Concluding Remarks

The foregoing analysis has provided extensive results for the developing laminar flow in annular ducts. Unfortunately, the authors were unable to uncover experimental data which might serve to check these results. However, strong support of the present analytical method is provided by excellent agreement between theory and experiment for the circular tube. It may also be noted that the present method may, in principle, be applied to ducts of arbitrary cross section.

References

1 T. S. Lundgren, E. M. Sparrow, and J. B. Starr, "Pressure Drop Due to the Entrance Region in Ducts of Arbitrary Cross Section," to be published in the JOURNAL OF BASIC ENGINEERING, TRANS. ASME.

2 N. A. Slezkin, "Dynamics of Viscous Incompressible Fluids" (in Russian), Moscow, Gostekhizdat, 1955.

3 J. G. Knudsen and D. L. Katz, *Fluid Dynamics and Heat Transfer*, McGraw-Hill Book Company, Inc., New York, N. Y., 1958.

APPENDIX

Expression for the Series Coefficients c_i

$$c_i = 2(F_1 - 2AF_2)/(1 + N^2 - 2A)[F_3(1) - F_3(N)]$$

$$F_1 = -[Z_Y(1)/Z_J(1)]G_J + G_Y + 0.25(1 - N^4)E$$

$$F_2 = -[Z_Y(1)/Z_J(1)]H_J + H_Y + 0.25(N^2 - 2N^2 \ln N - 1)E$$

$$F_3(\eta) = \frac{1}{2}\left[\frac{Z_Y(1)}{Z_J(1)}\right]^2 \eta^2[J_0{}^2(\alpha_i\eta) + J_1{}^2(\alpha_i\eta)] + \frac{1}{2}\eta^2[Y_0{}^2(\alpha_i\eta) + Y_1{}^2(\alpha_i\eta)] + \frac{1}{2}\eta^2E^2 - \left[\frac{Z_Y(1)}{Z_J(1)}\right]\eta^2[J_0(\alpha_i\eta)Y_0(\alpha_i\eta) + J_1(\alpha_i\eta)Y_1(\alpha_i\eta)] + \frac{2E}{\alpha_i}\left\{\eta Y_1(\alpha_i\eta) - \left[\frac{Z_Y(1)}{Z_J(1)}\right]\eta J_1(\alpha_i\eta)\right\}$$

$$\alpha_i{}^3G_J = (\alpha_i{}^2 - 4)J_1(\alpha_i) - (\alpha_i{}^2N^2 - 4)NJ_1(N\alpha_i) + 2\alpha_iJ_0(\alpha_i) - 2\alpha_iN^2J_0(N\alpha_i)$$

$$\alpha_i{}^2H_J = -\alpha_iNJ_1(N\alpha_i)\ln N + J_0(\alpha_i) - J_0(N\alpha_i)$$

$$\frac{1}{2}\alpha_i(1 - N^2)E = -\left[\frac{Z_Y(1)}{Z_J(1)}\right][NJ_1(N\alpha_i) - J_1(\alpha_i)] + NY_1(N\alpha_i) - Y_1(\alpha_i)$$

The terms G_Y and H_Y are obtained from G_J and H_J by replacing J by Y.

DISCUSSION

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The purpose of this discussion is to submit experimental results pertinent to the analysis of Sparrow and Lin. Velocity profiles were measured in the developing laminar flow of air in an annular duct. The annulus had a gap of 0.517 in., and a radius ratio, r_2/r_1, of 0.732. Measurements were made with a Flow Corporation hot-wire anemometer which was calibrated frequently during the experiments. Upstream of the test section converging walls were used to guide the air from the stilling chamber to the annulus. The contraction ratio was about 8 to 1. The studies were concerned with the developing flow along the inner wall. Consequently, boundary layer suction was applied at the entrance only at the inner wall. About 5 percent of the flow was removed at that point. These results are reported in a paper by the discusser.[3]

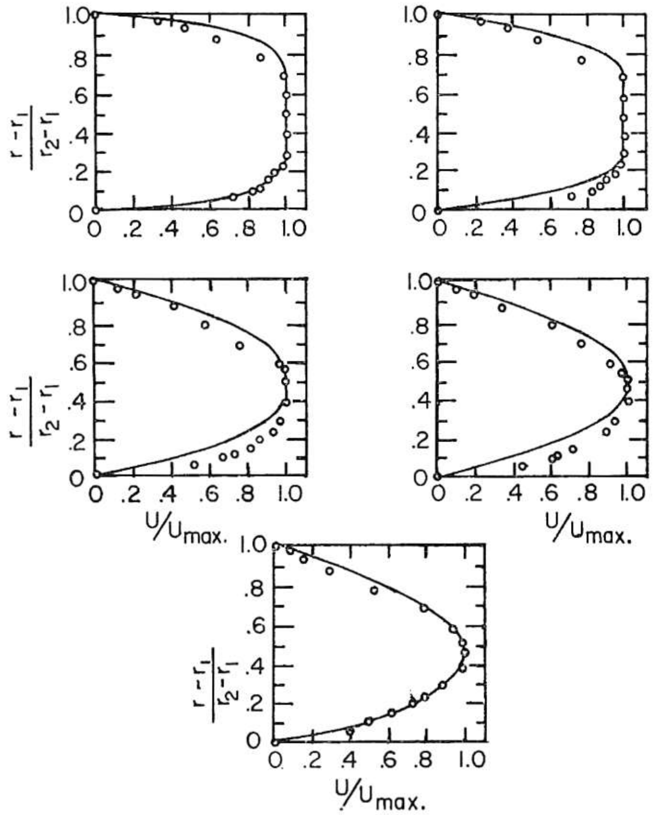

Fig. 4

Data were taken for Reynolds numbers of 200, 700, 1200, and 1700 at stations along the duct to a distance of 34 equivalent diameters from the entrance. Reynolds numbers here are defined as

$$\mathrm{Re} \equiv \frac{2(r_2 - r_1)\bar{u}}{\nu}$$

Several sets of data are compared with the results of Sparrow and Lin in Fig. 4. The solid lines are based on their results for an annulus having

$$r_1/r_2 = 0.8, \text{ Fig. 3}(a).$$

The dimensionless length is represented by L, and is taken from the paper as

$$L \equiv \frac{x/r_2 - r_1}{\frac{\bar{u}(r_2 - r_1)}{\nu}} = \frac{2x/(r_2 - r_1)}{\mathrm{Re}}$$

While the agreement between analysis and experiment is reasonably good, the experimental profiles are more skewed than the analytical profiles. One would expect the data to be skewed because the boundary layer suction was applied only at the inner wall. Thus the flow on the outer wall had more opportunity to develop. This skewing persists well down the duct.

A comparison was made for all the data available for values of $L < 0.05$. A dimensionless difference was defined as the difference between $u/\bar{u}$ for the data and analysis, divided by the $u/\bar{u}$ from the analysis. An rms of 0.2 was computed for these dimensionless differences, the value being about the same for the

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[3] K. N. Astill, "Modes of Adiabatic Flow in the Entrance Region of an Annulus With an Inner Rotating Cylinder," PhD thesis, M.I.T., Cambridge, Mass., 1961.

inner half of the gap as for the outer half. Some individual differences near the wall were quite large which tended to increase the rms value. A small absolute difference between velocities becomes a large dimensionless difference where the velocity is small, as near the wall.

Velocities generally were small for these experiments, the mean velocity at Re = 1200 being 2.35 ft/sec. It is difficult to make precise velocity measurements with air in this range of velocities. Many of the values were less than 0.5 ft/sec. It was estimated that errors in velocity readings in this range with the instrumentation used had a standard deviation of 0.13 from the true value.

While these comparisons are not conclusive they tend to support the results of the analysis of Sparrow and Lin. They should also serve to indicate areas of difficulty in making precise measurements of this sort.

Authors' Closure

Thanks are extended to Professor Astill for providing a comparison between theory and experiment. Generally, the level of agreement is quite satisfactory. The deviations that were noted in the neighborhood of the wall are very likely due to inevitable difficulties in measuring small velocities near solid surfaces.