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Fluctuations in the Waterlevel in Wells Due to Variations in Atmospheric Pressure

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Deviations from the classical theory of the barometric efficiency (Jacob 1940) is shown in some examples. A model with a semipermeable bed overlying the aquifer is proposed, and equations for the flow in this model, generated by changes in the total load, for instance changes in air pressure, are presented. Solutions are given for specified variations in the total load, and some data from nature are compared with one of these solutions. The data presented are from the archives of the Geological Survey of Denmark, Hydrogeological Department (DGU, HA).

Introduction

Drawdown data from aquifer tests in artesian aquifers are always influenced by changes in the atmospheric pressure, *P,* and it is often necessary to make a correction for this effect before the pumping test is analysed. Traditionally, the correction is based on the barometric efficiency *(BE)* of the well, which is defined by the elastic properties of the aquifer and the water on the assumption that the aquifer is over- and underlain by impermeable layers (Jacob 1940). In the aquifer, changes in load pressure induced by changes in atmospheric pressure are supported, partly by the water and partly by the aquifer skeleton. The fraction supported by the skeleton is constant (assuming Hooke's law valid for the elastic deformations), and is equal to the *BE* of a well screened in the aquifer. This relationship results in a direct proportionality between changes in P and changes of the well-water-level, h, corresponding to one of the straight lines through the origin in Figs. 1, 2, and 5. The direct proportionality is independent of the character of the changes and the selection of point of origin.

It is frequently observed, however, that the direct proportionality has only limited validity. Plots from a number of wells in Denmark illustrating this are shown in the Figs. 1, 2, and 5. The plots chosen represent periods where water-level variations induced by changes in P are believed to be absolutely dominating. The three plots to the right in Fig. 1 are from a chalk well, DGU file no. 187. 1016 in the northern part of Zealand, and the two plots to the left are from a sand-well, DGU file no. 106.638 west of Horsens, (Jutland). These plots show that the rate of change in P and the origin chosen will influence the computation of the *BE*. This can also be seen in Fig. 2 showing three plots from a sand well, DGU file no. 107.721 west of Horsens, Jutland. In this figure, however, it is important to notice the behaviour of W during oscillations in P. This behaviour clearly shows that the process has some kind of memory, a dependance of the past, comparable, for example, with the consolidation of clay-samples during load-unload-tests.

Some of the above mentioned effects have earlier been explained by somewhat more sophisticated modellings of the elastic properties of the system than used in the classical theory. Thus, some kind of time dependance must be introduced to account for the memory effect (Fig. 2). This may be done by considering hysteresis a part of the elastic properties of the aquifer. However, another way of introducing a hysteresis-like effect seems obvious and will be demonstrated in the following.

List of Symbols

a/b	thickness of aquifer/aquitard
B_1/B_2	barometric coefficient of aquifer/aquitard
C_1/C_2	tidal coefficient of aquifer/aquitard
E_1/E_2	modulus of elasticity of aquifer/aquitard
h	piezometric head
k_1/k_2	permeability of aquifer/aquitard
n_1/n_2	porosity of aquifer/aquitard
p	pore water pressure = γh
S_1	storage coefficient of aquifer
S'_2	specific storage coefficient of aquitard
γ	specific weight of water
σ	total load
σ_e	part of σ supported by the earth skeleton = $\sigma - p$

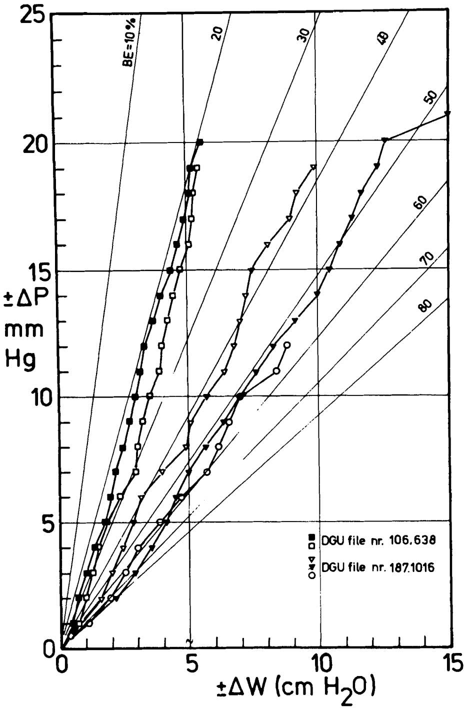

Fig. 1.

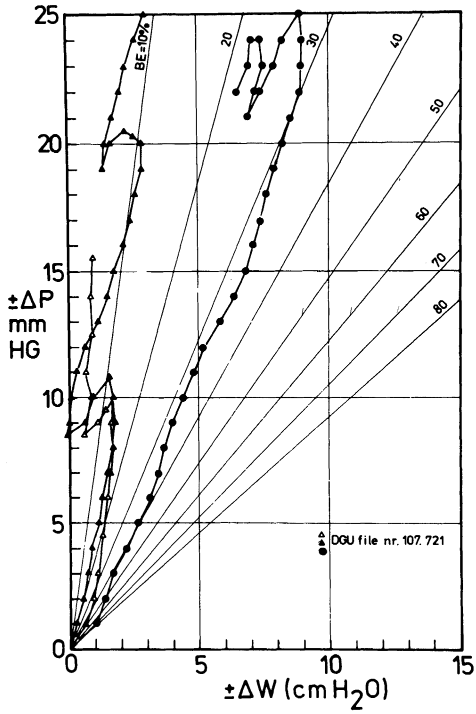

Fig. 2.

Basic Equations

Imagine an aquifer overlain by a saturated, semipervious bed on top of which some changes in the load takes place. The flow (vertical) in the semipervious layer will then be governed by the following equation: (y - direction positiv upward with zero at the top of the aquifer)

$$k_2 \frac{\partial^2 h}{\partial y^2} = \frac{n_2}{E_w} \frac{\partial p}{\partial t} - \frac{1}{E_2} \frac{\partial \sigma_e}{\partial t}$$

From $\sigma = \sigma(y, t) \quad \sigma_e + p$, we get

$$\frac{\partial \sigma_e}{\partial t} \quad \frac{\partial \sigma}{\partial t} - \frac{\partial p}{\partial t} = \frac{\partial \sigma}{\partial t} = \gamma \frac{\partial h}{\partial t}$$ which we introduce:

$$\frac{\partial^2 h}{\partial y^2} = \frac{S'_2}{k_2} \frac{\partial h}{\partial t} - \frac{1}{k_2 E_2} \frac{\partial \sigma}{\partial t} \qquad (1)$$

The boundary condition at $y = 0$ is:

$$k_2\frac{\partial h}{\partial y} \equiv S_1\frac{\partial h}{\partial t} - \frac{a}{E_1}\frac{\partial\sigma}{\partial t} \quad \text{or} \quad \frac{\partial h}{\partial t} = \frac{k_2}{S_1}\frac{\partial h}{\partial y} + \frac{C_1}{\gamma}\frac{\partial\sigma}{\partial t} \tag{2}$$

Using the same expression for $\partial\sigma_e/\partial t$.

The upper boundary condition is for simplicity in the computations chosen as

$$y \to \infty \,, \; h = \frac{C_2}{\gamma}\sigma(t) \,, \tag{3}$$

C_2 denoting the fraction of σ supported by the pore water. This form of the upper boundary condition ($b\to\infty$) is equivalent to the assumption that changes in h not significantly reaches the top of the aquitard.
Finally, at $t = 0$, we set

$$h \equiv \frac{C_2}{\gamma}\sigma(0) \,, \; y > 0 \qquad h \equiv \frac{C_1}{\gamma}\sigma(0) \,, \quad y = 0 \tag{4}$$

We now assume the solution to be of the form

$$h = \frac{C_2}{\gamma}\sigma(t) + A(y,t) \,, \tag{5}$$

A denoting the »deviations« caused by the flow in the system.
Introducing (5) in (1) - (4) yields:

$$\frac{\partial^2 A}{\partial y^2} - \frac{S'_2}{k_2}\frac{\partial A}{\partial t} \equiv 0 \tag{1a}$$

$$\frac{\partial A}{\partial t} - \frac{k_2}{S_1}\frac{\partial A}{\partial y} + \frac{(C_2-C_1)}{\gamma}\frac{\partial\sigma}{\partial t} = 0 \tag{2a}$$

$$\lim_{y\to\infty} A = 0 \tag{3a}$$

$$A(y,0) = 0, \quad y > 0 \qquad A(0,0) \equiv \frac{C_1-C_2}{\gamma}\sigma(0) \tag{4a}$$

Using Laplace-transformation, we get the transformed solution:

$$L[A] = \frac{C_1 - C_2}{\gamma}\,\frac{sL[\sigma]}{s+\sqrt{\dfrac{S'_2 k_2}{S_1^2}s}}\,\exp\left[-y\sqrt{\frac{S'_2}{k_2}s}\right] \tag{6}$$

$L[\sigma]$ denoting the Laplace transform of the load function σ.
Changing the general Eq. (5) into the special case of changes in atmospheric pressure, $P(t)$, h_w denoting the corresponding water-level changes in a well, can be done as follows:

$$h_w = -\frac{P(t)}{\gamma} + \frac{C_2}{\gamma}P(t) + A(0,t) = -\frac{1-C_2}{\gamma}P(t) - \frac{C_2-C_1}{\gamma}P(t)\beta(t) \quad ,$$

noting that $A(0,t) = -\frac{C_2-C_1}{\gamma}P(t)\beta(t)$ corresponds to the aquifer. Letting $C_2 = 1$ as will be the case for most claysoils, we get

$$h_w = -\frac{1-C_1}{\gamma}P(t)\beta(t) = -\frac{B_1}{\gamma}P(t)\beta(t) \tag{7}$$

which simplifies to the classical solution, when $\beta(t) = 1$, that is when $k_2 = 0$ in Eq. (6). On the basis of Eqs. (6) and (7) it is in principle possible to evaluate h_w for any variations in P or σ. In the following this will be demonstrated by some examples.

$$1: \quad \sigma_1(t) = \begin{cases} 0, & t < 0 \\ P_0, & t \geq 0 \end{cases} \quad , \quad L[\sigma_1] = \frac{P_0}{s}$$

$$L[A_1] = \frac{C_1-C_2}{\gamma}\frac{P_0}{s+\sqrt{\frac{S_2'k_2}{S_1^2}}\,s}\exp\left[-y\sqrt{\frac{S_2'}{k_2}s}\right] ,$$

$$A_1 = \frac{C_1-C_2}{\gamma}P_0\exp\left[\frac{S_2'}{S_1}y + \frac{k_2S_2'}{S_1^2}t\right]\operatorname{erfc}\left[\sqrt{\frac{k_2S_2'}{S_1^2}t} + \frac{y\sqrt{\frac{S_2'}{k_2}}}{2\sqrt{t}}\right]$$

$$h_w = -\frac{B_1}{\gamma}P_0\beta_1(t), \beta_1(t) = \exp\left[\frac{k_2S_2'}{S_1^2}\right]\operatorname{erfc}\left[\sqrt{\frac{k_2S_2'}{S_1^2}t}\right] \tag{8}$$

which is shown in Fig. 3.

$$2: \quad \sigma_2(t) \equiv \begin{cases} 0, & t < 0 \\ P_0t, & t \geq 0 \end{cases} \quad , \quad L[\sigma_2] = \frac{P_0}{s^2} \quad L[A_2] = \frac{1}{s}L[A_1]$$

$$A_2 = \frac{C_1-C_2}{\gamma}P_0\left\{\frac{2}{\sqrt{\pi}}\frac{\sqrt{t}}{\sqrt{\frac{k_2S_2'}{S_1^2}}}\exp\left[-\frac{1}{4}\frac{y^2S_2'}{tk_2}\right] - \left(\frac{S_1^2}{k_2S_2'} + \frac{S_1}{k_2}y\right)\right.$$

$$\operatorname{erfc}\left[\frac{y\sqrt{\frac{S_2'}{k_2}}}{2\sqrt{t}}\right] + \frac{S_1^2}{k_2S_2'}\exp\left[\frac{S_2'}{S_1}y + \frac{k_2S_2'}{S_1^2}t\right]$$

$$\left.\operatorname{erfc}\left[\sqrt{\frac{k_2S_2'}{S_1^2}t} + \frac{y\sqrt{\frac{S_2'}{k_2}}}{2\sqrt{t}}\right]\right\} ,$$

$$\beta_2(t) = \frac{S_1^2}{k_2 S_2' t} \left\{ \frac{2}{\sqrt{\pi}} \sqrt{\frac{k_2 S_2' t}{S_1^2}} - 1 + \exp\left[\frac{k_2 S_2' t}{S_1^2} \right] \right.$$

$$\left. \operatorname{erfc}\left[\sqrt{\frac{k_2 S_2' t}{S_1^2}} \right] \right\} \qquad (9)$$

which is also shown in Fig. 3.

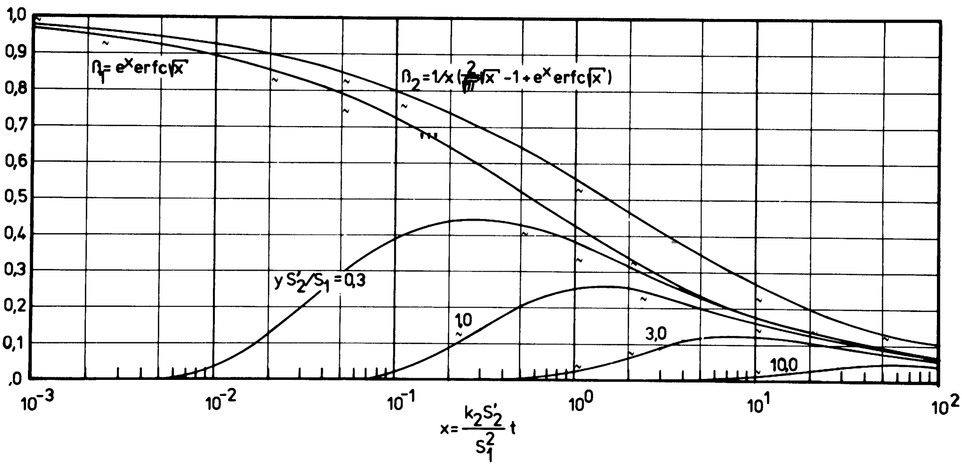

Fig. 3.

3: $\sigma_3(t) = \sum_{m=1}^{\infty} (d_m \cos\omega m t + e_m \sin\omega m t)$, (Fourier series), result in

$$A_3 = \frac{C_1 - C_2}{\gamma} \sum_{m=1}^{\infty} \frac{1}{b_m^2 + (1+b_m)^2} \exp[-a_m y]$$

$$\{(b_m e_m + (1+b_m) d_m)\cos(\omega m t - a_m y) - (b_m d_m - (1+b_m) e_m)$$

$$\sin(\omega m t - a_m y)\} \qquad (10)$$

which is another Fourier series with changed amplitude and phase.

$$a_m = \sqrt{\frac{m\omega S'_2}{2k_2}}, \quad b_m = \frac{1}{S_1}\sqrt{\frac{k_2 S'_2}{2m\omega}}$$, and d_m, e_m and ω

are standard Fourier constants.

4: $\sigma_4(t) = P_0 \sin\omega t$ yields therefore

$$h_w = -\frac{B_1}{\gamma} P_0 \frac{1}{b^2+(1+b)^2} (b\cos\omega mt + (1+b)\sin\omega mt) =$$

$$-\frac{B_1}{\gamma} \frac{P_0}{\sqrt{b^2+(1+b)^2}} \sin(\omega mt+\theta) , \qquad (11)$$

where $\tan\theta = \frac{b}{1+b}$ and $b = \frac{1}{S_1}\sqrt{\frac{k_2 S'_2}{2\omega}}$

The dependance of $\frac{1}{\sqrt{b^2+(1+b)^2}}$ (the ratio of the amplitude with semipervious upper stratum to the amplitude with impervious upper stratum)and $\theta/\pi/4$ on b is shown in Fig. 4. It is seen, that for $b > 10^{-1}$ the fluctuations of h_W will be clearly influenced by the semipervious stratum.

The upper boundary condition Eq. (3), (3a) will not always be satisfactory. If however we use the more correct boundary condition

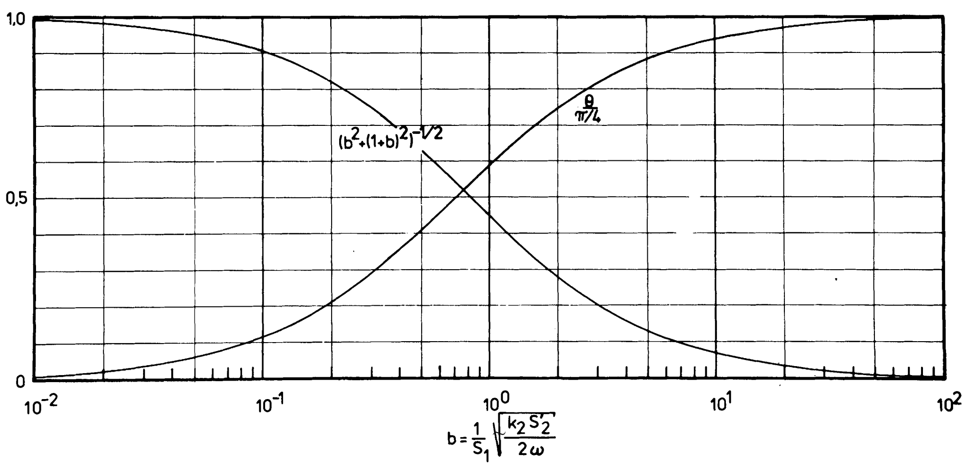

Fig. 4.

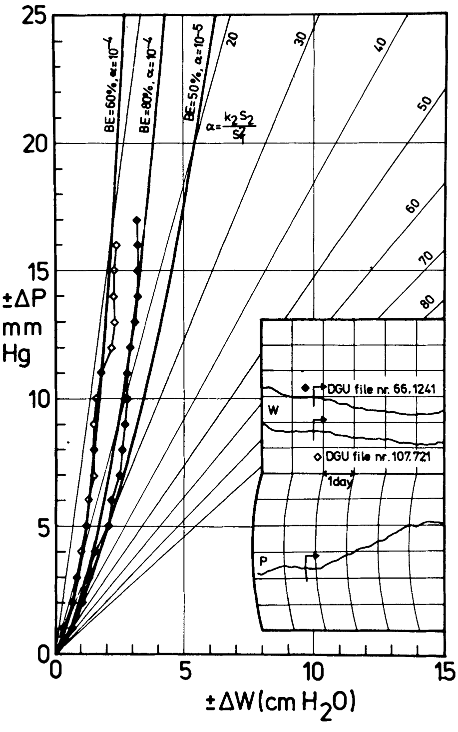

Fig. 5.

$$y = b\ ,\quad h = \frac{C_2}{\gamma}\sigma(t)\ , \qquad (3')$$

the solution becomes rather complicated to handle, and an additional parameter will show up in the final expression. Therefore the following simpler way to illustate the limitations in the »general« Eq. (6) has been used:

In case 1, where $\sigma(t)$ is the unit step function, the corresponding variations in piezometric head has been computed for four values of $y(S_2'/S_1)$. The resulting four curves are plotted in Fig. 3, showing the variation in piezometric head for different levels in the semipervious stratum. For instance, if $S_2'/S_1 = 0.1$, the changes in h will have reached a height of 10 meters in the clay when $x = S_2'k_2\,t/S_1^2 = 10^{-1}$. If $k_2/S_1 = 10^{-4}$ this will happen at $t = 10^4$ sec. or 2.78 hours. After this time, the flow pattern in the region $0 \leq y < 10$ m will depend on an eventual boundary condition at $y = 10$ m, though some additional time must be expected to pass, before the effect reaches y = 0 (aquifer).

Finally, water-level variations in two wells, DGU file no. 66.1241, and 107.721, observed during a period of almost linear increase in atmospheric pressure, has been plotted in Fig. 5 (the traditional way), to be compared with some curves, based on Eq. (9). These curves clearly provides a better fit than the classical straight lines. The upward deviation from the curves in one of the data-plots, is caused by the effect of an upper boundary condition, the semipervious layer being only a few meters thick in this locality.

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