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Fuzzy waste load allocation model: a multiobjective approach

Subimal Ghosh and P. P. Mujumdar

ABSTRACT

Fuzzy Waste Load Allocation Model (FWLAM), developed in an earlier study, derives the optimal fractional levels, for the base flow conditions, considering the goals of the Pollution Control Agency (PCA) and dischargers. The Modified Fuzzy Waste Load Allocation Model (MFWLAM) developed subsequently is a stochastic model and considers the moments (mean, variance and skewness) of water quality indicators, incorporating uncertainty due to randomness of input variables along with uncertainty due to imprecision. The risk of low water quality is reduced significantly by using this modified model, but inclusion of new constraints leads to a low value of acceptability level, λ, interpreted as the maximized minimum satisfaction in the system. To improve this value, a new model, which is a combination of FWLAM and MFWLAM, is presented, allowing for some violations in the constraints of MFWLAM. This combined model is a multiobjective optimization model having the objectives, maximization of acceptability level and minimization of violation of constraints. Fuzzy multiobjective programming, goal programming and fuzzy goal programming are used to find the solutions. For the optimization model, Probabilistic Global Search Lausanne (PGSL) is used as a nonlinear optimization tool. The methodology is applied to a case study of the Tunga–Bhadra river system in south India. The model results in a compromised solution of a higher value of acceptability level as compared to MFWLAM, with a satisfactory value of risk. Thus the goal of risk minimization is achieved with a comparatively better value of acceptability level.

Key words | conflicting objectives, fuzzy optimization, goal programming, multiobjective programming, Probabilistic Global Search Lausanne (PGSL), water quality

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INTRODUCTION

Waste load allocation (WLA) in streams refers to the determination of required pollutant treatment levels at a set of point sources of pollution to ensure that water quality standards are maintained throughout the stream. Water quality management problems are characterized by various types of uncertainties at different stages of the decision-making process to arrive at the optimal allocation of the assimilative capacity of the river system. The two types of uncertainties that influence the decision-making process are uncertainty due to randomness and uncertainty due to imprecision. Uncertainty due to randomness arises mainly due to the random nature of the input variables used in the water quality simulation model. Uncertainty due to imprecision or fuzziness is associated with describing the goals related to water quality and pollutant abatement.

There are three widely adopted approaches for addressing randomness in water-quality management models (Takyi & Lence 1999). These are (i) chance-constrained optimization (Lohani & Thanh 1978, 1979; Burn & McBean 1985, 1986; Ellis 1987; Fujiwara *et al.* 1986, 1987), (ii) combined simulation–optimization (Burn 1989;

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Takyi & Lence 1994) and (iii) multiple realization approach (Burn & Lence 1992; Takyi & Lence 1999). Another type of uncertainty prominent in the management of water quality systems is uncertainty due to imprecision or fuzziness associated with describing the goals related to water quality and pollutant abatement. Sasikumar & Mujumdar (1998, 2000) and Mujumdar & Sasikumar (2002) have addressed the uncertainty due to imprecision as well as randomness in a multiobjective framework. Fuzzy logic has been used for water quality management to model imprecision by Zhu *et al.* (2009) and Lermontov *et al.* (2009). Recently, uncertainty resulting from the inexactness of parameter values in water quality management models has been addressed in Karmakar & Mujumdar (2007) and Nie *et al.* (2008).

Starting with the FWLAM (Sasikumar & Mujumdar 1998) uncertainty due to randomness is incorporated in MFWLAM (Ghosh & Mujumdar 2006) by considering the basic statistics of the water quality indicator in the optimization model. The model considers the first three moments along with Chebyshev's inequality to derive the optimal fractional removal levels. Inclusion of skewness and Chebyshev's inequality in MFWLAM requires two additional set of constraints. Incorporation of the new constraints, however, leads to a low value of acceptability level, λ, which is interpreted as the maximized minimum satisfaction in a system with conflicting objectives. To improve the acceptability level, λ, a multiobjective model is developed in the present paper, allowing some violation in the new constraints, considering objectives of minimization of violations and maximization of acceptability level, λ. Fuzzy multiobjective programming (Zimmermann 1978), goal programming and fuzzy goal programming (Pal *et al.* 2003) are applied to solve the problem. A backward finite difference scheme of transport equation is used for the BOD-DO model. The basic statistics are derived from Monte Carlo simulation, which is intrinsic in the optimization model MFWLAM. Probabilistic Global Search Lausanne (PGSL), a direct stochastic algorithm for global search, developed by Raphael & Smith (2000), is used as an optimization tool for nonlinear optimization. The following sections provide a brief overview of FWLAM and MFWLAM, based on which the current work is developed.

FUZZY WASTE LOAD ALLOCATION MODEL

The fuzzy waste load allocation model (FWLAM) developed by Sasikumar & Mujumdar (1998) forms the basis for the optimization models developed in this study. The FWLAM is described using a general river system. The river consists of a set of dischargers that are allowed to release pollutants into the river after removing some fraction of the pollutants. These fractional removal levels of the pollutants are necessary to maintain an acceptable water quality condition in the river as prescribed by the pollution control agency (PCA). The acceptable water quality condition is ensured by checking the water quality in terms of water quality indicator levels (e.g. DO concentration) at a finite number of locations referred to as checkpoints. The following fuzzy optimization problem is formulated to take into account the fuzzy goals of the PCA and dischargers, which are in conflict with each other:

$$\text{Maximize } \lambda \tag{1}$$

$$\text{subject to} \quad \left[(c_{il} - c_{il}^{L})/(c_{il}^{L} - c_{il}^{D})\right]^{\alpha_{il}} \geq \lambda \quad \forall i, l \tag{2}$$

$$\left[\left(x_{imn}^{M} - x_{imn}\right)/\left(x_{imn}^{M} - x_{imn}^{\mathrm{L}}\right)\right]^{\beta_{imn}} \geq \lambda \quad \forall i, m, n \tag{3}$$

$$c_{il}^{L} \leq c_{il} \leq c_{il}^{D} \quad \forall i, l \tag{4}$$

$$\max\left[x_{imn}^{L}, x_{imn}^{\mathrm{MIN}}\right] \leq x_{imn} \leq x_{imn}^{\mathrm{MAX}} \quad \forall i, m, n \tag{5}$$

$$0 \leq \lambda \leq 1 \tag{6}$$

where c_{il} is the concentration level of water quality indicator i at the checkpoint l of the river system. The PCA sets a desirable level, c_{il}^{D} and a minimum permissible level, c_{il}^{L}, for the water quality indicator i at the checkpoint l ($c_{il}^{D} \leq c_{il}^{L}$) which form the bounds of c_{il} as shown in crisp constraint (4). Similarly, x_{imn} is the fractional removal level of the pollutant n from the discharger m to control the water quality indicator i in the river system. The aspiration level and maximum fractional removal level acceptable to the discharger m with respect to x_{imn} are represented as x_{imn}^{L} and x_{imn}^{M}, respectively. The PCA imposes minimum fractional removal levels that are also expressed as the lower bounds, x_{imn}^{MIN} in constraint (5). Constraint (2) presents the fuzzified goal of PCA, which is a representation of the fuzzy statement "the higher the water quality, the better the satisfaction of the PCA". The membership function presented in the left-hand side of constraint (2) may be interpreted as the variation of satisfaction level of the pollution control agency with respect to the water quality indicator concentration, c_{il}. The satisfaction of the pollution control agency increases as the concentration level, c_{il}, of the water quality indicator i at the checkpoint l approaches the desirable limit, c_{il}^D, starting with the minimum permissible level, c_{il}^L. The exponent α_{il} defines the shape of the membership function and is decided by the pollution control agency. Constraint (3) presents the fuzzified goal of the dischargers with the membership function, which may be interpreted as the variation of the satisfaction level of the discharger m in treating the pollutant n to control the water quality indicator i in the river system. The fuzzy constraint (3) can be treated as the mathematical representation of the statement "the lower the treatment, the better the satisfaction of the dischargers". The exponent β_{imn} defines the shape of the membership function, and is decided by the discharger m. The fuzzy constraints (2) and (3) are in conflict with each other. The decision variables are the fractional removal levels of different dischargers and the acceptability level λ. The exponents α_{il} and β_{imn}, appearing in constraints (2) and (3), respectively, are nonzero positive real numbers. Assignment of numerical values to these exponents is subject to the desired shape of the membership functions and may be chosen appropriately by the decision-maker. In the present study the value of the exponent is selected as 1 for deriving the linear membership function. The concentration of water quality indicator c_{il} in constraints (2) and (4) is determined using a water quality simulation model.

MODIFIED FUZZY WASTE LOAD ALLOCATION MODEL

The Modified Fuzzy Waste Load Allocation Model (MFWLAM) (Ghosh & Mujumdar 2005, 2006) incorporates randomness of input variables by introducing mean, variance and skewness of the water quality indicator. The goal of this model is not only to determine the fractional removal levels of the effluents considering the aspirations and conflicting objectives of the pollution control agency and dischargers, but also to improve the water quality by incorporating the skewness of the probability density function of the water quality indicator.

MFWLAM does not consider the base values of the input variables. It is a stochastic optimization model that includes the moments of the distribution. The model is based on fuzzy decision theory as does FWLAM. To improve the water quality, the membership function of the skewness of the water quality indicator is also incorporated in the model. The concept, "the higher the skewness the better" or "the higher the skewness the worse" is modeled through fuzzy logic by choosing appropriate membership functions for the skewness resulting from optimization. The nature of the membership function for skewness is selected depending on the water quality indicator. In the higher range of Dissolved Oxygen (DO) concentration, for example, a high frequency is desired and thus negative skewness is preferred for DO (Figure 1). A non-increasing membership function is thus assumed for the skewness of DO. Similarly for BOD, high frequency is desired in the lower range of BOD values and thus positive skewness is preferred. A non-decreasing membership function is used for BOD.

The bounds of the water quality indicator are determined from Chebyshev's inequality. According to Chebyshev's inequality, the proportion of observations lying k standard deviation outside the mean value is at most $1/k^2$, which can be mathematically stated by

$$P(|Z - \bar{Z}| \geq k\sigma) \leq \frac{1}{k^2} \quad (7)$$

where Z = a random variable; $\bar{Z}$ = mean value of Z; σ = standard deviation and $k \geq 0$.

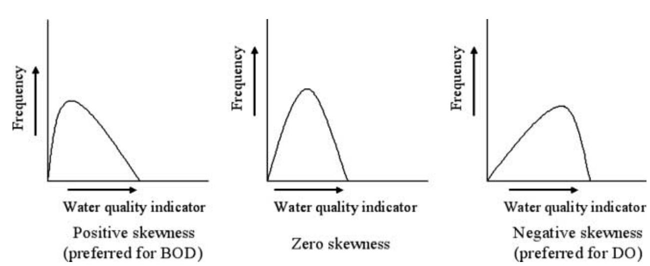

Figure 1 | Skewness of distribution of water quality indicators.

From Chebyshev's inequality:

$$P(Z \leq \bar{Z} - k\sigma) + P(Z \geq \bar{Z} + k\sigma) \leq \frac{1}{k^2} \quad (8)$$

$$P(Z \leq \bar{Z} - k\sigma) \leq \frac{1}{k^2}. \quad (9)$$

Replacing Z by the water quality indicator c_{il}:

$$P(c_{il} \leq \bar{c}_{il} - k\sigma_{c_{il}}) \leq \frac{1}{k^2} \quad \forall i, l \quad (10)$$

In the present model the lower bound of the water quality indicator is modified as follows:

$$c_{il}^L \leq (\bar{c}_{il} - k\sigma_{c_{il}}) \quad \forall i, l. \quad (11)$$

This ensures that the probability of water quality indicator level less than the acceptable level set by PCA is at most $1/k^2$:

$$P\left(c_{il} \leq c_{il}^L\right) \leq \frac{1}{k^2} \quad \forall i, l. \quad (12)$$

Finally the MAX–MIN formulation of the model can be given by

$$\text{Maximize} \quad \lambda \quad (13)$$

$$\text{subject to} \quad \left[\left(\bar{c}_{il} - c_{il}^L\right) / \left(c_{il}^L - c_{il}^D\right)\right]^{\alpha_{il}} \geq \lambda \quad \forall i, l \quad (14)$$

$$\left[\left(x_{imn}^M - x_{imn}\right) / \left(x_{imn}^M - x_{imn}^L\right)\right]^{\beta_{imn}} \geq \lambda \quad \forall i, m, n \quad (15)$$

$$\mu(s_{c_{il}}) \geq \lambda \quad \forall i, l \quad (16)$$

$$c_{il}^L \leq (\bar{c}_{il} - k\sigma_{c_{il}}) \quad \forall i, l \quad (17)$$

$$\bar{c}_{il} \leq c_{il}^D \quad \forall i, l \quad (18)$$

$$\max\left\lfloor x_{imn}^L, x_{imn}^{\text{MIN}} \right\rfloor \leq x_{imn} \leq x_{imn}^{\text{MAX}} \quad \forall i, m, n \quad (19)$$

$$0 \leq \lambda \leq 1 \quad (20)$$

where $\mu(s_{c_{il}})$ = membership function for the skewness of water quality indicator i at checkpoint l. The DO values generated with Monte Carlo simulations for a single checkpoint are used for the calculation of the skewness of DO at that checkpoint. For the solution of the water quality simulation model a backward finite difference method is used. The probability density function of the water quality indicator is derived from Monte Carlo simulations. For nonlinear optimization Probabilistic Global Search Lausanne (PGSL), a global search algorithm, is applied. Details of the algorithm may be found in Raphael & Smith (2003).

COMBINATION OF TWO MODELS: A MULTIOBJECTIVE APPROACH

In MFWLAM, some of the constraints of FWLAM are modified and new constraints are included. The constraints (2) and (4) are modified to (14) and (18). As the bias (difference between the base value obtained from the deterministic simulation model and the simulated mean obtained from Monte Carlo simulation) is very small for water quality indicators (Subbarao *et al.* 2004), constraints (14) and (18) are quite similar to (2) and (4) of FWLAM and thus will not make any difference to the results. But inclusion of extra constraints for the membership function for the skewness (constraint (16)) and Chebyshev's inequality (constraint (17)) lead to a low value of acceptability level. To satisfy the fuzzy constraints dealing with the membership function of skewness, λ will take a lower value. Due to the inclusion of Chebyshev's inequality the fractional removal level will increase which in turn reduces the value of λ. In the present model, allowing some violations in the above-mentioned two constraints, a multiobjective programming technique is developed to improve the λ value with minimum violations of the new constraints. The model has two objectives: (i) maximization of the acceptability level and (ii) minimization of the violation of the two constraints (16) and (17). These two objective functions are conflicting with each other as the consideration of constraints (16) and (17) reduces the value of the acceptability level. The formulation of the model can be given by

$$\text{Maximize} \; \lambda \quad (21)$$

$$\text{Minimize } v_1 \tag{22}$$

$$\text{Minimize } v_2 \tag{23}$$

$$\text{subject to } \left[\left(\bar{c}_{il} - c_{il}^{L}\right)/\left(c_{il}^{L} - c_{il}^{D}\right)\right]^{\alpha_{il}} \geq \lambda \quad \forall i, l \tag{24}$$

$$\left[\left(x_{imn}^{M} - x_{imn}\right)/\left(x_{imn}^{M} - x_{imn}^{L}\right)\right]^{\beta_{imn}} \geq \lambda \quad \forall i, m, n \tag{25}$$

$$c_{il} \leq c_{il}^{D} \quad \forall i, l \tag{26}$$

$$\max\left[x_{imn}^{L}, x_{imb}^{\text{MIN}}\right] \leq x_{imn} \leq x_{imn}^{\text{MAX}} \quad \forall i, m, n \tag{27}$$

$$0 \leq \lambda \leq 1 \tag{28}$$

where v_1 is the violation of constraint (16) and is given by

$$v_1 = \begin{cases} \lambda - \mu(s_{c_{il}}) & \text{when} \quad (\lambda - \mu(s_{c_{il}})) \geq 0 \\ 0 & \text{else} \end{cases} \tag{29}$$

Similarly, v_2 is the violation of constraint (17) and is given by

$$v_2 = \begin{cases} c_{il}^{L} - (\bar{c}_{il} - k\sigma_{c_{il}}) & \text{when} \quad (c_{il}^{L} - (\bar{c}_{il} - k\sigma_{c_{il}})) \geq 0 \\ 0 & \text{else} \end{cases} \tag{30}$$

As the newly introduced two constraints (16) and (17) are relaxed in the model, it will lead to a higher value of minimum acceptability level as compared to MFWLAM. Inclusion of the objective functions of minimization of violations of the two constraints will also lead to better water quality as compared to the FWLAM, as the skewness of water quality and Chebyshev's inequality are now involved in the model. Different multiobjective programming techniques are used to solve the problem. Fuzzy multiobjective programming (FMOP), goal programming and fuzzy goal programming are used in the present study. For the water quality simulation a finite backwards difference technique (Chapra 1997) is used. PGSL (Raphael & Smith 2000, 2003; Domer *et al.* 2003) is used to solve the nonlinear optimization problem, which is based on the assumption that better sets of points are more likely to be found in the neighborhood of good sets of points, therefore intensifying the search in regions that contain good solution (Raphael & Smith 2003). Tests on benchmark problems having multi-parameter nonlinear objective functions revealed that PGSL performs better than Genetic Algorithm and advanced algorithms for simulated annealing (Raphael & Smith 2003).

The PGSL algorithm consists of four nested cycles: sampling cycle, probability updating cycle, focusing cycle and subdomain cycle. In the sampling cycle a number of points (say NSC) are generated randomly by generating a value for each variable according to the probability density function (pdf). Among them the best sample is selected. In a probability updating cycle the sampling cycle is invoked for a number of times (say NPUC). After each iteration, the pdf of each variable is modified. The interval containing the best solution is first selected and then the probability of that interval is multiplied by a factor greater than 1. The pdf thus generated is then modified to make the area under the density function equal to unity. This ensures that the sampling frequencies in regions containing good points are increased. In a focusing cycle, the probability updating cycle is repeated for NFC times. After each iteration, the search is increasingly focused on the interval containing the current best point. The interval containing the best point is divided into uniform subintervals. 50% probability is assigned to this interval. The remaining probability is then distributed to the region outside this interval in such a way so that the pdf decays exponentially from the best interval. In the subdomain cycle, the focusing cycle is repeated NSDC times and, at the end of each iteration, the current search space is modified. In the beginning the entire space is searched, but in subsequent iterations a subdomain is selected for the search. The size of the subdomain decreases gradually and the solution converges to a point. PGSL is used in the present study with a penalty function (Ghosh & Mujumdar 2006) for constrained optimization. More details on this algorithm may be found in Raphael & Smith (2003).

AN APPLICATION

Application of the model is illustrated through a case study of the Tunga–Bhadra river system shown schematically in Figure 2. Details of the river system, effluent data, streamflow data and discretization may be found in Ghosh & Mujumdar (2006). The water quality simulation model is a finite-difference-based BOD-DO model (Chapra 1997).

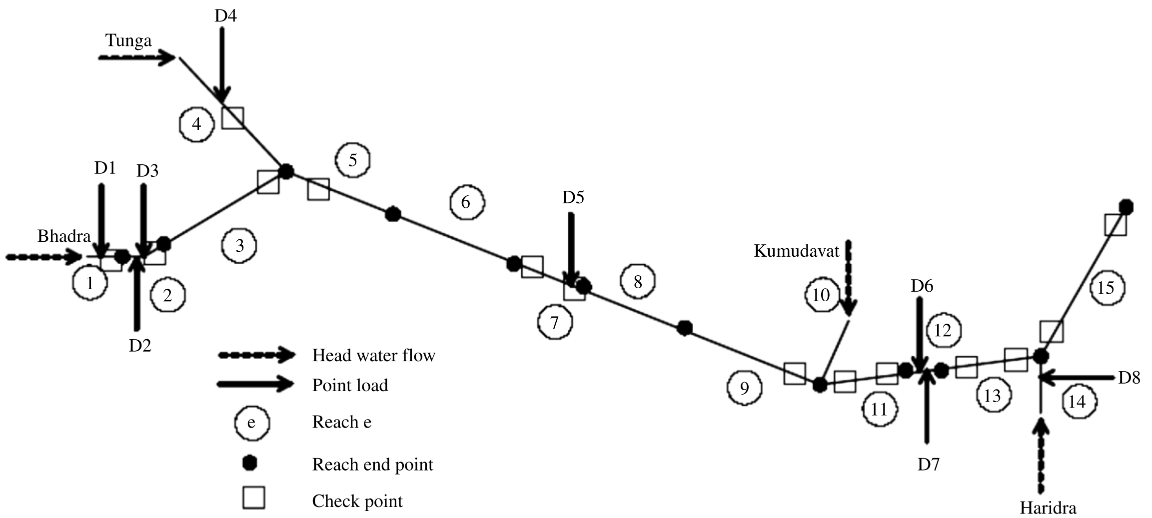

Figure 2 | Schematic diagram of the Tunga–Bhadra river system.

The uncertainty information of the basic variables is taken from Brown & Barnwell (1987) and Subbarao *et al.* (2004). Based on the literature (Fujiwara *et al.* 1987; Burn 1989; Melching & Yoon 1996; de Azevedo *et al.* 2000; Subbarao *et al.* 2004), all the input variables except headwater flow are assumed to follow a normal distribution for the purpose of analysis. A log-normal distribution is used for the headwater flow. A minimum fraction removal level of 35% and a maximum treatment level of 90% are assumed for the dischargers. Aspiration level, x_{imn}^{L}, and maximum permissible level, x_{imn}^{M}, of the dischargers are thus set to 35% and 90%, respectively. The maximum treatment level is imposed considering the technological constraints, whereas a minimum treatment level is imposed to ensure better quality conditions.

FWLAM is applied to the case study to derive the optimal fractional removal level (with $\alpha_{il} = 1$ and $\beta_{imn} = 1$), for the deterministic condition considering only the mean values of the input variables. In MFWLAM, the uncertainty analysis is performed to derive the mean, standard deviation and skewness coefficient of the water quality indicator with 2,000 numbers of Monte Carlo Simulations (MCS).

Multiobjective programming: background

If $f(y)$ is a real-valued function defined on $\Re^n$, a multi-objective model can be defined by

$$\text{Max/Min}\{Z_1, Z_2, \ldots, Z_n\} \tag{31}$$

where, $Z_j = f_j(y_1, y_2, y_3, \ldots, y_k)$ subject to

$$M = \{Y \in \Re^n; g_r(Y) \le 0\}. \tag{32}$$

A vector Y^* is said to be an efficient solution for the problem, where the objective functions are to be minimized, iff there is no $Y \in M$ such that $fj(Y) \le fj(Y^*)$ for each $j = 1, 2, \ldots, n$ with strict inequality for at least one j (Youness 1995). The different solution techniques for multiobjective programming are discussed in the following subsections.

Fuzzy multiobjective programming

Fuzzy multiobjective programming, developed by Zimmermann (1978), is based on the choice of appropriate membership functions for the objective functions. In this method first the model is solved for each of the objective functions at a time to derive the pay-off matrix. The pay-off matrix can be defined as the matrix which contains the solution for all the objective functions resulting from different runs of the multiobjective optimization model considering each of the objectives at a time. The ideal points (diagonal elements) are the best values of the objective functions. From the elements of the matrix, other than the diagonals, the worst values are selected for the objective functions. Using the best and worst values, appropriate membership functions are developed, assigning highest membership values to the best and lowest membership values to the worst. Finally the solutions are obtained by solving the following model:

$$\text{Maximize } \varepsilon \tag{33}$$

$$\text{subject to } \mu(Z_j) \geq \varepsilon \quad \forall j \tag{34}$$

$$M = \{Y \in \Re^n; g_r(Y) \leq 0\}. \tag{35}$$

In this model a new variable, overall acceptability level (ε), is introduced, making this less than or equal to the membership functions of the objectives using the fuzzy constraint (34). The objective is to maximize the overall acceptability level, which in turn finds out the non-inferior solution, making values of both the objectives as close as possible to their best values without violating the constraints.

The derivation of the best and worst values objective functions for the present study is as follows:

1. The solution of FWLAM gives the value of λ as 0.423, which is the best value of λ. As the constraints, whose violations are to be minimized, and which are responsible for a lower value of λ, are not considered in FWLAM, it gives the best value of λ.
2. The corresponding violation value for FWLAM will be the worst (maximum) value of violations. The worst values of v_1 and v_2 are 0.466 and 0.154, respectively.
3. The solution of MFWLAM gives the worst value of λ, as in this model, the inclusion of constraints reduces the minimum acceptability level, λ. The violation values are 0 for MFWLAM as the solution considers the new constraints. These are the best values of v_1 and v_2. The best and worst values thus derived are used to get appropriate membership functions. For the λ value a non-decreasing membership function is used as the model is maximizing λ:

$$\mu_\lambda = \left[(\lambda - \lambda^-)/(\lambda^+ - \lambda^-)\right]^{\Phi} \tag{36}$$

where μ_λ = membership function for λ, λ^+ = best value of λ and λ^- = worst value of λ.

Similarly, the violations at non-increasing membership functions are used because the objective is to minimize the violations of constraints:

$$\mu_{v_1} = \left[(v_1^- - v_1)/(v_1^- - v_1^+)\right]^{\eta} \tag{37}$$

where μ_{v_1} = membership function for v_1, v_1^+ = best value of v_1 and v_1^- = worst value of v_1:

$$\mu_{v_2} = \left[(v_2^- - v_2)/(v_2^- - v_2^+)\right]^{\eta} \tag{38}$$

where μ_{v_2} = membership function for v_1, v_2^+ = best value of v_2 and v_2^- = worst value of v_2.

The exponents, Φ and η, appearing in constraints (36), (37) and (38), are nonzero positive real numbers. Assignment of numerical values to these exponents is subject to the desired shape of the membership functions and may be chosen appropriately by the decision-maker. Finally the following MAX–MIN multiobjective programming formulation is developed:

$$\text{Maximize } \varepsilon \tag{39}$$

$$\text{subject to} \quad \left[(\lambda - \lambda^-)/(\lambda^+ - \lambda^-)\right]^{\Phi} \geq \varepsilon \tag{40}$$

$$\left[(v_1^- - v_1)/(v_1^- - v_1^+)\right]^{\eta} \geq \varepsilon \tag{41}$$

$$\left[(v_2^- - v_2)/(v_2^- - v_2^+)\right]^{\eta} \geq \varepsilon \tag{42}$$

$$\left[\left(\bar{c}_{il} - c_{il}^L\right)/\left(c_{il}^L - c_{il}^D\right)\right]^{\alpha_{il}} \geq \lambda \quad \forall i, l \tag{43}$$

$$\left[\left(x_{imn}^M - x_{imn}\right)/\left(x_{imn}^M - x_{imn}^L\right)\right]^{\beta_{lmn}} \geq \lambda \quad \forall i, m, n \tag{44}$$

$$c_{il} \leq c_{il}^D \quad \forall i, l \tag{45}$$

$$\max\left\lfloor x_{imn}^L, x_{imn}^{\text{MIN}} \right\rfloor \leq x_{imn} \leq x_{imn}^{\text{MAX}} \quad \forall i, m, n \tag{46}$$

$$0 \leq \lambda \leq 1 \tag{47}$$

$$0 \leq \varepsilon \leq 1. \tag{48}$$

In the present model a new variable overall acceptability level (ε) is introduced, making this less than or equal to the membership functions of the objectives using the fuzzy constraints ((40), (41) and (42)). The objective is to maximize the overall acceptability level, which in turn finds out the non-inferior solution, making all the objectives as best as possible.

Goal programming

Goal programming is a technique often used in engineering design activities primarily to find a compromised solution which will simultaneously satisfy a number of design goals. In this technique, the best values of the objective function are set to the target of the objective functions. The goal is to minimize the deviation of objective functions from their target. To minimize the deviations a MIN–MAX formulation is developed (Deb 1998). The resulting formulation for the problem stated in Equations (31) and (32) is as follows:

$$\text{Minimize}\, d \tag{49}$$

$$\text{subject to} \quad \zeta p_j + \delta n_j \le d_j \quad \forall j \tag{50}$$

$$f_j(Y) - p_j + n_j = t_j \quad \forall j \tag{51}$$

$$n_j; p_j \ge 0 \quad \forall j \tag{52}$$

$$M = \{Y \in \Re^n; g_r(Y) \le 0\}. \tag{53}$$

Here, the parameter d becomes the maximum deviation in any goal from the corresponding target t. Minimization of d leads to minimization of all the deviations. For the objective function of maximizing λ, the p value does not exist in the present case, as the resulting solution will always be less than the best or maximum value. Similarly for v_1 and v_2, an n value does not exist, as the resulting solution will always be greater than the best (minimum) value. The weights ζ and δ are set to 1, in Equation (50). Finally the following model is formulated:

$$\text{Minimize}\, d \tag{54}$$

$$\text{subject to} \quad n_1 \le d \tag{55}$$

$$p_2 \le d \tag{56}$$

$$p_3 \le d \tag{57}$$

$$\lambda + n_1 = \lambda^+ \tag{58}$$

$$v_1 - p_2 = v_1^+ \tag{59}$$

$$v_2 - p_3 = v_2^+ \tag{60}$$

$$\left[\left(\bar{c}_{il} - c_{il}^L\right)/\left(c_{il}^L - c_{il}^D\right)\right]^{\alpha_{il}} \ge \lambda \quad \forall i, l \tag{61}$$

$$\left[\left(x_{imn}^M - x_{imn}\right)/\left(x_{imn}^M - x_{imn}^L\right)\right]^{\beta_{imn}} \ge \lambda \quad \forall i, m, n \tag{62}$$

$$c_{il} \le c_{il}^D \quad \forall i, l \tag{63}$$

$$\max\left[x_{imn}^L, x_{imn}^{\text{MIN}}\right] \le x_{imn} \le x_{imn}^{\text{MAX}} \quad \forall i, m, n \tag{64}$$

$$0 \le \lambda \le 1. \tag{65}$$

The MIN–MAX model of goal programming thus derived is used in the present analysis.

Fuzzy goal programming

Fuzzy goal programming (Pal *et al.* 2003) is basically a combination of goal programming and fuzzy multiobjective programming. As FMOP, first the pay-off matrix is derived for the best and worst values of objective functions Z_j. In the field of fuzzy programming, the fuzzy goals are characterized by their associated membership functions $\mu(Z_j)$ and these memberships are derived as similar to the technique used in fuzzy multiobjective programming.

Now, in a fuzzy decision environment, the achievement of the objective goals to their aspired levels to the extent possible is actually represented by the possible achievement of their respective membership values to the highest degree. Regarding this aspect of fuzzy programming problems, a goal programming approach seems to be most appropriate for the problem. Finally, the following fuzzy goal programming model is formulated, for the problem stated in

Equations (31) and (32):

$$\text{Minimize} \sum u_j d_j \tag{66}$$

$$\text{subject to} \quad \mu(Z_j) + d_j = 1 \quad \forall j \tag{67}$$

$$d_j \geq 0 \quad \forall j \tag{68}$$

$$M = \{Y \in \Re^n; g_r(Y) \leq 0\} \tag{69}$$

where the objective function represents the fuzzy achievement function consisting of the weighted under deviational variables, where the numerical weights $u_j(\geq 0)$, represent the relative importance of achieving the aspired levels of the respective fuzzy goals subject to the constraints set in the decision situation. To assess the relative importance of the fuzzy goals properly, the weighting scheme suggested by Mohamed (1997) can be used to assign the values to u_j. The values of weights u_j, used in the present study as suggested by Mohamed (1997), are given below:

$$u_j = \frac{1}{|b_j - w_j|} \quad \forall j \tag{70}$$

where b_j and w_j are the best and worst values of the jth objective, respectively.

In the present study, suitable membership functions are assumed as fuzzy multiobjective programming (Equations (36), (37) and (38)). The targets of all the membership functions are set to 1. The objective is to minimize the weighted deviation from the target of each objective functions to derive the optimal fractional removal level. The model is as follows:

$$\text{Minimize}\,(u_1 \times d_1 + u_2 \times d_2 + u_3 \times d_3) \tag{71}$$

$$\text{subject to} \quad \mu_\lambda + d_1 = 1 \tag{72}$$

$$\mu_{v_1} + d_2 = 1 \tag{73}$$

$$\mu_{v_2} + d_3 = 1 \tag{74}$$

$$\left[\left(\bar{c}_{il} - c_{il}^L\right)/\left(c_{il}^L - c_{il}^D\right)\right]^{\alpha_{il}} \geq \lambda \quad \forall i, l \tag{75}$$

$$\left[\left(x_{imn}^M - x_{imn}\right)/\left(x_{imn}^N - x_{imn}^L\right)\right]^{\beta_{imn}} \geq \lambda \quad \forall i, m, n \tag{76}$$

$$c_{il} \leq c_{il}^D \quad \forall i, l \tag{77}$$

$$\max\left\lfloor x_{imn}^L, x_{imn}^{\text{MIN}}\right\rfloor \leq x_{imn} \leq x_{imn}^{\text{MAX}} \quad \forall i, m, n \tag{78}$$

$$0 \leq \lambda \leq 1. \tag{79}$$

The value of weights (u_j, $j = 1,2,3$) can be derived from Equation (70). An effort has been made to make the membership function values of different objectives as close as to 1 to obtain a non-inferior solution for the multi-objective model. The major advantage of this model is that it's a goal programming which considers the fuzzy membership functions, and the weights to the goals are predetermined.

RESULTS AND DISCUSSION

The PGSL method is used as an optimization tool to solve the nonlinear problem. Two sample cycles, 1 probability updating cycle, 80 focusing cycles and 30 subdomain cycles have been used in the present analysis.

The results of FWLAM and MFWLAM are shown in Table 1. Due to the inclusion of new constraints (16) and (17) in MFWLAM the λ value is decreased significantly, with an increase of fractional removal levels. In FWLAM the λ value is 0.423, but in MFWLAM it is reduced to 0.219. The constraints for skewness and Chebyshev's inequality

Table 1 | Optimal fractional removal level obtained from Modified Fuzzy Waste Load Allocation Model (MFWLAM)

	FWLAM	MFWLAM
λ	0.423	0.219
x_1	0.667	0.779
x_2	0.665	0.778
x_3	0.624	0.758
x_4	0.555	0.773
x_5	0.437	0.745
x_6	0.567	0.736
x_7	0.448	0.778
x_8	0.603	0.767

Table 2 | Statistics of DO concentration and fuzzy risk at checkpoints based on FWLAM and MFWLAM

	FWLAM				MFWLAM			
Check points	Mean	Standard deviation.	Skew-ness	Fuzzy risk (%)	Mean	Standard deviation.	Skew-ness	Fuzzy risk (%)
1	6.18	0.32	2.17	37.83	6.46	0.30	1.07	29.74
2	5.42	0.40	4.02	59.44	5.89	0.36	2.78	46.07
3	6.75	0.87	−5.63	19.95	6.78	0.81	−6.01	18.88
4	6.61	0.24	−1.03	57.16	6.62	0.25	−0.99	56.30
5	6.92	0.34	−2.50	35.16	6.96	0.33	−2.56	32.48
6	7.05	0.37	−3.01	26.77	7.07	0.36	−2.89	25.14
7	7.04	0.37	−3.25	27.11	7.06	0.36	−3.12	25.62
8	7.00	0.39	−4.20	29.78	7.02	0.38	−4.03	28.65
9	6.98	0.37	−4.50	30.84	7.00	0.36	−4.34	29.85
10	7.03	0.40	−4.50	27.42	7.04	0.38	−4.33	26.62
11	6.95	0.40	−4.24	28.47	6.98	0.39	−4.20	26.23
12	6.89	0.42	−4.52	32.94	6.95	0.40	−4.53	28.47
13	6.78	0.40	−4.39	44.32	6.86	0.39	−4.54	38.84
14	6.75	0.44	−4.74	46.24	6.85	0.42	−4.81	39.53

have increased the DO level with an increase in the fractional removal level of dischargers for MFWLAM. Here for comparison purposes the fuzzy risk of low water quality of low water quality (Subbarao *et al.* 2004) is taken as a measure of the performance of the model. Fuzzy risk is defined as the probability of a fuzzy event of low water quality. Denoting the fuzzy set of low water quality, DO concentration and fuzzy risk of low water quality by W_l, c_l and r_l, respectively, the fuzzy risk is rewritten in discrete form as

$$r_l = \sum_{c_{\min_l}}^{\mathrm{MIN}[c_{\mathrm{man}_l}, c_l^D]} \mu_{W_l}(c_l) p(c_l) \tag{80}$$

where $c_{\min_l}$ and $c_{\max_l}$ are the minimum and maximum concentration levels of DO obtained from MCS at checkpoint l. The subscript i for the water quality indicator may be added in Equation (80) in the case of multiple water quality indicators. A typical membership function of low water quality, $\mu_{W_l}(c_l)$, may be expressed as

$$\mu_{W_l}(c_l) = \left[\left(c_l^D - c_l\right)/\left(c_l^D - c_l^L\right)\right] \tag{81}$$

It is possible to reduce the fuzzy risk significantly by using MFWLAM. At locations 1–3 and 2–3 risks are reduced by 8.09% and 13.37%, respectively (Table 2). In the last three reaches the risks are reduced by 4.47%, 5.48% and 6.71%. For getting a compromise solution having a λ value higher than that of MFWLAM, and the risk value lower than that of FWLAM, a combination of the two models are used.

In the fuzzy multiobjective programming technique, appropriate membership functions have been assumed for the objective functions (Equations (36), (37) and (38)). The problem is solved for three different values of φ and η; 0.8, 1 and 1.25. Taking all the combinations, nine sets of analysis have been performed in the present study. As all the membership functions are greater than or equal to ε, so the minimum of the membership function values of objective functions are taken as ε (overall satisfaction level).

Table 3 shows the fractional removal levels obtained from the combined model for different values of φ and η. For $\varphi = 1$ and $\eta = 1$, the λ value is 0.336, whereas in MFWLAM it was as low as 0.219. So it is possible to increase the minimum acceptability level by using the combined model. Table 4 shows the risk values at different checkpoints. At the first two reaches the risk is reduced by 3.50% and 5.80% and at the last three reaches the risk is reduced by 2.39%, 3.00% and 3.73% as compared to FWLAM. So, using this

Table 3 | Optimal fractional removal level obtained from combined model using fuzzy multiobjective programming

	$\varphi = 0.80$ $\eta = 0.80$	$\varphi = 0.80$ $\eta = 1.00$	$\varphi = 0.80$ $\eta = 1.25$	$\varphi = 1.00$ $\eta = 0.80$	$\varphi = 1.00$ $\eta = 1.00$	$\varphi = 1.00$ $\eta = 1.25$	$\varphi = 1.25$ $\eta = 0.80$	$\varphi = 1.25$ $\eta = 1.00$	$\varphi = 1.25$ $\eta = 1.25$
ε	0.639	0.611	0.574	0.602	0.571	0.541	0.564	0.530	0.492
λ	0.336	0.330	0.321	0.342	0.336	0.330	0.348	0.342	0.336
x_1	0.715	0.719	0.723	0.712	0.715	0.719	0.708	0.712	0.715
x_2	0.715	0.718	0.723	0.712	0.715	0.718	0.708	0.712	0.715
x_3	0.681	0.683	0.717	0.707	0.681	0.683	0.684	0.707	0.681
x_4	0.629	0.662	0.608	0.636	0.629	0.663	0.646	0.636	0.629
x_5	0.487	0.628	0.486	0.450	0.487	0.628	0.457	0.498	0.487
x_6	0.636	0.704	0.664	0.666	0.636	0.704	0.651	0.666	0.636
x_7	0.644	0.623	0.705	0.690	0.644	0.623	0.536	0.690	0.644
x_8	0.698	0.717	0.714	0.666	0.698	0.717	0.700	0.665	0.698

model it is possible to get a higher value of acceptability level, with a satisfactory value of risk.

In the present case study the first two locations and the last three locations are the critical checkpoints. The first two locations are at the downstream of point loads 1 and 2 and the streamflow is less as compared to the main Tunga–Bhadra reach, making the checkpoints very much sensitive to the point loads. The last three locations are critical due to the cumulative effect of incremental flow. It is possible to reduce the fuzzy risk of low water quality by a significant amount by using the model at these critical locations, as compared to FWLAM.

Table 5 shows the fractional removal levels obtained from the combined model using goal programming and fuzzy goal programming. The λ is lowest (0.289) for fuzzy goal programming among all the three methods, with a high value of fractional removal level. So, the result for the fuzzy goal programming is most conservative. The goal

Table 4 | Fuzzy risk at different checkpoints

Location	Fuzzy risk (%) for different values of φ and η: $\varphi = 0.80$ $\eta = 0.80$	$\varphi = 0.80$ $\eta = 1.00$	$\varphi = 0.80$ $\eta = 1.25$	$\varphi = 1.00$ $\eta = 0.80$	$\varphi = 1.00$ $\eta = 1.00$	$\varphi = 1.00$ $\eta = 1.25$	$\varphi = 1.25$ $\eta = 0.80$	$\varphi = 1.25$ $\eta = 1.00$	$\varphi = 1.25$ $\eta = 1.25$
1	34.33	34.09	33.76	34.58	34.33	34.09	34.83	34.58	34.33
2	53.65	53.26	52.66	54.02	53.64	53.26	54.45	54.02	53.65
3	19.49	19.46	19.41	19.52	19.49	19.46	19.55	19.52	19.49
4	56.87	56.74	56.95	56.84	56.87	56.74	56.80	56.84	56.87
5	34.23	33.85	34.42	34.16	34.23	33.85	34.06	34.16	34.23
6	26.21	25.98	26.33	26.16	26.20	25.98	26.10	26.16	26.21
7	26.60	26.38	26.71	26.56	26.60	26.38	26.51	26.56	26.60
8	29.39	29.22	29.47	29.36	29.39	29.22	29.32	29.36	29.39
9	30.50	30.35	30.58	30.48	30.50	30.35	30.45	30.48	30.50
10	27.14	27.02	27.20	27.12	27.14	27.02	27.10	27.12	27.14
11	27.39	27.27	27.20	27.17	27.39	27.27	27.72	27.17	27.39
12	30.55	30.51	29.95	30.01	30.55	30.51	31.57	30.01	30.55
13	41.32	41.13	40.62	41.11	41.32	41.13	42.24	41.11	41.32
14	42.51	42.16	41.68	42.61	42.51	42.16	43.37	42.61	42.51

Table 5 | Optimal fractional removal level obtained from combined model using goal programming and fuzzy goal programming

	Goal programming	Fuzzy goal programming
λ	0.296	0.289
x_1	0.737	0.742
x_2	0.737	0.742
x_3	0.666	0.723
x_4	0.671	0.742
x_5	0.558	0.700
x_6	0.673	0.726
x_7	0.479	0.742
x_8	0.681	0.742

programming gives the λ value as 0.296, which lies in between the results of fuzzy multiobjective programming and fuzzy goal programming. Another important observation is that, compared to the goal programming technique, the fuzzy goal programming model increases the fractional removal levels of all the dischargers to its maximum possible value corresponding to the acceptability level (λ). So the maximum reduction of risk is possible by using the fuzzy goal programming (Table 6). Therefore, a general conclusion can be drawn, that an optimistic decision-maker will use the fuzzy multiobjective programming technique, and a pessimistic decision maker will use fuzzy goal programming for this case study.

Table 6 | Fuzzy risk (%) at different checkpoints for goal programming and fuzzy goal programming

Location	Goal programming	Fuzzy goal programming
1	32.76	32.40
2	51.09	50.44
3	19.28	19.23
4	56.70	56.42
5	33.70	32.92
6	25.89	25.41
7	26.31	25.87
8	29.17	28.83
9	30.30	30.01
10	26.99	26.75
11	27.79	26.51
12	31.99	28.99
13	42.80	39.52
14	44.09	40.40

CONCLUSIONS

The methodologies for waste load allocation in a river water quality control problem are presented. FWLAM is a deterministic model; it gives the fractional removal level only for the base flow condition and does not consider the random nature of the input variables of the water quality simulation model. The major advantage of MFWLAM is that these perform uncertainty analysis, considering all the input variables to be random. Consideration of Chebyshev's inequality and skewness makes MFWLAM more conservative and reduces the satisfaction of the dischargers, which results in a low acceptability level. The probabilistic constraints restrict the water quality in terms of skewness and Chebyshev's inequality with an increase in fractional removal levels, making the model slightly biased to the PCA. Therefore, strict probabilistic restrictions on water quality results in a low acceptability level, which is almost half of that of FWLAM. The restriction is fuzzified in terms of satisfying the probabilistic constraints 'as much as possible' with minimization of violations in the combination of FWLAM and MFWLAM to obtain a compromised solution which will give a higher value of acceptability level with a satisfactory value of risk. Use of Monte Carlo simulation with a finite difference formulation of water quality simulation, within the optimization makes the model complex and nonlinear and therefore it can not be solved using analytical or gradient-based methods. Moreover, the objective function, λ, being the minimum satisfaction level of all the stakeholders cannot be expressed analytically and is also not differentiable. Therefore PGSL, a nonlinear search algorithm, is applied to solve the problem. Apart from the uncertainties due to randomness and imprecision there is another source of uncertainty known as model uncertainty, resulting from the use of multiple water quality simulation models (e.g. Qual-2K, WASP, etc.). A limitation of the model developed is that such model uncertainty is not considered in the present study. Consideration of all the sources of uncertainties in a single optimization framework is a potential research area and will be a straightforward extension of the proposed model. It should be noted that the model is not applied to a benchmark problem. The model is applied to a real case study of the Tunga–Bhadra river system. FWLAM, MFWLAM and the proposed models are applied to the case study and the improvement of the results is presented.

Modified fuzzy waste load allocation model and the combined model do not limit their application to any particular pollutant or water quality parameter in the river system. Given an appropriate transfer equation for spatial and temporal distribution of the pollutant in a water body, the methodologies can be used to derive the optimal fractional removal levels. In a general sense, they are adaptable to various environmental systems where a sustainable and efficient use of the environment is of interest.

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