Effects of Coriolis Perturbation on Normal-Parity Rotational Bands of Odd-Mass Nuclei

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The puzzling behavior of the $M1$ transitions in a normal-parity band of $^{163}$Dy observed recently is studied in the framework of the particle-rotor model. Characteristic features of the $M1$ transitions as well as the energy spectrum are well reproduced by the present calculation. We have found that the Coriolis interaction gives rise to an appreciable amount of rotation alignment. A remarkable point of the result is that the alignment of a level comes not from all the $j$'s around but from a few specific $j$-orbitals. The resultant rotational bands are well characterized by the $j$'s which contribute to their rotation-alignment.

§ 1. Introduction

Recently a great deal of experimental and theoretical efforts have been devoted to the study of nuclear rotational motion. A central problem is how the collective rotation affects the internal structure. A prominent effect of the nuclear rotation as a whole is to cause particles to align their spin with the total angular momentum. The particles moving in unique-parity orbitals couple most strongly with the collective rotation and most easily align their angular momenta with the total spin. The backbending of yrast bands of many deformed even nuclides is explained as a crossing of a band of the ground state configuration with a band in which two particles in unique-parity orbitals are rotation-aligned. We mean by unique-parity orbitals the ones which are shifted downward in energy by strong spin-orbit interaction into the region of orbitals with principal quantum number less by one and with the opposite parity. Typical examples are $h_11/2$ and $i_13/2$. We will call other orbitals the normal-parity orbitals.

A rotational band based on a quasiparticle moving in unique-parity orbitals (a unique-parity rotational band) in odd-mass nuclides is also characterized by rotation-alignment. Without the Coriolis interaction, it would constitute a regular rotational band characterized by a specific value of $Q$. Actually such a band splits into two sequences, each of which is composed of levels differing in spin by $2\hbar$. These two sequences are discriminated by the quantum number called signature, which is defined as $\frac{1}{2}(-1)^{I-j-\frac{1}{2}}$. Signature is a quantum number associated with the invariance of a system with quadrupole deformation under rotation by $180^\circ$ around one of the principal axes.

The significance of signature in a unique-parity rotational band of an odd-mass nuclide is seen in the following facts: (1) As noted above, such a band consists of two sequences of levels. The one in which $I-j=$ even is shifted downward in energy and called the favored band, and the other the unfavored. Therefore the signature
quantum number has one-to-one correspondence to the characteristic energy splitting. (2) Magnetic dipole \( (M1) \) transitions from a level of \((\text{spin}=I, f \ [\text{favored}])\) to a level of \((I-1, u \ [\text{unfavor­ed}])\) are enhanced over the ones from \((I, u)\) to \((I-1, f)\). This is another one-to-one correspondence between signature and physical observable.

In a rotational band based on a quasiparticle moving in normal-parity orbitals (a normal-parity rotational band), the meaning of signature becomes less evident, since such a band involves various \( j \)'s and receives different contributions to the energy and \( M1 \) transitions from them. Only \( \Omega=1/2 \) bands are affected by the Coriolis interaction in a predictable way. It seems to have been accepted that normal-parity rotational bands with \( \Omega\neq1/2 \) are not noticeably affected by the Coriolis interaction.

The energy spectrum of \(^{163}\text{Dy}\) observed is in accordance with what is usually expected.\(^{1,2}\) The ground state, \( 5/2^- \), and the rotational band on it are supposed to heavily involve \( 5/2 \) \([523]\). They may involve also \( 3/2 \) \([521]\) as a result of the rotational perturbation. These orbitals largely contain the \( f_{7/2} \) and \( h_{9/2} \) shells. The energy spectrum is quite smooth with spin. We find by a close look, however, that the levels with spin \( I=9/2\,+\text{even} \) are energetically favored compared with the other levels. The spectrum is illustrated in Fig. 1 in the form of \( \Theta_I=(E_I-E_{I-1})/2I \). This may mean that predominance of the rotation alignment of \( h_{9/2} \) characterizes the band. However, this conjecture seems to be inconsistent with the behavior of the \( M1 \) transitions. The transitions from \( I=7/2\,+\text{even} \) to \( I-1 \) are stronger than the ones from \( I=7/2\,+\text{odd} \) to \( I-1 \).\(^{1,2}\) This feature seems to indicate a characteristic of an \( f_{7/2} \) rotation-aligned band. This is the problem which we are going to investigate in this paper. This problem was previously studied\(^{3}\) in the framework of the cranking model and the strange behavior of the \( B(M1) \)'s was attributed to the interference of the spin and orbital contributions. The treatment was applied to different configurations in other nuclei.\(^{3}\)

We will study this problem in the framework of the particle-rotor model. It is an advantage of this model over the cranking model that we can rigorously conserve the angular momentum in this model. Our objective is to find out a consistent view for the seemingly inconsistent features of the experimental data, to see in detail what effects the Coriolis interaction introduces into the energy spectra and \( M1 \) transitions of normal-parity rotational bands, and to find out characteristic properties, if any, in the resultant wave functions.

§ 2. The model

We employ the model in which a quasiparticle couples to an axially symmetric
rotator. After the BCS transformation being applied and only the one-body terms being retained, our hamiltonian reads

\[ H = \sum_{k=1}^{2} \hbar^2 \left( I_k - j \right)^{2} + \sum_{\alpha} \epsilon_{\alpha} c_{\alpha}^{\dagger} c_{\alpha}, \tag{2.1} \]

where \( k \) denotes the principal axes of the intrinsic frame, \( c^\dagger \) and \( c \) are creation and annihilation operators of quasiparticles specified by Nilsson quantum numbers, \( \alpha \). The quasiparticle energies, \( \epsilon \), are given as

\[ \epsilon_{\alpha} = \sqrt{(\epsilon_{\alpha} - \lambda)^2 + \Delta^2}, \tag{2.2} \]

where \( \lambda \) is the Fermi energy, \( \Delta \) the energy gap and \( \epsilon \)'s are the Nilsson energies determined by solving the equation

\[ H_{\text{Nilsson}} = \hbar \omega_0 (\delta) \left[ \frac{1}{2} (-\nabla^2 + r^2) - \beta r^2 Y_{20}(\theta, \phi) \right] \]

\[ - \kappa \hbar \omega_0 [2 \mathbf{l} \cdot \mathbf{s} + \mu \mathbf{l}^2] + \hbar \omega_0 \cdot \delta \epsilon(j), \tag{2.3} \]

which is diagonalized in the space of \( N=5 \) orbitals. Thus our model contains a few parameters to specify the Nilsson field, the rotational motion and the pairing correlation. The values frequently used are adopted for \( \kappa \) and \( \mu : \kappa = 0.0637, \mu = 0.42 \). The average of the deformations determined\(^4\) from the \( B(E2)'s \) of \(^{162}\)Dy and \(^{164}\)Dy is assigned to \( \beta : \beta = (0.3407 + 0.3481)/2 \). We assign 0.8 MeV to \( \Delta \), which is expected to be representative in this region. The reciprocal moment of inertia, \( \hbar^2/2J \), is assumed to be 12.7 keV, which appropriately reproduces the energy spectrum of \(^{163}\)Dy. Single-particle energy shifts, \( \delta \epsilon \)'s, are set equal to zero except for \( \delta \epsilon(h_{11/2}) \), which is assumed to be \(-0.11\). If we do not shift \( h_{11/2} \) downward, \( 11/2 [505] \) appears too close to the Fermi energy and the correct spin value cannot be obtained for the ground state. We have located the Fermi energy \( \lambda \) on \( 5/2[523] \) in the present calculation.

The Coriolis interaction, when taken into account in the ordinary particle-rotor model calculations, affects \( M1 \) transition strengths very much but \( E2 \) transition strengths only a little. We experience this lack of influence of the Coriolis interaction on \( B(E2)'s \) not only in normal-parity rotational bands but also in unique-parity ones.\(^5\) This applies to the present case also and \( B(E2)'s \) are given to good approximation as

\[ B(E2: I \rightarrow I') = \frac{5}{16\pi} (IK 20|I'K)^2 Q_t^2, \tag{2.4} \]

where \( Q_t \) is related with deformation \( \beta \) in the Bohr-Mottelson model as

\[ Q_t = -\frac{3}{\sqrt{5\pi}} Z (0.12A^{1/3})^2 \beta \text{ (eb)} \tag{2.5} \]

The present choice of \( \beta \) gives rise to \( Q_t = 7.39 \text{ eb} \). This is satisfactorily compared with \( Q_t = 7.2 \text{ eb} \), which is the one extracted from the experimental data. Here we remark that the value obtained in Ref. 2) for \( \beta \) in the cranked Hartree-Bogoliubov calculation, 0.258, seems to be unrealistically too small.

The operator of \( M1 \) transitions is of the form
\[ \mathcal{O}(M1, m) = \sqrt{\frac{3}{4\pi}} \mu_m \] (2.6)

with

\[ \mu_m = \sum_l g_l^{(l)} l_{i,m} + \sum_l g_s^{(l)} s_{i,m} + g_R R_m \]

\[ = \sum_l (g_l^{(l)} - g_R) l_{i,m} + \sum_l (g_s^{(l)} - g_R) s_{i,m} + g_R I_m, \] (2.7)

where \( I \) is the angular momentum of the total system and \( R \) is that of the rotor, which is equal to \( I - J \) where \( J \) is the angular momentum of the quasiparticles. The term proportional to the total spin, \( I \), does not contribute to \( M1 \) transitions. It is convenient for later discussion to decompose the operator into the orbital and the spin part:

\[ \mathcal{O}^{(l)}(M1, \mu) = -\sum_l g_R l_{i,n} \] (2.8)

and

\[ \mathcal{O}^{(s)}(M1, \mu) = \sum_l (g_s^{(l)} - g_R) s_{i,n}. \] (2.9)

Since we intend to describe an odd-\( N \) nucleus in the space spanned by one neutron quasiparticle configurations, \( g_l \) is set to be zero while \( g_R \) and \( g_s \) are supposed to take on finite values. It should be noted that the \( M1 \) operator (2.7) contains a term proportional to \( I \) even with vanishing \( g_l \). We assume the bare value of the neutron for \( g_s \), i.e., \( -3.82 \) in units of \( (e\hbar/2M_{pc}) \). \( g_R \) is expected to be \( Z/A \) from a simple physical picture. This may not hold strictly, but we expect as a working hypothesis that it is around this value.

§ 3. The results and discussion

The energy spectrum calculated is presented in Fig. 2. The correct spin value is obtained for the ground state. The band of interest to us, which we will call band ‘A’, is properly reproduced. The levels of \( I=9/2+ \) even and the ones of \( I=9/2+ \) odd deviate from a smooth line downward and upward as is shown in Fig. 1, respectively, in accordance with the experimental data. The shift is somewhat too large, and this may be related with the Coriolis attenuation problem. In addition to band ‘A’, there are two other bands which have not been identified yet but which we will see have characteristic properties. Band ‘A’ has its largest probability at \( 5/2 \) \([523]\), which decreases in an oscillatory way from 0.99 at 5/2 to 0.60 at 29/2 as spin increases. The second largest probability is at 3/2 \([521]\), increasing as spin. The opposite applies to band ‘B’.

In Fig. 3 illustrated are the calculated \( B(M1) \)'s in band ‘A’. They are well compared with the experimental data. The experimentally observed variation of the \( B(M1) \)'s with spin and their smallness are properly reproduced. We have assumed \( g_R = 0.385 \) instead of \( Z/A = 0.405 \). The assumed number for \( g_R \) is a phenomenological one and may be correlated with the adopted values of the other parameters. It, therefore, should not be taken too strictly, but it certainly demonstrates that the present model
is able to give a reasonable description of the data with a slight adjustment of $g_R$. The calculated $M1$ transition probabilities are sensitive to the ratio $g_R/(g_S - g_R)$. We remark that the data were reproduced similarly well in the framework of the cranking model by Oshima et al.\textsuperscript{2)}

Now that we have the wave functions which have good correspondence to the experimental data, we will analyze them to find out what structure they have. Since the structure of rotation-alignment is expected to have developed in them, it is interesting to see how much alignment is produced in them. We define the aligned angular momentum and decompose it into contributions from various $j$'s as follows:

$$i^{(l)} = \sum_R i_R^{(l)}$$

with

$$i_R^{(l)} = \langle \Phi^{(l)}_l | R \cdot j^{(l)}_+ | R \rangle,$$

where $\Phi^{(l)}_l$ denotes the eigenfunctions of the Hamiltonian (2·1). The aligned angular momentum defined above is just the projection of the particle spin on rotor's angular momentum which is regarded as the collective angular momentum. The denominator $|R|^{-1}$ is defined to be

$$\frac{1}{|R|} |R\rangle = \frac{1}{\sqrt{R(R+1)}} |R\rangle,$$

where $|R\rangle$ stands for an eigenstate of $R^2$ with eigenvalue $R(R+1)$. Since it is not an easy task to transform the strong coupling basis functions to the weak coupling ones,
we adopt the approximation to replace it with the expectation value,

\[ \frac{1}{|R|} \rightarrow \frac{1}{\sqrt{\langle \Phi_{IM}:R^2:\Phi_{IM} \rangle}}. \quad (3.4) \]

This approximation is poor when the fluctuation, \( \Delta R \), is comparable to the mean value \( \bar{R} \). This may happen when \( I \) is small. In Eq. (3.2) \( j_p(\rho) \) is defined as

\[ j_p(\rho) = \sum_{a,a'} \langle j_p \rho | \sum_i j_i \rho a_i a_i \rho a_i \rangle, \quad (3.5) \]

where \( a_i^\dagger \) and \( a_i \) are creation and annihilation operators of a nucleon with the indicated quantum numbers. It is understood that the index \( \rho \) refers to the \( \rho \)-th \( j \)-shell. In Eqs. (3.2) and (3.4), \( : \) represents the normal product of the operators with respect to the intrinsic BCS vacuum. This is necessary to exclude the vacuum contribution to the alignment. All the \( j \)-components of the aligned angular momenta are calculated and illustrated for bands 'A' and 'B' in Fig. 4. Immediately we notice a remarkable feature in the figure: The aligned angular momentum is large in levels of high spin, but it does not come from all the \( j \)-orbitals lying around. In band 'A', the \( h_{9/2} \) shell predominantly contributes to rotation-alignment, while the \( f_{7/2} \) shell gives small alignment, which is negative except at low spins. On the other hand, band 'B'

Fig. 4. \( j \)-decomposed rotation-aligned angular momenta for band 'A' and band 'B'. We should bear in mind that they are not quite accurately evaluated at low spins. See the text for the definition of the \( j \)-decomposed rotation-aligned angular momenta.

Fig. 5. Magnetic dipole transition amplitudes coming from the intrinsic spin and the orbital angular momentum. These are matrix elements of operators (2.8) and (2.9).
receives major portion of its alignment from the \( f_{7/2} \) shell and some from the \( h_{11/2} \) shell, and small but negative alignment from the \( h_{9/2} \) shell. It should also be noted that the alignment from the \( h_{9/2} \) shell in band ‘A’ is larger in \( I = 9/2 + \text{even} \) and smaller in \( I = 9/2 + \text{odd} \). The alignment from \( f_{7/2} \) and \( h_{11/2} \) is larger in \( I = 7/2 + \text{even} \) and smaller in \( I = 7/2 + \text{odd} \). This structure has been recognized for the first time and may be useful to understand various features of normal-parity rotational bands in the future.

We can evaluate the contribution from each \( j \) to the energy gain due to the rotation alignment as

\[
-\frac{\hbar^2}{2\mathcal{J}} \langle \Phi_{MM}| \mathbf{j} \cdot \overrightarrow{p} | \Phi_{MM} \rangle = -\frac{\hbar^2}{2\mathcal{J}} (i_p(\mathbf{j}) + \langle \mathbf{j} \cdot \overrightarrow{p} \rangle_i) \sqrt{\langle \mathbf{R}^2 \rangle_i},
\]

where \( \mathbf{I} \) denotes the projection of \( \mathbf{I} \) onto the 1-2 plane. Band ‘A’ gains about \(-10\) keV at \( 5/2 \) up to \(-450\) keV at \( 29/2 \) through the rotation alignment of the \( h_{9/2} \) shell. Band ‘B’ receives about \(-290\) keV and \(-100\) keV through the rotation alignment of the \( f_{7/2} \) shell and the \( h_{11/2} \) shell, respectively, at \( I = 31/2 \). The energy gain coming from the rotation alignment oscillates regularly with spin as a result of the oscillation of the alignment. This explains the variation of energy spectrum with spin observed in the experiment.

We have found that band ‘A’ is characterized by predominant contribution of \( h_{9/2} \) to the rotation alignment. In § 1, however, it was remarked that a naive argument would predict the \( M1 \) transitions to behave opposite to the experimental data for a band with predominant alignment of \( h_{9/2} \). In order to see the details, the contributions from the spin and orbital angular momenta to the \( M1 \) transition amplitudes are calculated separately and illustrated in Fig. 5. The matrix elements of operators \((2.8)\) and \((2.9)\) are denoted by \( \langle l \rangle \) and \( \langle s \rangle \). It is seen that the naive argument is actually correct in the sense that the spin and the orbital part of the \( M1 \) transition amplitudes oscillate in such a way that they are larger in magnitude for the transitions from \( I = 9/2 + \text{even} \) to \( I - 1 = 9/2 + \text{odd} \) than for the transitions of the opposite direction for \( I > 15/2 \). However, since the spin and the orbital angular momentum are antiparallel in \( h_{9/2} \), the contributions from them to the \( M1 \) transitions cancel out each other to a large extent. This cancellation has already been pointed out by Oshima et al.\(^{2}\) by doing numerical calculations in the cranking model. Still, if only the \( h_{9/2} \) shell were involved in band ‘A’, the rigorous proportionality would hold between the spin and the orbital part of the \( M1 \) transition amplitudes, and the characteristics of their variation with the total angular momentum as a pure \( h_{9/2} \) rotation-aligned band would survive as expected from the naive argument. Actually, however, various shells other than \( h_{9/2} \), especially \( f_{7/2} \) among them, are involved in band ‘A’. After the major two amplitudes from \( h_{9/2} \) being canceled out to a large extent, we are left with transition amplitudes coming from them. They give rise to transition probabilities which are larger for the transitions from \( I = 9/2 + \text{odd} \) to \( I - 1 = 9/2 + \text{even} \) over the ones from \( I = 9/2 + \text{even} \) to \( I - 1 = 9/2 + \text{odd} \). The extent of cancellation shown in Fig. 5 is such that

\[
\left| \frac{\langle l \rangle + \langle s \rangle}{\langle l \rangle} \right|^2 < \frac{\langle l \rangle + \langle s \rangle}{\langle s \rangle} < 0.026.
\]
The situation is quite different in band \('B'\) from in band \('A'\). The \(M1\) transition amplitudes coming from the spin and the orbital angular momentum are additive for \(f_{7/2}\), which predominantly contribute to the rotation-alignment in band \('B'\). Therefore the \(M1\) transitions in band \('B'\) are predicted to be strong and to have similar variation with the total spin as a pure \(f_{7/2}\) rotation-aligned band has (see Fig. 6).

The \(B(M1)'s\) in band \('C'\) are also presented in Fig. 6. This band is largely based on \(11/2 \[505\]\) and the calculated alignment is less than 1 even at high spins, e.g., at \(I=31/2\). The \(B(M1)'s\) change smoothly with spin and are well represented by squared Clebsch-Gordan coefficients as prescribed by the strong coupling model. Therefore there seems not much to discuss this band. However, it should be noted that the contribution of \(h_{11/2}\) to the \(B(M1)'s\) in band \('A'\) is small but not negligible after the strong cancellation of major amplitudes. This suggests that the magnetic dipole matrix elements between \(h_{9/2}\) and \(h_{11/2}\) play a role in the \(B(M1)'s\) in band \('A'\). Its evaluation is correlated with the parameter \(\delta \epsilon(h_{11/2})\), which in turn affects the position of band \('C'\) in a straightforward manner. Thus the experimental identification of this band would help our understanding of band \('A'\).

The calculated results show another interesting aspect of these bands. There are rather strong interband \(M1\) transitions. The \(B(M1)'s\) of the transitions from \([I, \text{band } 'A']\) to \([I-1, \text{band } 'B']\) are of appreciable size and change regularly with spin as seen in Fig. 7. On the other hand, the calculated transitions from \([I, \text{band } 'B']\) to \([I-1, \text{band } 'A']\) are weak and do not show regular variation with spin. In view that these two kinds of transitions involve rather similar wave functions, namely, that the initial (final) state wave function of the first transitions is very similar in structure to the final (initial) state wave function of the second kind of transitions, they seem to suggest coherent interference in the first kind and destructive interference in the second kind of transitions.
§ 4. Summary

We have applied the particle plus symmetric-rotor model to describe the energy spectrum and $B(M1)$ values of the ground state rotational band of $^{163}\text{Dy}$. The calculation has reproduced the characteristic features of the data rather well. The band in question has the largest amplitude at $5/2$ [523] and the secondly largest at $3/2$ [521] (band 'A'). In addition to this, two other low-lying bands are predicted: one based on $3/2$ [521] with substantial coupling to $5/2$ [523] (band 'B') and the other based on $11/2$ [505] (band 'C'). We have calculated the $j$-decomposed rotation-alignment and found that the alignment grows in these normal-parity rotational bands as the total spin increases.

A noteworthy point of the results is that the rotation-alignment produced in a state by the action of the Coriolis interaction does not come from all the $j$'s lying around but from only a specific few of them. This is a feature of a normal-parity rotational band recognized for the first time in the present study. We may refer to this feature as $j$-discrimination of rotation-alignment. The rotation-alignment of the ground state rotational band of $^{163}\text{Dy}$ is found to come solely from the $h_{9/2}$ shell. The magnetic dipole transition matrix elements calculated in the band clearly show the expected characteristics of the $h_{9/2}$ rotation-aligned band, if we look at the spin and the orbital contribution separately. The two contributions, however, have opposite signs and are similar in size, and cancel out each other to a large extent. This explains the extreme smallness of the transition probabilities. Then contributions from other $j$'s give rise to $B(M1)$'s which look opposite to what would be expected for an $h_{9/2}$ rotation-aligned band and which compare well with the data.

We should note that $j$-discrimination of rotation-alignment is expected not only in band 'A' but also in band 'B', and that the two bands interact rather strongly. In this sense band 'B' can be considered to be a counterpart of band 'A'. We have seen that the aspect of band 'A' as an $h_{9/2}$ rotation-aligned band cannot be seen through the $M1$ transitions. In band 'B', however, the rotation-alignment of $f_{7/2}$ and $h_{11/2}$ is expected to be seen through the $M1$ transitions, since the spin and the orbital angular momentum contribution are additive for the $M1$ transition amplitudes in this band. Thus the observed $M1$ transitions of the 'A' and 'B' bands are completely different in nature from each other. We have found that the $M1$ transitions of the 'A' band suffer from heavy cancellation and do not reflect at all the important characteristic property of rotation-alignment which is produced by the Coriolis interaction and which is actually observed in the energy spectrum. On the other hand the $M1$ transitions of the 'B' band do not suffer from such strong cancellations and display the characteristic property of rotation-alignment consistently with its energy spectrum. The inter-band $B(M1)$'s between these two bands are also calculated and their behavior is found very characteristic.

We have seen that the calculated energy spectrum shows signature-dependence stronger than the experiment (Fig. 1). This imprecision would be seen in the calculated $M1$ transition probabilities also. It is interesting to see how serious it is. From Fig. 1 we can approximately express the energy spectrum as
where \( \alpha_t \) is the term which does not involve the signature phase explicitly. We see \( \alpha_{\text{theor}} \approx 0.1 \text{ keV} \) to 1 keV while \( \alpha_{\exp} \approx 0.1 \text{ keV} \). In the present model \( \alpha \) may be written to a good approximation as

\[
\alpha = \frac{1}{2I} \frac{\hbar^2}{2} \sum_{\xi, \eta} c_{\xi, \eta} a \langle \xi, \eta, \Omega = 1/2 | i \rangle i | \xi, \eta, \Omega = -1/2 \rangle ,
\]

where \( \xi \) and \( \eta \) stand for Nilsson quantum numbers beside \( \Omega \). Thus probably \( c_{\alpha = 1/2} \)'s may have been overestimated approximately by \( \sqrt{\alpha_{\text{theor}}/\alpha_{\exp}} \approx 3 \) at worst. The \( M1 \) transition probabilities can be similarly written as

\[
B(M1: I \rightarrow I - 1) \approx [\beta_{I, I - 1} + \beta_{I, I + 1}(-1)^{I - 1/2}]^2 .
\]

Here \( \beta \) may be written as

\[
\beta_{I, I - 1} = \sum_{\xi, \eta} c_{\xi, \eta} a \langle \xi, \eta, \Omega = 1/2 | (t_i - g_i) t_i + (s_i - g_i) s_i | \xi, \eta, \Omega = -1/2 \rangle ,
\]

and \( \beta_{I, I - 1} \) is the part which does not involve the signature phase explicitly. It is expected that the \( \beta \) term may be overestimated by factor 10 at worst because of the possible imprecision contained in \( c_{\alpha = 1/2} \)'s. However, as we have already seen, the contributions from spin and orbital angular momenta cancel out to a very large extent and the \( \beta \) term becomes extremely small. This means the possible imprecision in \( c_{\alpha = 1/2} \)'s manifests itself with very small numbers multiplied. It is therefore expected that the inaccuracy is fortuitously less serious in the \( B(M1) \)'s calculated.

The energy calculated for the 3/2– level of band 'B' is 183 keV, which is a little too low compared with the experimental value, 422 keV. The fit would have been improved by varying the parameters in a systematic way. The present fit, however, is not too bad and we hope the conclusion derived here would remain valid.

Finally, there should be a word about the coupling to \( \gamma \)-vibration. It is reasonably expected that \( \gamma \)-vibration is excited with non-vanishing probability in the levels of interest and we know that it influences \( E2 \) transitions to a noticeable extent.5–7 It affects \( M1 \) transitions and energy spectra also. In the present case, however, we have seen that the \( B(M1) \)'s of interest suffer very strong cancellation and that the experimental data can be reproduced rather well without introducing \( \gamma \)-vibration. In view of these circumstances, we feel that we need more experimental information in addition to these \( B(M1) \)'s to discuss the effects of \( \gamma \)-vibration.

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