Light Curve of SN1987A and the Pulsar Cavity Model

Humitaka SATO and Yoshiyuki YAMADA

Department of Physics, Kyoto University, Kyoto 606

(Received August 4, 1990)

Assuming the pulsar cavity model, the dissipation rate of the energy from the pulsar is computed. In this model, the energy liberated from the pulsar is assumed to be confined in the cavity and the cavity expands by pushing the supernova ejecta. Through the shock compression just outside the cavity surface, a fraction of the pulsar energy compiled in the cavity is dissipated. The energy supply to the light curve by this dissipation may become dominant in the stage after the radioactive energy source has diminished. Application to the SN1987A is discussed.

§ 1. Introduction

The light curve of the Supernova 1987A has shown a deviation from that expected by the radioactive energy supply after November, 1989.1) In order to explain this deviation, a new source of energy is necessitated and it is considered to be a newly born pulsar. Sometimes the luminosity of the light curve after the above deviation is simply interpreted to be the energy liberated from the pulsar. However this assumption would not be simply true because a mode of the energy emitted by the pulsar could be the one which does not dissipate into thermal energy so easily. We have to consider more carefully how much fraction of the pulsar energy is converted into the light curve.

In the early 1970s, the pulsar cavity model was proposed and has been explored in relation to the Crab nebula and the “plerion” type supernova remnants.2)–6) This model was successful in explaining many observed features in the Crab nebula. In this model, the energy liberated from the pulsar is completely confined in the cavity formed around the pulsar in the expanding ejecta of the supernova. The mode of the energy emitted by the pulsar is supposed to be magnetic field and relativistic plasma and the pressure of the confined energy compresses the matter around the cavity to form the shock wave, in which some fraction of the pulsar energy is dissipated and it will work as a thermal energy source of the luminosity observed by the light curve.

In the papers on the cavity model, the density distribution in the supernova ejecta was assumed to be uniform and an elegant similarity solution was derived.2) According to this result, the dissipation rate is given by \( \frac{L_d}{L_p} = \frac{1}{66} \), where the pulsar luminosity \( L_p \) is assumed to be constant with time. Then, if the source of the light curve is solely due to this dissipated energy, we are obliged to equate \( L_d \) to the observed luminosity of the light curve \( L_l \) and \( L_p \) will be equal to 66 \( L_l \), that is, the pulsar power can be much larger than the “pulsar component” of the light curve. This opens a possibility to assume a more powerful pulsar in the SN1987A compared with that resulting from the assumption \( L_p = L_l \).

In this paper, we compute the dissipation rate in the ejecta with more realistic density profile. It is shown that the dissipation coefficient of \( L_d/L_p \) varies with the
density profile from the (1/66) in the uniform case. When the cavity surface (or the shock front) sweeps a steep region of the density profile the dissipation rate is suppressed less than (1/66) and this implies a more powerful pulsar than $L_p = 66L_0$.

In §2, we write down the basic equations for the expansion of the cavity surface using the comoving radial coordinates. In §3, the expansion is analyzed numerically adopting the density profile given by Nomoto et al. and the dissipation coefficient will be estimated. In §4, we will discuss the various limitations of this model, which must be kept in mind in case of the comparison with the observation.

§2. Expansion of the cavity surface

The energy liberated from the pulsar is assumed to be a relativistic entity such as magnetic field and relativistic plasma and to be confined uniformly in the cavity without penetrating into the surrounding ejecta. The ejecta matter is evacuated by the pressure of the pulsar energy to form a dense shell around the cavity. The uniformity will be justified because the sound velocity in the cavity is large. Writing a radius of the cavity as $R$ and the luminosity of the pulsar as $L_p$, the conservation laws of momentum, energy and mass are written respectively by the following equations:

\[ \frac{dM_s}{dt} = 4\pi R^2 \rho (v - V), \]  
\[ \frac{d}{dt} (4\pi R^4 P) = L_p R, \]  
\[ \frac{d}{dt} (4\pi R^2 \rho (v - V)). \]

where $v = dR/dt$, $M_s$ the mass of the shell, $V$ the ejecta’s expansion velocity, $L_p$ the pulsar luminosity and $3P$ the energy density in the cavity. We represent the density profile in the ejecta by $f(x)$ as

\[ \rho(t, x) = \frac{3M_0}{4\pi(V_0t)^3} f(x) \quad \text{and} \quad \int_{V}^{x} f(x)x^2 dx = 1/3, \]

where $M_0$ is the mass of the ejecta within $x=1$, $x=R/V_0 t$ is the comoving coordinate of the shell and $V(x) = V_0 x$ with $V_0 = V(1)$. Introducing the nondimensional quantities such as

\[ \tau = t/t_0, \quad m(x) = M_s/M_0, \quad p = P/(M_0/4\pi V_0 t_0^3), \]

\[ l = L_p/L_0 \quad \text{and} \quad \alpha = t_0/T_0 \quad \text{with} \quad T_0 = M_0 V_0^2/L_0, \]

Eqs. (2·1)~(2·4) are rewritten respectively as

\[ m(x' + \frac{2x'}{\tau}) = x^2 \tau p - m' x', \]
\[ m' = 3f(x)x^2 x', \]
\[ p' + 4p \left( \frac{1}{r} + \frac{x'}{x} \right) = \alpha \left( \frac{t}{r x^3} \right) \]  
\[ (2.8) \]

where \( m' = dm/dx \) and so on. If \( t_0 \) is chosen \( T_0, \alpha = 1 \) and the time scale is the order of

\[ T_0 = 15 \text{years} \left( \frac{M_0}{1 M_\odot} \right) \left( \frac{V_0}{500 \text{km/s}} \right)^2 / \left( \frac{10^{46} \text{erg/sec}}{L_0} \right) \].  
\[ (2.9) \]

The pulsar energy liberated up to \( t \) has contributed to the following three modes, the energy \( W \) stored in the cavity, the kinetic energy \( K \) of the shell and the dissipated energy \( E_d \) at the shock wave just in front of the shell. \( W, K \) and \( E_d \) are given as

\[ W = 3P^4 \frac{4\pi}{3} (V_0 t x)^3, \]  
\[ (2.10) \]

\[ K = \frac{M_\odot (dR/dt)^2}{2} - 4\pi \int_0^x \rho \left( \frac{V_0 x}{2} \right)^3 (V_0 t)^3 x^2 dx \] \[ \text{and} \]  
\[ (2.11) \]

\[ E_d = \int_0^t L_\rho(t') dt' - W - K. \]  
\[ (2.12) \]

And the dissipation rate is given by

\[ L_d = dE_d/dt. \]  
\[ (2.13) \]

Though we have assumed that the energy liberated from the pulsar does not penetrate into the surrounding ejecta, there is an energy release into heat at the surface of the cavity shell. The energy dissipation always occurs in this kind of snowplow mechanism, since the mass of the cavity shell is fed through a compression by the shock front in front of the cavity shell. This energy dissipation is analogous to that in case of a perfect inelastic collision of two bodies. As in this case, the dissipation rate is computable without treating a shock front explicitly but it is totally determined by the conservation of mass and momentum.

As told in § 1, the uniform model with \( f(x) = 1 \) and \( L_\rho = \text{const} \) permits the similarity solution such as

\[ R = V_0 t \left( \frac{125}{99} \right)^{0.2} \left( \frac{t}{T_0} \right)^{0.2} \]  
\[ (2.14) \]

and the energy allotment is \( W = (5/11)L_\rho t \), \( K = (35/66)L_\rho t \) and \( E_d = (1/66)L_\rho t \).

§ 3. Expansion of the cavity and dissipation rate

The density profile in the ejecta was computed by the hydrodynamic code of the explosion. As the progenitor model of SN1987A, we adopted the model given by Nomoto and Shigeyama.\(^7\) The hydrodynamic computation was done by the spherically symmetric explicit Eulerian code with the second order accurate flux split scheme. We have adopted the case of the explosion energy \( E_{\text{exp}} = 10^{51} \text{ erg} \).

Figure 1 is a profile realized after the supernova shock wave has passed away. Our main concern is the inner part of the ejecta. We have taken \( x = 1 \) at the mass
As seen in Fig. 1, the ejecta has a hollow in the central part surrounded by the compressed broad shell. The profile is shown also in Fig. 2 relative to the mass \( M_r \) from the center. The composition in each mass shell of the progenitor is shown in this figure. Outside this broad shell, the profile decreases more slowly with radius.

Adopting the above profile as \( f(x) \), we computed the ordinary differential equations (2.6)~(2.8) with Runge-Kutta method taking \( L_p \) a constant with time. As mentioned in the above, \( L_p \) determines the time scale of the cavity expansion. The expansion is shown in Fig. 3, in which the expansion given by the similarity solution is also shown. The uniform density adopted in this similarity solution is the density averaged within \( x=1 \). Since the density in the hollow is much smaller than the average density, the initial velocity of the cavity shell in the early stage is large compared with that of the similarity solution. But, as the cavity surface catches up the broad shell of the compressed C+O and He core, the acceleration of the expansion suddenly decreases and the mass of shell begins to increase rapidly.

The sweep time of the cavity’s shell in the hollow is the order of

\[
t_1 = 1.9 \times 10^4 \left( \frac{10^{40} \text{erg/sec}}{L_p} \right) \text{years}.
\]

The cavity’s shell passes through the broad shell of the ejecta at \( t_2 \), which is about

\[
t_2 = 3.2 \times 10^2 \left( \frac{10^{40} \text{erg/sec}}{L_p} \right) \text{years}.
\]

After that time, the acceleration of the cavity surface increases again.

The time scale is inversely proportional to the energy liberation rate of the pulsar. As discussed in the next section, \( L_p \) cannot be longer than \( 10^{41} \text{erg/s} \) for the SN1987A and, then, the cavity shell is supposed to lie still within the hollow or in the broad shell.

The energy dissipation rate given by (2.13) is obtained as shown in Fig. 4. The
Fig. 3. Time evolutions of mass $m$, velocity $v = \dot{R}/V_0$, and radius of the shell $x$. Normalization of these quantities are the same with that in Fig. 1. Solid lines are solutions for M11. In Eq. (2.5), $M_0 = 10 M_*$, $V_0 = 2500$ km/sec, $L_0 = 10^{36}$ erg sec$^{-1}$ and $T_0 = 4 \times 10^3$. Behavior of these quantities for the similarity solution is also shown by the dash-dotted lines.

Fig. 4. Time evolution of the energy dissipation rate. The solid line is energy dissipation rate for the model M11, and the dashed line is $f(x(t))$. The energy dissipation rate for the similarity solution is also shown by the dash-dotted line. The dissipation rate (1/66) $L_\rho$ for the similarity solution is denoted also by the horizontal line. The dissipation rate is smaller than this value of the similarity solution when the cavity surface shell propagates in the steeply descending region of the density profile. As the cavity surface shell has swept almost all mass and the increase of $M_r$ saturates, the dissipation rate suddenly decreases in a monotonic manner.

§ 4. Discussion

1) In the basic equations, it has been assumed that the thermal pressure in the freely expanding ejecta is negligible compared with the ram pressure due to the cavity expansion. The validity of this assumption should be checked by estimating how...
long the heat supply from the radioactive source keeps the ejecta hot enough. The analysis shows that the assumption will be verified except the initial few years after the supernova explosion. 6)

2) The dissipation rate $L_d$ computed in this paper should not be identified directly to the light curve $L_l$. $L_d$ is a source to $L_l$. The transport of the energy must be solved to get $L_l$ supplied by $L_d$ if the photosphere of the ejecta is far outside the cavity’s surface. In this respect, Chevalier has given an interesting discussion about SN1986j, 8) whose light curve cannot be due to the radioactive energy in four years after the explosion and is supposed to be due to the pulsar. Since the spectrum feature of the SN1987j indicates an emission from a dense layer, he speculated that this dense layer might be the shocked shell surrounding the pulsar cavity. In this case, the location of the energy dissipation and the photosphere is very close and $L_d$ may roughly represent $L_l$. The spectrum feature of the SN1987A may give some information on this problem.

3) If we may assume $L_d$ roughly gives $L_l$, the allowed value of $L_p$ is estimated as $L_p < (L_p/L_d)_{\text{computed}} \times (L_l)_{\text{observed}}$. This will be the “allowed maximum” because additional modes of energy supply to $L_l$ can exist. According to the observation of the SN1987A, $L_l$ is less than $10^{39}$ erg/s at the time of the deviation from the radioactive source supply and $L_p$ cannot exceed $10^{41}$ erg/s at most.

4) As the additional routes to $L_l$ from the pulsar activity, we can list up many possibilities such as the absorption of the pulsar energy by the ejecta, the synchrotron emission in the cavity, the high energy particle injection from the magnetosphere and so on. The radiation by the cooling of the neutron star may also contribute to $L_l$ but that is not classified to the pulsar energy. Theoretical argument about the above possibilities is still very uncertain. Our computation in this paper has presumed the extreme case and our result would be valuable even for the study of the other possible routes to $L_l$ from $L_p$.

Acknowledgements

We would like to thank Professor K. Nomoto and Dr. T. Shigeyama for showing us a computer output of stellar evolution models. The numerical computation of this work is processed using FACOM M780 of the data processing center of Kyoto University.

References

1) IAU Circ. 4933.
5) More references are given in Ref. 6).