Electroweak Gauge Theory out of $SU(2/1)$ Gauge Theory

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(Received November 30, 1990)

The $SU(2) \otimes U(1)$ electroweak gauge theory derived out of an $SU(2/1)$ gauge theory is investigated from a point of view that the $SU(2/1)$ symmetry is an underlying supersymmetry realized in internal lines of the Feynman diagrams of an $SU(2) \otimes U(1)$ gauge theory. The action with such an underlying supersymmetry is derived from one for the $SU(2/1)$ symmetry by eliminating ghost fields which are peculiar to the gauge theories of internal supersymmetry. In particular, it is shown that in this theory, the Faddeev-Popov fields coming from the gauge fixing term for fermionic gauge transformations play the role of Higgs-like scalar fields.

§ 1. Introduction

The well-established standard electroweak theory proposed by Weinberg and Salam$^1$ is known as a prototype of the later attempts to unify all forces based on gauge theories. A lot of those attempts of unification were tried to embed the gauge group $SU(2) \otimes U(1)$ of the electroweak theory to a simple group which includes the symmetry group of strong interaction.$^2$ Then the ratio of the weak $SU(2)$ coupling constant $g$ to that of weak hypercharge $g'$ is fixed by the symmetry, so that the Weinberg angle $\tan \theta_W = g'/g$ can be determined theoretically.

Another line of attempt embedding $SU(2) \otimes U(1)$ to a simple Lie supergroup, such as $SU(2/1)$, was proposed by Ne'eman, Taylor and other authors$^3$ by considering the supertraceless nature of the leptonic fields: $Q(\nu_e L) + Q(e_L) - Q(e_R) = 0$. Here $Q$ stands for the charge or the weak hypercharge of those fields. The $SU(2/1)$ symmetry can naturally accommodate generations of quarks in its four-dimensional multiplets and the Weinberg angle in $SU(2/1)$ gauge theories can be determined as $\theta_W = 30^\circ$, under a specific normalization of the group generators, without depending on the nature of strong interactions.$^4$ This type of gauge theories was also discussed in order to find the possibility unifying the Higgs-scalar fields and gauge fields into a gauge-field multiplet.$^5$ We, further, remark that the Kaluza-Klein theory associated with such an internal supersymmetry is adequate to remove the problem of the Planck scale fermion mass and that of huge cosmological constant.$^6$

In practice, however, such an internal supersymmetry requires that either the $SU(2)$ doublet $(\nu_e, e)_L$ or the singlet $(e)_R$ must obey abnormal commutation relations in the sense of the spin statistics relation. In addition, in the Lagrangian for the $SU(2/1)$ gauge fields, the signs of the kinetic terms come to be indefinite because of the noncompactness of the $SU(2/1)$ group.$^7$ Therefore, in spite of its interesting features, this line of extension of the $SU(2) \otimes U(1)$ to a simple group is not always adequate to apply to realistic weak interactions, directly.

The purpose of this paper is to study the possibility regarding the $SU(2/1)$ as an
underlying symmetry in a $SU(2) \otimes U(1)$ invariant Lagrangian $\mathcal{L}_{\text{eff}}$, which is obtained from an $SU(2/1)$ invariant Lagrangian $\mathcal{L}$ by performing the path integral over the ghost fields in the partition function associated with $\mathcal{L}$.

In the next section, we shall derive an $SU(2) \otimes U(1)$-invariant Lagrangian $\mathcal{L}_{\text{eff}}$ which exhibits the underlying $SU(2/1)$ symmetry in the internal lines of the Feynman diagrams. This will be done by eliminating ghost fields from an $SU(2/1)$ Lagrangian $\mathcal{L}$ by the method of path integral. In particular, it is shown that the Faddeev-Popov fields which define the gauge fixing term for fermionic gauge transformations in $\mathcal{L}$ play the role of Higgs-like-scalar fields. The resultant effective Lagrangian consists of one-loop quantum effects of ghost fields in addition to the terms with ordinary fields for $SU(2) \otimes U(1)$ subgroup in the starting Lagrangian. However, since the terms coming from the quantum effect are divergent, we have to regularize those terms by an appropriate method in order to compare the resultant Lagrangian with that of the standard model. This will be done in § 3 and there, a low energy effective theory, which should be compared with the standard model, is derived.

Section 4 is devoted to a summary and discussion.

§ 2. Effective action with underlying $SU(2/1)$ symmetry

In a simple version of the $SU(2/1)$ symmetry, the weak $SU(2)$ doublet $(\nu_e, e)_L$ and the singlet $(e)_R$ are incorporated into a multiplet of the fundamental representation of $SU(2/1): (\nu_e, e)_L, (e)_R)$. In this case, however, the Lorentz group cannot commute with the $SU(2/1)$ gauge group. In this paper, we deal with the realistic leptonic fields as elements in a pair of $SU(2/1)$ multiplets $\Psi = ((\nu_e, e)_L, (e')_R)^T$ and $\Phi = ((\nu'_e, e')_R, (e)_R)^T$. Here, the fields with "prime" stand for the ghosts which spoil the spin statistics relation. In this case, the Lorentz group is commutable with the gauge group.

Now, in the fundamental representation of $SU(2/1)$, the generators $\lambda^A (A = 1, \cdots, 8)$ have the same form as that of Gell-Mann's $\lambda$ matrices except $\lambda_8$, since the generators must be supertraceless instead of traceless. The $\lambda^A$'s are, then, characterized by the super-commutation relations and the normalization:

\[
\begin{align*}
[\lambda^A, \lambda^B] &= \lambda^C \delta_AB - (-1)^{AB} \lambda_B \lambda^A = i f_{ABC} \lambda^C \\
\text{str}(\lambda_A) &= (\lambda_A)_{11} + (\lambda_A)_{22} - (\lambda_A)_{33} = 0 \\
\text{tr}(\lambda_4 \lambda_8) &= \frac{1}{2} \delta_{AB} ,
\end{align*}
\]

(2.1)

where the structure constants have nonzero elements for $^*$

\[
f_{\dot{a} \dot{b} \dot{c}} , f_{\dot{a} \dot{b} \dot{d}} \text{ and } f_{\dot{a} \dot{b} \dot{c}} .
\]

(2.2)

In particular, the explicit forms of $\lambda_5$ and $\lambda_8$ are (Appendix A):

$^*$ The vector indices of the internal symmetry are grouped as $(A) = (\dot{a}, \dot{i})$: $\dot{a}, \dot{i}, \cdots$ run over 1, 2, 3, 8, while $i, j, \cdots$ run over 4, 5, 6, 7. The suffixes $a, b, \cdots$ also take on values 1, 2, 3. Further, $(-1)^a$ stands for the Grassmann parity defined so that $(-1)^2 = 1$ and $(-1)^3 = -1$. 

where the row and the column indices of $\eta_{AB}$ are placed in the order $A, B=1, 2, 3, 8, 4, 5, 6, 7$.

In terms of these generators, the $SU(2/1)$ covariant derivative acting on $\Psi_L, \Phi_R$ can be defined as

$$D_\mu = \partial_\mu - ig(W_\mu(x) + \xi_\mu(x)) = D_\mu - ig\xi_\mu(x), \quad (D_\mu = \partial_\mu - igW_\mu(x))$$

where $W_\mu(x) = W^a_\mu(x)\lambda^a$ and $\xi_\mu(x) = \xi^i_\mu(x)\lambda_i$ are bosonic and fermionic gauge bosons, respectively. The fermionic property of the $\xi^i_\mu(x)$'s is owing to that the $\lambda^i$'s cause the mixing between the fermionic (bosonic) weak $SU(2)$ doublet and the bosonic (fermionic) singlet of matter fields.

The Lagrangian invariant under local $SU(2/1)$ transformations, now, can be constructed easily and is given by*)

$$\mathcal{L} = -\frac{1}{2} \text{str}(G_{\mu\nu}G_{\mu\nu}) + \overline{\Psi}_L i\gamma^\mu D_\mu \Psi_L + \overline{\Phi}_R i\gamma^\mu D_\mu \Phi_R = \mathcal{L}_G + \mathcal{L}_M$$

with the definition of the field strength

$$G_{\mu\nu} = (i/g)[D_\mu, D_\nu] = F_{\mu\nu} + [D_{[\mu}, \xi_{\nu]}] - ig[\xi_\mu, \xi_\nu]$$

and $F_{\mu\nu} = (i/g)[D_\mu, D_\nu]$. In Eq. (2·6), we have written the Lagrangian density of pure gauge fields and that of matter fields as $\mathcal{L}_G$ and $\mathcal{L}_M$, respectively.

In what follows, we deal the above Lagrangian with an additional gauge fixing term with a view to eliminating the ghost fields $\xi_\mu = \xi^i_\mu(x)\lambda_i$ by means of the path integral. For this purpose, it is sufficient to fix the gauge symmetry caused by $\lambda_i, (i=4, 5, 6, 7)$ only; we put the gauge fixing term and the F. P.-fields term as (Appendix B)

*) $g_{\mu\nu} = (\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, (\mu, \nu=0, 1, 2, 3)$. We also read the $\gamma$-matrices as $\gamma^a = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, $(\gamma^a) = (\frac{1}{2}, \sigma)$, $(\gamma^a) = (\frac{1}{2}, -\sigma)$ and $\gamma^a = i\gamma^a\gamma^b\gamma^c$. \[\]
where
\[ (D_{\mu})^j = \partial_\mu \delta^j - ig W_{\mu}^\alpha (t_\alpha)^j, \quad (t_\alpha)^j = \frac{i}{\alpha}; \quad \alpha = 1, 2, 3, 8. \]

As shown in Appendix A, the \( t_\alpha \)'s are a \( SU(2) \otimes U(1) \) subset of \( SU(2/1) \) (adjoint-representation) generators \( t_\beta \), \( (t_\beta)^c = if_{\alpha \beta}^c \), to which matrix indices are restricted to \( A, C = 4, 5, 6, 7 \) and so, \( \mathcal{L}_{CF+FP} \) in Eq. (2.8) still preserve the \( SU(2) \otimes U(1) \) symmetry. We also note that the Faddeev-Popov field \( C_i \)'s, here, are ordinary scalar fields unlike the case in the usual gauge theory.

Now, the effective Lagrangian invariant under the local \( SU(2) \otimes U(1) \) transformation is obtained by carrying out the path integral with respect to the ghost fields in the partition function associated with \( \mathcal{L} \). To do this, it is convenient to rewrite the \( \mathcal{L}_G \), by introducing auxiliary tensor fields \( Q_{\mu} = Q_{\mu}^\alpha t_\alpha \), in the following form:

\[
\mathcal{L}'_G = -\frac{1}{2} \text{str} [F^\mu\nu F_{\mu\nu} + [D^\mu, \xi^\nu][D_{\mu}, \xi_{\nu}]] \\
+ \Omega^\mu\nu \Omega_{\mu\nu} - 2ig (F^\mu\nu - i\Omega_{\mu\nu}) [\xi^\mu, \xi_{\nu}] \\
= -\frac{1}{4} F_{\mu\nu}^\alpha F_{\alpha\nu}^\alpha - \frac{1}{4} \Omega_{\mu\nu} \Omega_{\mu\nu} + \frac{1}{2} \xi^{\mu \nu} i(M_{\mu\nu})^j \xi^j, \\
\]

Here, "boldface" are used for writing the fields associated with the generators \( (t_\alpha) \); such as, \( W_\mu = W_\mu^\alpha t_\alpha, \quad F_{\mu\nu} = F_{\mu\nu}^\alpha t_\alpha \). One can see that the right-hand side of Eq. (2.10) can be reduced easily to \( \mathcal{L}_G \) by using equations of motion of \( \Omega_{\mu\nu} \) and by taking Eqs. (2.2) and (2.4) into account. In addition, the equivalence between Eqs. (2.10) and (2.11) can also be verified with the help of properties of the "supertrace".

Keeping the later path integral over \( \nu'_{ER}, e'_L \) and \( e'_R \) in mind, we further rewrite the Lagrangian density \( \mathcal{L}_M \) in the form decomposed into its ordinary and ghost components:

\[
\mathcal{L}_M = i\bar{\mathcal{L}}^\gamma_\mu (-\partial_\mu - ig W_\mu) L + i\bar{\mathcal{R}}^\gamma_\mu (-\partial_\mu - ig W^\mu \lambda_\mu) R \\
+ i\bar{\mathcal{L}}'^\gamma_\mu (-\partial_\mu - ig W^\mu \lambda_\mu) L' + i\bar{\mathcal{R}}'^\gamma_\mu (-\partial_\mu - ig W_\mu) R' \\
+ g(\bar{\mathcal{L}}' \xi^\mu \gamma_L + \bar{\mathcal{L}} \xi^\mu \gamma_R + \bar{\mathcal{L}}' \xi^\mu \gamma_L + \bar{\mathcal{R}}' \xi^\mu \gamma_R),
\]

where \( L = (\nu_{EL}, e_L, 0)^T, \quad R = (0, 0, e_R)^T, \quad L' = (\nu'_{ER}, e'_L, 0)^T \) and \( R' = (\nu'_{ER}, e'_R, 0)^T \). One can see that the Lagrangian densities \( \mathcal{L}'_G \) and \( \mathcal{L}_M \) given in Eqs. (2.11) and (2.13) are bi-linear forms of ghost fields and as such, it is not difficult to integrate over those fields.

First, the path integral with respect to \( L' \) and \( R' \) gives
Here, $\overline{W}_\mu$ is the restriction of $W_\mu$ to a $2 \times 2$ matrix in the $SU(2)$ subspace,

$$L'' = L' + L_{GF+FP} + i\overline{L}(\partial - i g W) L + i\overline{R}(\partial - i g \lambda_5 W^8) R + \frac{1}{2} \xi_{\mu\nu} (N_{\mu\nu})^2 \xi^\nu$$

(2·15)

and

$$i (N_{\mu\nu}) = -2 i g^2 (\overline{L} \gamma_\mu \lambda_5 (\partial - i g W^8) \lambda_5 + \overline{R} \gamma_\mu \lambda_5 (\partial - i g W)^{-1} \gamma_\nu \lambda^5).$$

(2·16)

Second, carrying out the path integral with respect to the $\xi_{\mu\nu}$'s, we get

$$e^{i S_{\text{eff}}} = \exp \left[ i \int d^4 x \left( i \overline{L}(\partial - i g W) L + i\overline{R}(\partial - i g W^8) R \right) \
\left. \right. \
- \left. \left. \frac{1}{4} F_{\mu\nu} F_{\mu\nu} + D^\mu C^\nu + D_\mu C \right. \right] \
\left. \right. \
\times \det \left( i \overline{L} (\partial - i g W) \right) \det \left( i \overline{R} (\partial + i g W^8 / \sqrt{3}) \right) \
\left. \right. \
\times \int \mathcal{D}Q \left[ \det \left( M'_{\mu\nu} + N_{\mu\nu} \right) \right]^{1/4} \exp \left[ -(i/4) \int d^4 x \Omega_{\mu\nu} \Omega_{\mu\nu} \right],$$

(2·17)

where

$$i (M'_{\mu\nu}) = i (g_{\mu\nu} D^2 - i g (2 F_{\mu\nu} - i \Omega_{\mu\nu})) t^i + 2 g^2 g_{\mu\nu} (C^+ t^i) t^i (t_a C).$$

(2·18)

From Eq. (2·17), one can get finally the formal expression to the effective action in the following form:

$$S_{\text{eff}} = \int d^4 x \left[ i \overline{L}(\partial - i g W) L + i\overline{R}(\partial - i g \lambda_5 W^8) R - \frac{1}{4} F_{\mu\nu} F_{\mu\nu} + D^\mu C^\nu + D_\mu C \right]$$

$$+ i \log \det \left( i \overline{L} (\partial - i g W) \right) + i \log \det \left( i \overline{R} (\partial + i g W^8 / \sqrt{3}) \right)$$

$$- i \log \int \mathcal{D}Q \left[ \det \left( M'_{\mu\nu} + N_{\mu\nu} \right) \right]^{1/4} \exp \left[ -(i/4) \int d^4 x \Omega_{\mu\nu} \Omega_{\mu\nu} \right].$$

(2·19)

The right-hand side of Eq. (2·19) is invariant under the $SU(2) \otimes U(1)$ gauge group; the $SU(2/1)$ is an underlying symmetry of $S_{\text{eff}}$. The expression of $S_{\text{eff}}$ also suggests that the (bosonic) F.P.-fields $(C^i, C^\ast)$ play the role of the Higgs fields. Hereafter, we shall regard the $S_{\text{eff}}$ as the fundamental action in this formalism, in which we are not worried by the problem of ghosts which spoil the spin statistics relation.

§ 3. Low energy effective theory

The effective action (2·19) comes to have a definite meaning under an appropriate regularization for the determinant terms, since those terms are divergent. In what follows, we try to regularize those terms, simply, by introducing cutoff parameters.
and compare the result to the standard low energy theory.

For this purpose, it is convenient to use an approximation formula for $\det H$, which is applicable for a hermitian operator $H$ such as

$$H = p^2 + V(p, x), \quad (V(p, x) = V^{(0)}(x) + p^a V^{(1)}(x), p_\mu = i\partial_\mu).$$

To derive the formula, we first write the log($\det H$) in the following form:

$$i\log(\det H) = \lim_{s_0 \to 0} \int_{s_0}^\infty ds (is)^{-1} \int d^4x \text{tr} \langle x | e^{isH} | x \rangle,$$  \hspace{1cm} (3·1)

where $\{|x\rangle\}$ are the eigenstates of $x^a$ normalized so that $\langle x | x' \rangle = \delta^a(x - x')$ and "tr" denotes the trace with respect to the finite matrix indices of $H$. In Eq. (3·2), it is also understood that $H \to H + i\varepsilon$, with $\varepsilon$ a small positive constant; $\varepsilon$ must be taken to zero after all calculations to remove the infrared divergence in (3·2).

Assuming, here, the expansion

$$\text{tr} \langle x | e^{isH} | x \rangle = i(4\pi is)^{-2}[f_0(x) + (is)f_1(x) + (is)^2f_2(x) + \cdots],$$

Eq. (3·2) becomes

$$i\log(\det H) = \lim_{\Lambda \to \infty} \frac{1}{(4\pi)^2} \int d^4x \left[ \frac{1}{2} \Lambda f_0(x) + \Lambda f_1(x) + \log(\Lambda/\Lambda_0)f_2(x) + O(\Lambda^{-1}) \right].$$ \hspace{1cm} (3·3)

where $is_0 = \Lambda^{-1}$ and $\Lambda_0$ is an arbitrary scale constant. Equation (3·4) says that the divergence part of log($\det H$) is determined by $f_0(x)$, $f_1(x)$ and $f_2(x)$. Since the cutoff parameter $\Lambda$ is sufficiently large, we, hereafter, define the effective action with those three terms. Substituting Eq. (3·1) for the left-hand side of Eq. (3·3), we can get $f^{(0)}$, $f^{(1)}$ and $f^{(2)}$, within the approximation up to the second order of $V$ in the following forms (Appendix D):

$$f_0 = \text{tr}(1),$$

$$f_1 = \text{tr}(V^{(0)} - \frac{1}{4} V^{(1)2}),$$

$$f_2 = \text{tr}\left( \frac{1}{2} V^{(0)2} + \frac{i}{2} V^{(0)}(\partial \cdot V^{(1)}) - \frac{1}{12}(\partial^a V^{(1)}_a)^2 + \frac{1}{24} V^{(1)a} \partial^a V^{(1)}_a \right).$$ \hspace{1cm} (3·4)

We are, now, ready to evaluate the determinants in Eq. (2·19).

i) First, as for $\det(i\sigma \cdot (\partial - ig\overline{W}))$, we read

$$\det(i\sigma \cdot (\partial - ig\overline{W})) \sim \det(i\gamma \cdot (\partial - ig\overline{W})\frac{1+\gamma^5}{2}) \sim [\det(i\partial + g\overline{W})^2]^{1/4} e^{\theta(W)},$$ \hspace{1cm} (3·5)

where $\sim$ is the equality disregarding unimportant constants. The $\theta(W)$ is a topological term, which depends rather on global structure of gauge fields. In what follows, we simply disregard the phase factor $\theta(W)$: That is, we put simply as $\det(i\sigma \cdot (\partial - ig\overline{W})) \sim [\det H]^{1/4}$ with $H = (\partial + g\overline{W})^2$. Then, $V^{(0)}$ and $V^{(1)}$ from this $H$ are obtained as
Electroweak Gauge Theory out of SU(2/1) Gauge Theory

\[ V^{(0)} = -ig(\partial \cdot \bar{W}) - g\sigma^{\mu\nu}(\partial_{\nu} \bar{W}_{\mu}) + g^2 \bar{W}^2, \quad (\sigma^{\mu\nu} = (i/2)[\gamma^{\mu}, \gamma^{\nu}]) \]
\[ V^{(1)} = 2g \bar{W}^2. \]

Substituting Eq. (3.7) for Eq. (3.5), and using \( \text{tr}(\sigma^{\mu\nu}\sigma^{\alpha\beta}) = 4(g^{\mu\rho}g^{\nu\beta} - g^{\mu\beta}g^{\nu\rho}) \), we get
\[ f_1 = 0 \quad \text{and} \quad f_2 = (2/3)g^2 \text{tr}(F_{\mu\nu}F_{\mu\nu}), \]
from which follows:
\[ \frac{i}{4\pi} \log[\det(i\partial + g\bar{W}^8)]^{1/4} \sim \frac{g^2}{6\pi} \log(\Lambda_1/\Lambda_0) \int d^4 x \text{tr}(F_{\mu\nu}F_{\mu\nu}) \]
within the approximation up to the order of \( g^2 \).

ii) Second, for \( \det[i\sigma \cdot (\partial + igW^8/\sqrt{3})] \), as in the previous case, we also read \( \det[i\sigma \cdot (\partial + igW^8/\sqrt{3})] \sim \det[H]^{1/4} \) with \( H = (\partial - gW^8/\sqrt{3})^2 \). Then, the \( V^{(0)} \) and \( V^{(1)} \) become
\[ V^{(0)} = ig(\sqrt{3}/\partial \cdot W^8) + (g/\sqrt{3})\sigma^{\mu\nu}(\partial_{\nu} W^8_{\mu}) + (g/\sqrt{3})^2(W^8)^2, \]
\[ V^{(1)} = -2(g/\sqrt{3})W^8^2, \]
and so, we have
\[ f_1 = 0 \quad \text{and} \quad f_2 = (2/3)(g/\sqrt{3})^2F_{\mu\nu}^8F_{\mu\nu}^8. \]

Therefore,
\[ \frac{i}{4\pi} \log[\det(i\partial - gW^8/\sqrt{3})]^{1/4} \sim \frac{g^2}{18\pi} \log(\Lambda_2/\Lambda_0) \int d^4 x F_{\mu\nu}^8 F_{\mu\nu}^8. \]

iii) Finally, let us evaluate \( \det[(M_{\mu\nu}^\prime + N_{\nu\mu}^\prime)] \) and carry out the path integral over \( Q_{\mu\nu} \) in succession. In this case, the \( M_{\mu\nu}^\prime + N_{\nu\mu}^\prime \) is not a local operator by the presence of the \( N_{\nu\mu} \) term; one cannot use the formula (3.5) directly. However, if we read
\[ \log[\det(M^\prime + N)]^{1/2} \]
then we can use Eq. (3.5) to evaluate \( \log[\det(M^\prime)] \). In this case, the \( V^{(0)} \) and \( V^{(1)} \) defined from \( iH_{\mu\nu} = -iM_{\mu\nu}^\prime \) become
\[ i[V^{(0)}(x)]_{\mu\nu} = i[(-ig(\partial \cdot W(x)) + g^2W^8(x))g_{\mu\nu} + ig(2F_{\mu\nu}(x) - iQ_{\mu\nu}(x))]_{\mu\nu} \]
\[ -2g^2g_{\mu\nu}(C^+(x)t^a)(t_{a\dot{a}}C(x)), \]
\[ i[V^{(1)}(x)]_{\mu\nu} = 2g_i[W_\rho(x)]_{\mu\nu}g_{\mu\nu}, \]
from which we get
\[ f_1 = -4g^2C^+ C, \]
\[ f_2 = \frac{g^2}{2} \text{tr}(2F_{\mu\nu} - iQ_{\mu\nu})(2F_{\mu\nu}^\prime - iQ_{\mu\nu}^\prime) - \frac{g^2}{3} \text{tr}F_{\mu\nu}F_{\mu\nu}, \]
with the help of \( t^a t_\dot{a} = 1/2 \). Hence,
\[-\frac{i}{2} \log[\det(M')] = -\frac{g^2}{2(4\pi)^2} \int d^4x \left[ -4A_3C^+C \
ol intentionally left blank \nol intentionally left blank \right] + \log(\Lambda_3/\Lambda_0) \left\{ \frac{1}{2} \text{tr}(2F - i\Omega)^2 - \frac{1}{3} \text{tr} F^2 \right\} \right]. \quad (3.16)\]

On the other hand, the divergent part of \( \text{Tr} \log (1 + M'^{-1}N) \) can also be calculated by the standard manner and we have

\[-\frac{i}{2} \text{Tr} \log (1 + M'^{-1}N) = -\frac{g^2}{2(4\pi)^2} \log(\lambda_1/\lambda_0) \int d^4x \left( \frac{1}{2} \bar{L}_i \partial L - \bar{R}_i \partial R \right) + o(g^2) \quad (3.17)\]

within the approximation up to the order of \( g^2 \). Here, use has been made of the relations: \((\lambda_{\alpha}')(SU(2)) = -1/2\) and \((\lambda_{\alpha}')_U(1) = 1\).

Since Eq. (3.16) is a bi-linear form of \( \Omega \), the path integral over those fields, in Eq. (2.19), can be carried out easily and yields

\[-i \log \int D\Omega [\det(M' + N)]^{1/2} \exp \left[ -\frac{i}{4} \int d^4x \partial \bar{\Omega} \Omega \partial \bar{\Omega} \right] = \int d^4x \left[ \frac{g^2}{4(4\pi)^2} 2A_3C^+C - \frac{5g^2}{6(4\pi)^2} \log(\Lambda_3/\Lambda_0) \left( F^{\alpha\mu\nu} F_{\alpha\mu\nu} + \frac{1}{3} F^{\mu\nu\rho} F_{\mu\nu\rho} \right) \right] + o(g^2). \quad (3.18)\]

Substituting Eqs. (3.9), (3.12) and (3.18) to Eq. (2.19), the \( S_{\text{eff}} \) becomes

\[S_{\text{eff}} = \int d^4x \left[ i\bar{L}(\partial - ig W)L + i\bar{R}(\partial - ig \partial_8 W^8)R + (D^+C^+)(D_\mu C) - M^2 C^+C \right] - \frac{1}{4} \left( F^{\mu\nu} F_{\mu\nu} + F^{\mu\nu} F_{\mu\nu} \right) + \left( \frac{1 - 5D_3}{3} \right) F^{\mu\nu} F_{\mu\nu} + \Delta L \right] + o(g^2), \quad (3.19)\]

where

\[\Delta L = \frac{D_1}{2} \left( F^{\mu\nu} F_{\mu\nu} + \frac{1}{3} F^{\mu\nu} F_{\mu\nu} \right) + \frac{D_2}{3} F^{\mu\nu} F_{\mu\nu} - 5D_3 F^{\mu\nu} F_{\mu\nu} - 6D_4 \left( \frac{1}{2} \bar{L}_i \partial L - \bar{R}_i \partial R \right) \quad (3.20)\]

and

\[D_i = \frac{5g^2}{6(4\pi)^2} \log \left( \frac{A_i}{\Lambda_0} \right), \quad (i = 1, 2, 3, 4) \quad \text{and} \quad M^2 = -2g^2 \Lambda_3/(4\pi)^2. \quad (3.21)\]

From Eq. (3.20), one can see that \( \Delta L \) has the form of counterterms which can remove the divergence coming from one-loop diagrams of ordinary fields by adjusting cutoff parameters \( D_i (i = 1, 2, 4) \) suitably. On the other hand, the term \( (1 - 5D_3) \times F^{\mu\nu} F_{\mu\nu}/3 \), the one-loop effect of \( \xi_\mu \) to \( W^8 \), has no counterpart of one-loop divergent diagrams of physical fields; hereafter, we use the field \( W^8 \) which is obtained by the rescaling.
We also emphasize that if we take into account the higher order corrections to F.P.-fields, the fourth order potential of those fields comes into the effective action through $\frac{f_2}{2} = \text{tr} \left( V(0) \right)^2$ term; then the mass term $-M^2 C^+ C$ suffers the modification such as

$$-M^2 C^+ C \to -M^2 C^+ C - \lambda (C^+ t^a t_b C)(C^+ t^b t_a C). \quad \lambda = \frac{4 g^4}{(4\pi)^2} \log \left( \frac{\Lambda_3}{\Lambda_0} \right)$$

(3.23)

Since F.P.-fields can be decomposed into two-component $SU(2)$ spinor, $H = (h^+, h^0)^T$ with $Y = 1$ and $U (u^0, u^-)^T$ with $Y = -1$, one can rewrite the Lagrangian density for F.P.-fields with quadratic potential (3.23) in the following form (Appendix C):

$$\mathcal{L}_c = (D^\mu H)^+ D_\mu H + (D^\mu U)^+ D_\mu U - M^2 (H^+ H + U^+ U)$$

$$-\frac{g^4}{4} [ (H^+ H)^2 + 4(H^+ U)(U^+ H) + (U^+ U)^2].$$

(3.24)

Namely, after eliminating $\xi$-fields, the F.P.-fields $C$ behave as a hypercharge doublet of Higgs-like-scalar fields, though they appear only in internal loop diagrams. Therefore, if we include the higher order terms of F.P.-fields in the effective action, the Lagrangian density becomes

$$\mathcal{L}_{\text{eff}} = i \mathcal{L} \left[ \partial - i g \lambda \sigma \partial \right. + i \left. \frac{g'}{2} W^a \right] L + i \mathcal{R} \left( \partial + i g' W^a \right) R - \frac{1}{4} \left( F^{a\mu\nu} F_{a\mu\nu} + F^{8\mu\nu} F_{8\mu\nu} \right)$$

$$+ \mathcal{L}_c + \Delta \mathcal{L}. \quad (g' / g = \tan \theta_w = 1 / \sqrt{20D_3 - 1})$$

(3.25)

Now, in order to see the mechanism corresponding to the spontaneous symmetry breaking in the standard model, we give attention to that the potential for $H$ and $U$ has the minimum at neutral directions $H = (0, v/\sqrt{2})^T$ and $H = (v/\sqrt{2}, 0)^T$ with $v^2 = -4M^2 / \lambda$. Since $(H^+ U) = 0$ at that minimum, we assume that the fields $H$ and $U$ are expanded under the condition $(H^+ U) = 0$ around the minimum in such a form as

$$H = e^{i \theta_a \sigma_a + i \theta} \left[ \begin{array}{c} 0 \\ (v + h) / \sqrt{2} \end{array} \right], \quad U = e^{i \theta_a \sigma_a - i \theta} \left[ \begin{array}{c} (v + u) / \sqrt{2} \\ 0 \end{array} \right],$$

(3.26)

where $h$ and $u$ are real fields. Then, together with a gauge transformation with respect to $W^a_\mu$ and $W^8_\mu$, the $\Theta_a$ and $\theta$ are transformed away and the resultant form of $\mathcal{L}_c$ becomes

$$\mathcal{L}_c = \frac{1}{2} \left[ (\partial h)^\ast (\partial h) + \left( \frac{g}{2 \cos \theta_w} \right)^2 Z^a Z_\mu (v + h)^2 - M^2 (v + h)^2 \right]$$

$$+ \frac{1}{2} \left[ (\partial u)^\ast (\partial u) + \left( \frac{g}{2 \cos \theta_w} \right)^2 Z^a Z_\mu (v + u)^2 - M^2 (v + u)^2 \right]$$

$$+ \frac{g^2}{16} W^+ \mu W^- \mu [(v + h)^2 + (v + u)^2] - \frac{\lambda}{16} \left[ (h + v)^4 + (u + v)^4 \right],$$

(3.27)

where $W^\pm_\mu = (W^1_\mu \pm i W^2_\mu) / \sqrt{2}$ and $Z_\mu = W^8_\mu \cos \theta_w - W^3_\mu \sin \theta_w$, as usual. If we regard
(3.27) as tree Lagrangian density, the constant $v$ in $\mathcal{L}_c$ gives rise to mass terms for $W^\pm_\mu$ and $Z_\mu$ such that

$$M_w^2 = M_Z^2 \cos^2 \theta_w = \frac{g^2 v^2}{2} = \frac{\Lambda_3}{\log(\Lambda_3/\Lambda_0)}.$$  \hspace{1cm} (3.28)

In practice, however, there remains the procedure of one more path integral over the fields $h$ and $u$ in the effective action (3.25), since they are the fields defining F. P. -determinant. If we disregard higher order terms of $g^2$, this procedure adds $i \log[\det \{p^2 + (g/2 \cos \theta_w)^2 Z^2 + (g^2/2) W^+ W^- - M^2\}]$ to the effective action instead of $\mathcal{L}_c$. In this case, the formulae (3.4)~(3.5) have to be slightly modified by the presence of $M^2$-term; however, under the condition $M^2/\Lambda \ll 1$, which we assume, leading terms are again represented by those formulae. Thus, we get finally the following form of $\mathcal{L}_{\text{eff}}$:

$$\mathcal{L}_{\text{eff}} = \mathcal{L} + \frac{i}{4\pi^2} \int d^4x \, \left( \frac{g^2}{2} F_{\mu\nu}^+ F_{-\mu\nu} + \frac{1}{4} F_{\mu\nu}^+ A_{\mu
u} F_{\lambda\rho} - \frac{1}{2} \frac{g^2}{2} W^\pm + \frac{1}{2} W^\pm W^- \right) + o(g^2),$$

where $\Lambda_2 = \Lambda_2 \pm i \varepsilon_0$, $A_\mu = W^3_\mu \sin \theta_w + W^8_\mu \cos \theta_w$ and $e = g \sin \theta_w$. One can see that the above procedure reproduces the same mass terms for $W^\pm_\mu$ and $Z_\mu$ as in Eq. (3.28) provided that $A_4 = (4\pi)^2$; that is, in this sense, the minimum of the potential for $H$ and $U$ is stable against their quantum effect. Then we need not distinguish masses of vector bosons in Eq. (3.28) from those in Eq. (3.29).

We finally remark the relation between the cutoff parameters $(D_3, \Lambda_3)$ and the Weinberg angle $\theta_w$ and $M_w^2$. From the definition of $\theta_w$ in Eq. (3.25), we can get

$$D_3 = (20 \sin^2 \theta_w)^{-1}. \hspace{1cm} (3.30)$$

On the other hand, the definition of $D_3$, Eq. (3.21) with $g = e/\sin \theta_w$, gives $\log(\Lambda_3/\Lambda_0) = 6(4\pi)^2/20 e^2/\approx 516$ by the use of $e^2/4\pi = 1/137$ and so, Eq. (3.28) leads to

$$\sqrt{\Lambda_3} \approx \sqrt{516} M_w. \hspace{1cm} (3.31)$$

If we substitute $M_w = 81 \text{(GeV)}$ for Eq. (3.31), then the energy scale which causes the change of the theory based on the effective action (3.29) is determined as $\sqrt{\Lambda_3} \approx 1840$ (GeV).

§ 4. Summary and discussion

We have derived the low energy effective action (3.29) by taking two steps. In
the first step, the $SU(2)\otimes U(1)$ gauge invariant action (2·19) was derived out of an $SU(2/1)$ gauge invariant action by carrying out the path integral over the ghost fields and by adding the result of integration, the quantum effect due to the ghost loops, to the $SU(2)\otimes U(1)$ part in the original action. Since the quantum effect gives rise to non-local terms, the resultant action has different structure from that of standard Weinberg-Salam theory. In particular, the F.P.-fields, which are introduced through the gauge fixing term for fermionic gauge transformations, become ordinary scalar fields in contrast to the usual case.

In the second step, the low energy effective action with cutoff parameters was derived, by using a proper time method to evaluate ghost loop diagrams. To do this, we first extract ultraviolet-divergent terms out of the divergent terms and discarded the remainder. Then, we regularized them by means of cutoff parameters within the approximation up to the order of $g^4$ except F.P.-fields. As for F.P.-fields, we evaluated $g^4$ term and found that those fields behave like Higgs fields, though they appear only in internal lines of loop diagrams. The introduction of cutoff parameters, unfortunately, spoils some properties of the original $SU(2/1)$ gauge theory but can get rid of the bad sign of the term $F^{\mu\nu}F_{\mu\nu}$ in that theory. In addition, the Higgs-like fields really produce mass terms for vector bosons $W$ and $Z$ associated with those cutoff parameters. In this case, redefinition of Higgs-like fields (3·26) under a condition reflecting the structure of potential for those fields played an essential role. We also emphasize that many of those cutoff parameters work to define counterterms $\Delta L$ which can remove the divergence coming from one-loop diagrams of ordinary fields; only two of those cutoff parameters $(D_5, \Lambda_5)$ remain free parameters, by which the Weinberg angle $\theta_W$ and the $W$-boson's mass $M_W$ are determined. As a result, our approach to the electroweak theory has the same structure as that of the standard model until the energy scale $\sqrt{516} M_W$.

Finally, we remark the following:

i) The quantum effect of ghost fields in the action (2·19) cannot seem to produce mass terms for leptonic fields; accordingly the masses of those fields should be introduced as parameters. In this case, the Dirac mass term $\bar{\Psi}_L M \Psi_R$ spoils $SU(2/1)$ symmetry, since the matrix $M$ must be proportional to $\lambda_i$ ($i=4, 5, 6, 7$) in order to get $\bar{e} L e_R$ from this term. Therefore, a Majorana mass term $m_L \bar{\Psi}_L \Psi_L + m_R \bar{\Psi}_R \Psi_R$ may be suitable in the present formalism and then, massive neutrinos will get Majorana masses.

ii) The present formalism can be extended to a more realistic model including more generations of leptons or quarks. This will be possible in two ways. One way is to extend the group $SU(2/1)$ to a higher rank one such as $SU(5/1)$.

iii) In the original $SU(2/1)$ action (2·6), there are the same number of bosonic and fermionic chiral fields; we can see that there appears no chiral anomaly in the starting theory. On the other side, the low energy effective action looks like to yield chiral anomaly. This means that such an anomaly is expected to be compensated by the $\theta(W)$-terms, which have been disregarded in getting the effective action.
iv) In this paper, the elimination of ghost fields has been done by means of the path integral over those fields. However, in order to verify the unitarity in the resultant theory, it is necessary to study if the degrees of freedom of ghosts are suppressed consistently by conservative constraints \((G_4 + iG_5 \approx 0, \ G_6 + iG_7 \approx 0\) with \(SU(2/1)\) generators \(G_k\) are candidates for these ones).

The approach to the electroweak theory presented in this paper is not complete and contains problems which are the subject for a future study. We believe, however, that the study on the electroweak theory, especially on the Higgs fields, from various points of view is still meaningful to get deeper understanding on that theory.

**Acknowledgements**

The authors wish to express their sincere thanks to Professor O. Hara, Professor S. Ishida, Professor S. Y. Tsai and Dr. S. Deguchi for their interest and discussions. They are also grateful to Professor J. Otokozawa and other members of their laboratory for encouragement.

**Appendix A**

By definition, the explicit forms of the \(\lambda_a\)'s are

\[
\lambda_1 = \frac{1}{2} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad \lambda_2 = \frac{1}{2} \begin{bmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad \lambda_3 = \frac{1}{2} \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad \lambda_8 = \frac{1}{2\sqrt{3}} \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -2 \end{bmatrix},
\]

\[
\lambda_4 = \frac{1}{2} \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad \lambda_5 = \frac{1}{2} \begin{bmatrix} 0 & 0 & -i \\ i & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad \lambda_6 = \frac{1}{2} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad \lambda_7 = \frac{1}{2} \begin{bmatrix} 0 & 0 & 0 \\ 0 & i & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad \text{(A.1)}
\]

under the normalization \((2\cdot 1)\). Then the super-commutation relation \((2\cdot 1)\) determines the structure constants so as to be

\[
f_{ab}^c = F_{ab}^c, \quad f_{ab} = 0, \quad f_{ai} = F_{ai}^i, \quad f_{bi} = F_{bi}^i, \quad f_{ij} = D_{ij}^i, \quad f_{ij} = -\frac{1}{\sqrt{3}} \delta_{ij}, \quad \text{(A.2)}
\]

where the \(F_{ab}^c, D_{ab}^c\) are the usual \(SU(3)\) structure constants defined out of Gell'\text{m}ann's \(\Lambda\)-matrices by \([\Lambda_A, \Lambda_B] = 2iF_{AB}^C \Lambda_C\) \((\Lambda_A, \Lambda_B) = (4/3)\delta_{AB} + 2D_{AB}^C \Lambda_C\) . With those structure constants, the generator in the adjoint representation are given as

\[
\Lambda(T_b)^C_{ab} = iF_{ab}^C, \quad \Lambda(T_b)^C_{ab} = (A, B, C = 1, \cdots, 8), \quad \text{to which one can find}
\]

\[
\Lambda(T_b)^C_{ab} = -(-1)^{AB} (T_a)^C_{ab}, \quad \Lambda(T_b)^C_{ab} = (-1)^{A(B+C)} (T_c)^C_{ab} \quad \text{etc.} \quad \text{(A.3)}
\]

under the rules for raising and lowering the suffixes:

\[
\Lambda(T_b)^D_{ab} = \eta_{DC}, \quad \Lambda(T_b)^C_{ab} = \eta^{AD} (T_a)^D_{bc}. \quad \text{(A.4)}
\]

Now, from Eq. \((A\cdot 2)\), one can see that \(T_a^b\) \(a = 1, 2, 3, 8\) form a sub-algebra \(SU(2) \otimes U(1)\) and satisfy \((T_a^b)_{ab} = (T_a^b)_{ba} = 0\). In other words, the \(4 \times 4\) matrices defined
by \( \{ t^a \} = \{ T^a \} = \{ i \delta_{ij} \} \) form the generators of \( SU(2) \otimes U(1) \). The explicit forms of the \( t^a \)’s are

\[
t_1 = \frac{1}{2} \begin{bmatrix} 0 & \sigma_2 \\ \sigma_2 & 0 \end{bmatrix}, \quad t_2 = \frac{1}{2} \begin{bmatrix} 0 & -iI \\ iI & 0 \end{bmatrix}, \quad t_3 = \frac{1}{2} \begin{bmatrix} 0 & 0 \\ 0 & -\sigma_2 \end{bmatrix}, \quad t_8 = \frac{1}{2\sqrt{3}} \begin{bmatrix} \sigma_2 & 0 \\ 0 & \sigma_2 \end{bmatrix}, \]

where \( \sigma_i (i=1, 2, 3) \) and \( I \) are Pauli’s matrices and \( 2 \times 2 \) unit matrix, respectively.

From these expressions, one can verify, directly,

\[
[t_a, t_b] = i\varepsilon_{abc} t_c, \quad [t_a, t_8] = 0 \quad \text{and} \quad \text{tr}(t_\alpha t_\beta) = |\eta_{\alpha\beta}|. \tag{A.6}
\]

### Appendix B

The form of \( \mathcal{L}_{GF+FP} \) in Eq. (2.8) is determined according to the standard method by Faddeev and Popov. First, let us consider the variation of gauge fields \( (W^A_\mu) = (W^a_\mu, \xi^i_\mu) \) under infinitesimal gauge transformations:

\[
\delta W^\mu_A = \delta \omega^\mu = \lambda_a \phi^b (\partial_\mu \phi^c - ig^a_\mu (T^a_\mu \phi)_c) \delta \omega^c. \tag{B.1}
\]

Putting, here \( \delta \omega^a = 0 \) \((a=1, 2, 3, 8)\), Eq. (B.1) yields the specific transformations caused by \( \lambda_a (i=4, 5, 6, 7) \):

\[
\delta W^\mu_a = -ig^a_\mu (\xi^i_\mu T^i_\mu) \delta \omega^i, \quad \delta \xi^i_\mu = [\partial_\mu \delta^i - ig^i_\mu (W^a_\mu T^a_\mu)] \delta \omega^i = i(D^a \delta \omega). \tag{B.2}
\]

Since we are going to fix the symmetry under those transformations, we impose the gauge condition \( F^i = -i(D^a \xi^a) - \rho^i = 0 \), to which one can verify

\[
\delta F^i = i(\delta D^a \xi^a + D^a \delta \xi^a) = [g^a_\mu \xi^a_\mu (T^a_\mu T^a_\mu) \xi^a_\mu + D^a] \delta \omega^i. \tag{B.3}
\]

with the help of \( \delta (T^a_\mu) = \delta^{a}_b (T^b_\mu) = -i(T^b_\mu T^a_\mu) \).

Now, the Faddeev-Popov determinant \( \Delta[\xi] \) is defined, as usual, by

\[
1 = \Delta[\xi] \int D(\delta \omega) \delta[F_\omega] = \Delta[\xi] \det \left( \frac{\delta F^i(x)}{\delta \omega^j(y)} \right), \quad (F_\omega = F + \delta F). \tag{B.4}
\]

where use has been made of the fermionic property of the \( \omega^a \)’s; then, we get

\[
\Delta[\xi] = \left[ \det \left( \frac{\delta F^i(x)}{\delta \omega^j(y)} \right) \right]^{-1} = \int DC^* D\!C \exp \left[ -i \int d^4x C^* (D^2 + g^a_\mu \xi^a_\mu (T^a_\mu T^a_\mu) \xi^a_\mu) C \right]. \tag{B.5}
\]

Here, \((C^*, C)\) \((i=4, 5, 6, 7)\) are bosonic variables owing to the inverse character of the determinant in Eq. (B.5). Furthermore, since

\[
'(T^a_\mu T^a_\mu) = -i(T^b_\mu) \delta^{a}_b (T^b_\mu) = -i(T^a_\mu T^a_\mu) = -i(T^a_\mu T^a_\mu), \tag{B.6}
\]

the exponent in the resultant integral coincides with the F. P. term in Eq. (2.8).
Finally, the G. F. term in Eq. (2·8) is obtained simply by smearing the \( \delta \)-function \( \delta[F_o] \) with the Gaussian function exp\[\frac{-i/2}{d^4} x \rho_i\].

**Appendix C**

The quadratic potential in Eq. (3·20) is obtained from \((C^+ t^d t^e C)(C^+ t^e t^d C)\) by the Fierz rearrangement. In order to see this, we first decompose \( C \) into the direct sum of \( Y(=2\sqrt{3} t\delta) \)'s eigenstates:

\[
C = C(+) + C(-), \quad (YC(\pm) = \pm C(\pm)). \quad (C·1)
\]

Second, let us carry out the unitary transformations:

\[
C(+) = U_1 C(+), \quad \left( U_1 = \frac{1}{\sqrt{2}} I \otimes (I + i\sigma_3), \quad I = 2 \times 2 \text{ unit matrix} \right),
\]

\[
C(-) = U_1 U_2 C(-), \quad (U_2 = -i\sigma_2 \otimes I) \quad (C·2)
\]

to which one can verify

\[
U_1^{-1} Y U_1 = I \otimes \sigma_3 \quad \text{and} \quad (U_1 U_2)^{-1} Y (U_1 U_2) = I \otimes \sigma_3. \quad (C·3)
\]

From Eqs. (C·1)~(C·3), we can write \( \bar{C}(\pm) \) as

\[
\bar{C}(+) = (h^+, 0, h^0, 0) ^T \quad \text{and} \quad \bar{C}(-) = (0, u^0, 0, u^-)^T. \quad (T; \text{transpose}) \quad (C·4)
\]

If we notice \( U_2^{-1}(\sigma_1, \sigma_2, \sigma_3) U_2 = (-\sigma_1, \sigma_2, -\sigma_3) \), one can also verify that

\[
t_\alpha C(+) = U_1 \frac{\sigma_2 \otimes I}{2} \bar{C}(+) \quad \text{and} \quad t_\alpha C(-) = U_1 U_2 \frac{\sigma_2 \otimes I}{2} \bar{C}(-). \quad (C·5)
\]

Therefore, \( \bar{C}(+) \) and \( \bar{C}(-) \) suffer the same transformations under the \( SU(2) \) group but they suffer inverse transformations under the \( U(1) \) group.

Now, by taking \( (t^a = t_a, \ t^8 = -3 t_8) \) into account, one can find

\[
(C^+ t^d t^e C)(C^+ t^e t^d C) = (C^+ t_\alpha t_\beta C)(C^+ t_\beta t_\alpha C) - 6(C^+ t_\delta t_\alpha C)(C^+ t_\alpha t_\delta C) + \frac{1}{16} (C^+ C)^2
\]

\[
= \frac{1}{16} (H^+ \sigma_a \sigma_b H + U^+ \sigma_a \sigma_b U) (H^+ \sigma_a \sigma_b H + U^+ \sigma_a \sigma_b U)
\]

\[
- \frac{6}{4(2\sqrt{3})^2} (H^+ \sigma_a H - U^+ \sigma_a U) (H^+ \sigma_a H - U^+ \sigma_a U)
\]

\[
+ \frac{1}{16} (H^+ H + U^+ U)^2, \quad (C·6)
\]

where \( H = (h^+, h^0)^T \) and \( U = (u^0, u^-)^T \). Using, here, the Fierz rearrangement with respect to the Pauli matrices:

\[
(a^+ \sigma_i b)(c^+ \sigma_i d) = 2(a^+ d)(c^+ b) - (a^+ b)(c^+ d), \quad (C·7)
\]

\[
(a^+ \sigma_i \sigma_j b)(c^+ \sigma_j \sigma_i d) = 4(a^+ d)(c^+ b) + (a^+ b)(c^+ d), \quad (C·8)
\]
Electroweak Gauge Theory out of SU(2/l) Gauge Theory

(i, j = 1, 2, 3, a = (a_1, a_2)^T, etc.),

Eq. (C·6) leads to

\[(C^+ \mathbf{t}^i \mathbf{t}_5 C)(C^+ \mathbf{t}^j \mathbf{t}_2 C) = \frac{1}{4} [(H^+ H)^2 + 4(H^+ U)(U^+ H) + (U^+ U)^2]. \tag{C·9} \]

Appendix D

Here, we shall derive the forms of \(f^{(0)}, f^{(1)}\) and \(f^{(2)}\) given in Eq. (3·5). In order to obtain the explicit forms of those functions, let us use the following formula:

\[e^{iSH} = e^{iSPZ} T \exp \left[ i \oint_0^s ds' V(s'; p, x) \right] = e^{iSPZ} \left[ 1 + i \oint_0^s ds' V(s'; p, x) + \frac{i^2}{2} \oint_0^s ds' \oint_0^{s''} ds'' V(s'; p, x) V(s''; p, x) + \cdots \right], \tag{D·1} \]

where “\(T\)” stands for the chronological ordering with respect to \(s\) and

\[V(s; p, x) = e^{-ispz} V(p, x) e^{ispz} = \sum_{n=0}^{\infty} \frac{(-is)^n}{n!} [p^2, [p^2, \cdots [p^2, V(p, x)] \cdots]] = \frac{1}{n!} (2i\not{p} \partial_x + \partial^2)^n V(p, x). \tag{D·2} \]

In consideration of Eqs. (D·1) and (D·2), we can rewrite the expectation value in Eq. (3·2), by using the eigenstates of \(p_\mu = i \partial_\mu\) normalized so that \(\langle p | p' \rangle = \delta^{(4)}(p - p')\), in the following form:

\[\langle x | e^{iSH} | x \rangle = \int d^4 p \langle x | p \rangle \langle p | e^{iSH} | x \rangle = \frac{1}{(2\pi)^4} \int d^4 p e^{ispz} :T \exp \left[ i \oint_0^s ds' V(s'; p, x) \right]:, \tag{D·3} \]

where the colons are the operation which rewrites the products of \(p\) and \(x\) in “normal form” with operators \((p_\mu)\) standing to the left of all operators \((x_\mu)\). Writing, here, the \(n\)th order term of \(V\) in \(T \exp[\cdots]\) as \(F^{(n)}(s; p, x)\), we have

\[\langle x | e^{iSH} | x \rangle = \frac{i}{(4\pi)^2} \sum_{n=0}^{\infty} \langle x | F^{(n)}(s; s^{-1/2} \bar{p}, x) \rangle, \tag{D·4} \]

where

\[\langle \cdots \rangle = \frac{i}{\pi^2} \int_{-\infty}^{\infty} d^4 p e^{ispz} (\cdots), \quad (p = s^{-1/2} \bar{p}). \]

The next task is to calculate each term in Eq. (D·4). By definition, \(F^{(0)} = 1\) obviously and, since we may disregard the surface terms, the first order term \(F^{(1)}\) is obtained easily as
\[ F^{(1)}(s; \bar{p}, x) = -\sum_{n=0}^{\infty} \frac{(-is)^{n+1}}{(n+1)!} (2i\bar{p} \cdot \partial + \partial^2)^n (V^{(0)} + \bar{p} \cdot V^{(1)}) \]
\[ \approx (is) (V^{(0)} + \bar{p} \cdot V^{(1)}) , \quad (D\cdot 5) \]

where \( \approx \) is the equality disregarding surface terms. Hence, we can get
\[ \langle F^{(1)}(s^{-1/2} \bar{p}, x) \rangle = (is) V^{(0)} \quad (D\cdot 6) \]

by taking \( \langle \bar{p} \rangle = +1 \) and odd power of \( \bar{p}_\mu = 0 \) into consideration.

Now, substituting Eq. (D\cdot 2) for the second order term on the right-hand side of Eq. (D\cdot 1) and carrying out the integration with respect to \( s' \) and \( s'' \), the second order term \( F^{(2)} \) becomes
\[ F^{(2)}(s; \bar{p}, x) = \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} \frac{(-is)^{n+m+2}}{n!(m+1)!(n+m+2)} ((2i\bar{p} \cdot \partial + \partial^2)^n (V^{(0)} + \bar{p} \cdot V^{(1)}) \]
\[ \times \{ (2i\bar{p} \cdot \partial + \partial^2)^m (V^{(0)} + \bar{p} \cdot V^{(1)}) \} \]
\[ \approx \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} \frac{(-1)^m(-is)^{n+m+2}}{n!(m+1)!(n+m+2)} \{ (2i\bar{p} \cdot \partial + \partial^2)^n (V^{(0)} + \bar{p} \cdot V^{(1)}) \} V^{(0)} \]
\[ + \{ -(i\partial^\mu + \bar{p}^\mu)(2i\bar{p} \cdot \partial + \partial^2)^{n+m} (V^{(0)} + \bar{p} \cdot V^{(1)}) \} V^{(1)} , \quad (D\cdot 7) \]

Since we are interested in \( s^0, s^1, s^2, \) and \( s^3 \) terms to get the effective action, we may disregard the \{ \cdots \} \( V^{(0)} \) terms with \( n+m \geq 2 \) and the \{ \cdots \} \( V^{(1)} \) terms with \( n+m \geq 3 \) on the right-hand side of (D\cdot 7). Then, after rescaling \( \bar{p} = s^{-1/2} \bar{p} \), effective terms become
\[ F^{(2)}(s; s^{-1/2} \bar{p}, x) = \frac{is^2}{2} V^{(0)} + \frac{is^2}{3} \bar{p} \cdot \partial (2 \bar{p} \cdot V^{(1)}) \]
\[ \frac{-is^2}{2} (2 \bar{p} \cdot \partial \bar{p} \cdot V^{(1)}) V^{(0)} + \frac{is^2}{2} \{ -(i\partial^\mu V^{(0)}) + s^{-1}(\bar{p}^\mu \bar{p} \cdot V^{(1)}) \} V^{(1)} \]
\[ + \frac{is^2}{3} \{ (2 \bar{p} \cdot \partial \bar{p}^\mu V^{(0)}) - 2i(\bar{p} \cdot \partial \partial^\mu \bar{p} \cdot V^{(1)}) - i(\partial^\mu \partial^\nu \bar{p} \cdot V^{(1)}) \} V^{(1)} \]
\[ - \frac{is^2}{2} \{ (2 \bar{p} \cdot \partial \partial^\mu \bar{p} \cdot V^{(0)}) - i(2 \bar{p} \cdot \partial \partial^\nu \bar{p} \cdot V^{(1)}) - i\partial^\mu \partial^\nu \bar{p} \cdot V^{(1)} \} V^{(1)} \]
\[ + \frac{is^2}{3!} \{ (2 \bar{p} \cdot \partial)^2 \bar{p}^\mu \bar{p} \cdot V^{(1)} \} V^{(1)} + \text{(disregarded terms)}. \quad (D\cdot 8) \]

In Eq. (D\cdot 8), the terms with odd power of \( \bar{p} \) have also been discarded. Therefore, using the relations
\[ \langle \bar{p}_\mu \bar{p}_\nu \rangle = (i/2) g_{\mu \nu} \quad \text{and} \quad \langle \bar{p}_\mu \bar{p}_\nu \bar{p}_\rho \bar{p}_\sigma \rangle = (i/2)^2 (g_{\mu \nu} g_{\rho \sigma} + g_{\mu \rho} g_{\nu \sigma} + g_{\mu \sigma} g_{\nu \rho}) , \quad (D\cdot 9) \]
we obtain finally the following:
\[ \langle F^{(2)}(s; s^{-1/2} \bar{p}, x) \rangle = -\frac{is}{4} V^{(1)} \cdot V^{(1)} \]
\[ + \frac{(is)^2}{2} V^{(0)} + \frac{i}{2} V^{(0)} (\partial \cdot V^{(1)}) - \frac{1}{12} (\partial \cdot V^{(1)})^2 + \frac{1}{24} V^{(1)} \partial^2 V^{(1)} \mu . \quad (D\cdot 10) \]
It is obvious that Eqs. (D-6) and (D-10) are nothing but Eq. (3-5).

References


   There are a number of models unifying weak, electromagnetic and strong interactions based on
   simple gauge groups. See the following review and papers they quote.

3) Y. Ne'eman, Phys. Lett. 81B (1979), 190.
   Y. Ne'eman and J. Thierry-Mieg, in Differential Geometrical Methods in Mathematical Physics, ed.
   P. H. Dondi and P. D. Jarvis, Phys. Lett. 84B (1979), 75.
   J. G. Taylor, Phys. Lett. 84B (1979), 79.

4) The $\theta_w = 30'$ can also be obtained by assuming $SU(3)$ instead of $SU(2/1)$ as the gauge group for

   As for the treatment of Higgs scalars relevant to those approaches, see also,


10) L. Parker, Recent Development in Gravitation, ed. S. Deser and M. Lévy (New York, Plenum, 1979),
    p. 219.


    Y. Ne'eman and S. Sternberg, in High Energy Physics-1980, XXth International Conference,