\xi}\) in the expression of the Feynman rules.

The above treatment actually removes the presence of \(\xi\) from the superconformal invariant part of the action providing, at the same time, an implicit perturbative
expansion in the effects of the non-trivial background. With respect to the expansion in $\varphi$ formerly used to calculate explicitly the one-loop renormalization of the superfield model,\(^7\) the $\xi$-approach described above presents the advantage that infrared singularities are manifestly absent to all perturbative orders and no place is left for ambiguities in identifying the counterterms of the quantum theory.

In fact, the only counterterms compatible with the supersymmetry realized in AdS\(_4\) are

\begin{equation}
\alpha \int d^4x d^2 \varphi^3 \eta = \int d^4x d^4 \varphi \bar{\eta} = \int d^4x d^4 \varphi \bar{\xi} \eta , \tag{13}
\end{equation}

\begin{equation}
\alpha \int d^4x d^2 \varphi^3 \eta^2 = \int d^4x d^4 \varphi^{-1} \bar{\xi} \eta^2 , \tag{14}
\end{equation}

\begin{equation}
\alpha \int d^4x d^2 \varphi^3 \eta^3 = \int d^4x d^4 \varphi^{-2} \bar{\xi} \eta^3 . \tag{15}
\end{equation}

We already know that the first one is actually obtained explicitly at the one-loop level.\(^7\) The others are the object of analysis in the present paper. It is worthwhile remarking that, while the renormalization of the cubic interaction term can be excluded on dimensional grounds, a renormalization of the mass lagrangian is still possible as long as the usual line of argumentation that leads to the nonrenormalization theorem in flat space time is not applicable in the present situation.

It is straightforward to generalize the above treatment to the case of interacting chiral and real gauge superfields. This model is described (in the gauge-chiral representation) by the action\(^6\)

\begin{equation}
S(\eta, \bar{\eta}, V) = \int d^4x d^4 \varphi \bar{\varphi} \eta e^\eta + \left( \int d^4x d^4 \varphi^3 W^2 + \text{h.c.} \right) + \left( \frac{m}{2} \int d^4x d^2 \varphi^3 \eta^2 + \frac{\lambda}{3!} \int d^4x d^4 \varphi^2 \eta^3 + \text{h.c.} \right) , \tag{16}
\end{equation}

where $W_a = i(\bar{\cal{D}}^a + a)e^{-V}V_a e^V$ and $V = V^A T_A$, $T_A$ being a matrix representation of the generators of the gauge group of invariance of the action (16). With respect to the expression (2), we have only introduced new terms which are superconformally invariant, and we can introduce the transformation of the source

\begin{equation}
\tilde{J}_V = \varphi \bar{\varphi} J_V \tag{17}
\end{equation}

and the rescaling

\begin{equation}
\tilde{W}_a = \varphi^{3/2} W_a \tag{18}
\end{equation}

together with (5) in order to reduce the superconformal part of the action (16) to its flat space time analogue in terms of $\tilde{\eta}$ and $\tilde{W}_a = i\bar{\cal{D}}^a e^{-V}D_a e^V$ (we remember at this point the expression of the background covariant derivatives\(^5\) $\cal{D}_a = \bar{\varphi} \varphi^{1/2} D_a$, $\bar{\cal{D}}_a = \varphi^{-1} \bar{\varphi}^{1/2} \bar{D}_a$ in terms of the flat space time ones $D_a$, $\bar{D}_a$).

In splitting the action we have now, in the interaction part, the extra term
so that the expression of the generating functional \( \tilde{Z}(\tilde{J}) \) in terms of the free-field functional integral \( Z_0 \) is

\[
\tilde{Z}(\tilde{J}) = \exp \int d^4 x d^4 \theta \sum_{m=0}^{\infty} \frac{V^m}{m!} \tilde{J}_m \times (\text{non-abelian interactions})
\]

\[
\times \exp \left[ \frac{m}{2} \int d^4 x d^2 \theta \varphi^3 \bar{\varphi}^3 \left( \frac{\delta}{\delta \tilde{J}_{\nu}} \right)^2 + \frac{\lambda}{3!} \int d^4 x d^2 \theta \varphi^3 \left( \frac{\delta}{\delta \tilde{J}_{\nu}} \right)^3 + \text{h.c.} \right] Z_0.
\]

Taking into account the covariant functional derivative\(^5\)

\[
\frac{\delta \tilde{J}_{\nu}(z)}{\delta \tilde{J}_{\nu}(z')} = \frac{\varphi(z) \bar{\varphi}(z)}{\varphi(z') \bar{\varphi}(z')} \frac{\delta \tilde{J}(z)}{\delta \tilde{J}(z')} = \frac{1}{\varphi(z) \bar{\varphi}(z)} \delta^4(z-z')
\]

we conclude that all the \( \varphi \)'s and \( \bar{\varphi} \)'s are absent in any vertex with chiral and gauge superfields. Once again, this is a consequence of the partial superconformal invariance of the model and leads to the conclusion that, after the superconformal redefinition, the system of \( N=1 \) super Yang-Mills coupled to a chiral superfield in AdS\(_4\) is equivalent to the corresponding flat space time theory provided the quadratic vertex \( (m/2)\xi \) is introduced in the expression of the Feynman rules.

\section*{§ 3. The no-renormalization theorem}

From what we have expressed in the preceding section it is clear that the renormalization of the mass or the cubic self-interaction term in the lagrangian demands, in terms or our perturbative approach in the curvature effects, recovering a very definite structure in the \( \xi \) and \( \bar{\xi} \) fields when computing the radiative corrections to these terms in the effective action. To be more precise, what we know is that the presence of a covariant divergent contribution to the \( \tilde{J} \) two-point function, for example, can be manifest only through a sum of \( \xi \) and \( \bar{\xi} \) insertions

\[
\varphi^{-1} \bar{\xi} = \sum_{n=0}^{\infty} \bar{\xi} (-1)^n \xi^n.
\]

Then, to prove at a given level in coupling constant perturbation theory that there is no divergent contribution with only a \( \bar{\xi} \) insertion (the first term in the expansion) is equivalent to show that the mass term cannot get renormalized.

For the sake of clearness in the exposition, we start giving the argumentation for the model only with chiral self-interactions, and then we extend the proof to the case in which gauge interactions are taken into account. We will suppose that some supersymmetric regularization procedure can be implemented in the computation of the radiative corrections and, in what follows, reference is made to a given order in the loop expansion of the \( \tilde{J} \) two-point function (the consideration of the \( \tilde{J} \) three-point function follows afterwards). Then, we begin by remarking that, in
general, inserting a $\xi$ or $\overline{\xi}$ field in the internal line of a flat space time Feynman graph improves the ultraviolet behavior of the diagram, while the $D$, $\overline{D}$-structure of the graph does not suffer any change. In fact, one can easily see that only when introducing a $\xi (\overline{\xi})$ vertex in a $\overline{\eta} \eta (\eta \overline{\eta})$ internal line the degree of divergence of the graph remains unchanged. This leads to the conclusion that a sufficient condition for the non-renormalization of the mass to hold is the absence of any Feynman graph without $\xi$ and $\overline{\xi}$ vertices that, after having performed the $D$-algebra, could lead to a divergent structure in momentum space (it is obvious that the chirality of the integrand that is used in flat space time to disregard these diagrams does not help in $\text{AdS}_4$ since now one has the possibility of building diagrams with $\overline{\xi}$ insertions that would make the integrand not chiral).

At this point, we would like to pose the problem of the search of these divergent structures in momentum space in terms of a similar problem for an equivalent scalar quantum field theory with cubic vertices. There are, however, two things that could prevent us from doing this identification. The first one is the presence of propagators for the $\overline{\eta} \eta$ or $\overline{\eta} \eta$ internal lines with abnormal ultraviolet behaviour (for what one would expect of a scalar field theory). And the second is the generation of some $p^2$ factors ($\square$ in $x$-space) when performing the $D$-algebra to contract the whole graph to a point in $\theta$, $\overline{\theta}$-space. Fortunately, one of the troubles helps to solve the other, since one can always do the $D$-algebra so that every $\overline{\eta} \eta$ or $\overline{\eta} \eta$ internal line ends with the corresponding $p^2$ factor. This converts these propagators into the ones with the expected ultraviolet behaviour. Furthermore, we remain in general with some other $p^2$ factors placed over $\overline{\eta} \eta$ internal lines. This ultraviolet behaviour is equivalent to that in which the internal line has been contracted to a point, leading to the generation of quartic vertices from the two cubic vertices that were joined by the internal line. Thus, we see that the ultraviolet behaviour of the original superfield graph is equivalent to the corresponding graph of a scalar field theory with cubic and quartic vertices.

Then, the last part of the proof for the model with chiral self-interactions consists in checking that none of these graphs can be divergent when the wave-function renormalization of the original superfield theory is taken into account. In principle, we can classify these graphs in two categories: primitive graphs, i.e., graphs that do not contain another $n$-point function as an insertion, and graphs that contain an insertion. To check the finiteness of the first class of graphs is a matter of triviality, since by assumption the corresponding contribution in momentum space has to be dimensionless (is a contribution to a $\int d^4 \theta \overline{\eta} \eta$ term in the effective action) and we know that to build an $\overline{\eta} \eta$ graph we need at least one $\overline{\eta} \eta$ (or $\overline{\eta} \eta$) internal line that carries a factor $m$. Then, the finiteness of these primitive graphs is ensured by a naive power counting of dimensions in the momentum integrals. Furthermore, we know that the graphs that are not primitive can be divergent only if they contain the insertion of a divergent $n$-point function. At this point we have to recall the equivalent scalar theory with cubic and quartic vertices to conclude that the only primitive divergent insertions that we can have are two, three and four-point functions. It is obvious, however, that, if the $D$-algebra was performed in a convenient way, the last types of insertions could be considered as contributions to terms of the type $\int d^4 \theta \overline{\eta}^3$ and $\int d^4 \theta \overline{\eta}^4$ respectively in the effective action. Once again, the power counting of
dimensions shows that these contributions are finite. Then, we are led to the conclusion that only the insertion of a two-point function can make a graph divergent and, moreover, by the argumentation given at the beginning of the paragraph we see that this is possible only if the insertion was an original $\bar{\eta}\eta$ two-point function. But this is the kind of divergence that is removed by the presence of the same graph in which the insertion has been replaced by the corresponding term of the wave-function renormalization counterterm. This leads to the end of the argument.

Up to this point, what we have given is an argument for the no-renormalization of the mass term

$$m\left(\int d^2 \theta \phi \bar{\eta} \eta + \int d^2 \bar{\theta} \bar{\phi} \bar{\eta} \bar{\eta}\right)$$

(23)

to any order in coupling constant perturbation theory, relying on the fact that there are no divergent contributions to the $\bar{\eta}\eta$ or $\bar{\eta}\bar{\eta}$ two-point functions in the flat space time model (before performing the superspace integration). A similar argument can be given for the $\bar{\eta}\eta \bar{\eta}$ or $\bar{\eta}\bar{\eta} \bar{\eta}$ three-point functions of the model. In fact, the only difference is that now primitive divergent contributions are excluded on dimensional grounds from the fact that they are contributions to a term of the type $\int d^4 \theta \bar{\eta} \eta \bar{\eta}$ in the effective action. Apart from this, one is led in the same way as before to the conclusion that is impossible to obtain a divergent expression in momentum space for any contribution to the $\bar{\eta}\eta \bar{\eta}$ or $\bar{\eta}\bar{\eta} \bar{\eta}$ three-point functions of the flat space time theory, which in turn implies the no-renormalization of the interaction term

$$\lambda \left(\int d^2 \theta \bar{\eta}^3 + \int d^2 \bar{\theta} \bar{\eta}^3\right)$$

(24)

to any order in coupling constant perturbation theory.

Next, we extend our argument for the no-renormalization of the mass and chiral self-interaction lagrangians when gauge interactions are switched on. The argumentation follows the same way as before, up to the point in which insertions in a $\bar{\eta}\eta$ two-point function or $\bar{\eta} \eta \bar{\eta}$ three-point function are considered. In this sense, we have now to take care of insertions of $n$-point functions with external gauge fields, but there is the obvious remark that the wave-function renormalization neither of the gauge supermultiplet nor of the chiral supermultiplet can be affected by the external background, as long as $\int d^4 \theta \bar{W}_a \bar{W}^a$ and $\int d^4 \theta \bar{\phi} e^\gamma \bar{\bar{\eta}}$ (in which $\phi$ or $\bar{\phi}$ do not appear) are the only covariant structures, in the sense of the gravitational background, that can be built. Even the possibility of generating a term of the type

$$\int d^4 \theta E^{-1} V^2$$

(25)

is ruled out, from the fact that the first term in the expansion

$$\int d^4 \theta \bar{\phi} \bar{\eta} V^2 = \int d^4 \theta (1 + \xi + \bar{\epsilon} + \xi \bar{\epsilon}) V^2$$

(26)

cannot be present by the cancellation of quadratic divergencies in the gauge super-
multiplet self-energy that takes place in flat space-time. Then, we can argue, as we did before, that the divergent part of every one of these wave-function renormalization insertions can be cancelled by the inclusion of a similar graph in which the insertion has been replaced by the corresponding term of the wave-function renormalization counterterm. Finally, we conclude again that, in the presence of the gauge superfield, there is no divergent structure in momentum space that can come out of a graph contributing to the \( \tilde{\eta} \tilde{\eta} \) two-point function or \( \tilde{\eta} \tilde{\eta} \tilde{\eta} \) three-point function in flat space-time. This leads, by means of the same argumentation given for the chiral model, to the no-renormalization of the mass and self-interaction lagrangians in AdS4.

§ 4. Discussion

In this work we have implemented an expansion in the effects of the AdS4 background curvature in terms of the interaction vertices of the chiral self-interacting model coupled to super Yang-Mills, in the superfield formulation of the theory. This is made possible by the existence of a partial superconformal invariance of the superfield action. Transforming both the chiral superfield and the field strength of the real gauge superfield (and the corresponding sources) according to their canonical weights, reduces the action of the interacting model to the flat space time action, plus a deviation from the exact superconformal invariant behaviour which would be absent in the massless case. The conclusion of the no-renormalization of the mass and the chiral self-interaction action is then attained, to every order in perturbation theory, by simple power counting analysis. The linear superfield insertion needed to renormalize the one-loop effective action\(^{2,3}\) appears to constitute then the only deviation (and a minor one) of the quantum features of the model in AdS4 from the flat space time model.

We have argued that the no-renormalization theorem holds in AdS4, in what concerns the mass and the chiral self-interaction action, as well as in flat background. In fact, in the context of perturbation theory, the only warning is the same as that usually given in flat space time about the possibility of invalidating the theorem through a pathological infrared-type behavior.\(^5\) It is worthwhile to remark that the no-renormalization of the mass of the chiral supermultiplet interacting with a Yang-Mills superfields, in the non-trivial background, opens the possibility of implementing the solution of the hierarchy problem in the context of the AdS4 supersymmetric field theories. Finally, we also mention that, in the light of the present work, the quantum features of an exact superconformal invariant theory like \( N=4 \) Yang-Mills in AdS4\(^5\) cannot suffer any modification with respect to the flat space-time theory. The transformations (5) and (17), in particular, would reduce the action of the model to the action in flat background, so that the conclusions that the \( \beta \)-function vanishes to all orders in perturbation theory and that \( N=4 \) Yang-Mills is a finite four-dimensional field theory hold in AdS4 as well as in flat space time.

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